When do organisms gravitate to the vertices of a regular simplex?

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Animals forming patterns: flocking, milling, swarming

and schooling

bottom view

Led scientists to make models...

e.g. (very incomplete)
Parr (1927)
Breder (1954) attractive-repulsive power law interaction for fish separation
Keller-Segal (1971) purely attractive, 1st order (2d cell chemotaxis)
.
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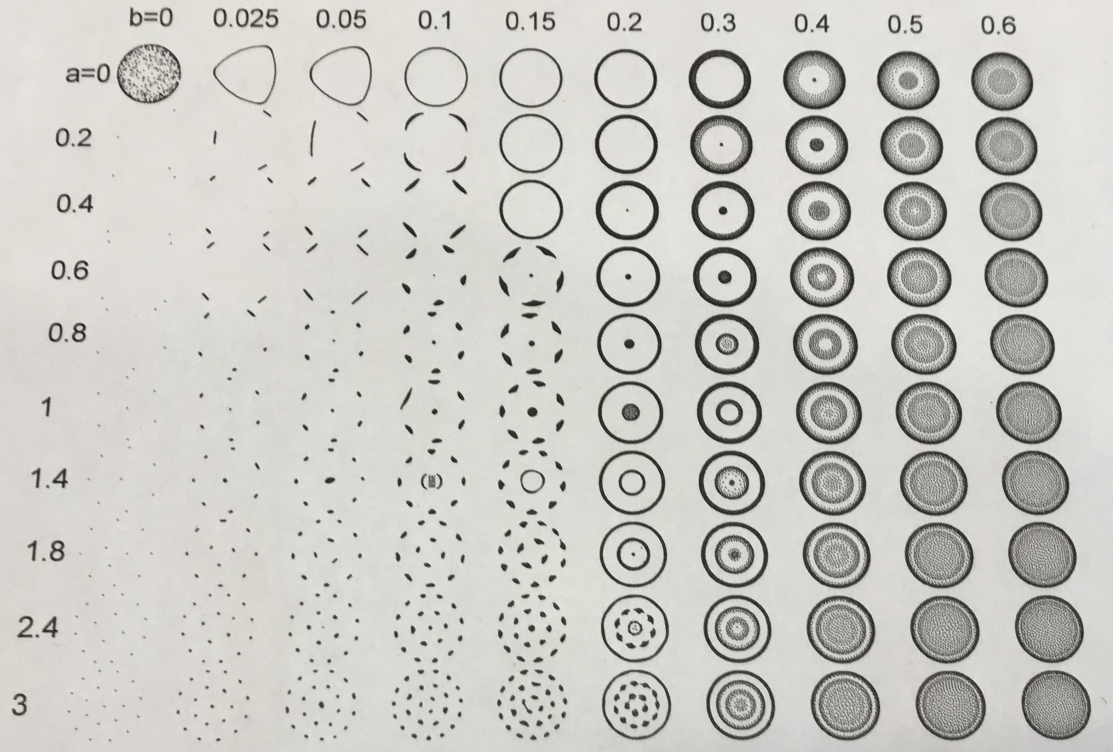
Mogilner and Edelstein-Keshet (1999) 1d attractive-repulsive + diffusion
Levine, Rappel and Cohen (2000) 2nd order, preferred speed
Topaz, Bertozzi and Lewis (2006)
Cucker and Smale (2007) 2nd order, matched speeds
.
.
... analyze and simulate
e.g. (just as incomplete) Albi, Balague, Bertozzi, Burchard, Carrillo, Choksi, Craig, Fetecau, Figalli, Frank, Huang, Kolokolnikov, Laurent, Lieb, Lopes, Pavlovski, Pattachini, Raoul, Sun, Topaloglu, Uminsky, von Brecht, ...

Piecewise linear force laws

Kolokolnikov, Huang, Pavlovski (2013) Phys. D.



Piecewise quadratic potentials

Figure 1: Steady states of (1) with $F(r) = \min(ar + b, 1 - r)$, using $N = 1000$ particles and with a, b as indicated. A snapshot at $t = 10,000$ is shown. Integration was performed using forward Euler method with stepsize 0.5.

Power law potentials

Albi, Balagué, Carrillo, von Brecht (2014) SIAM

STABILITY OF FLOCK AND MILL RINGS 815

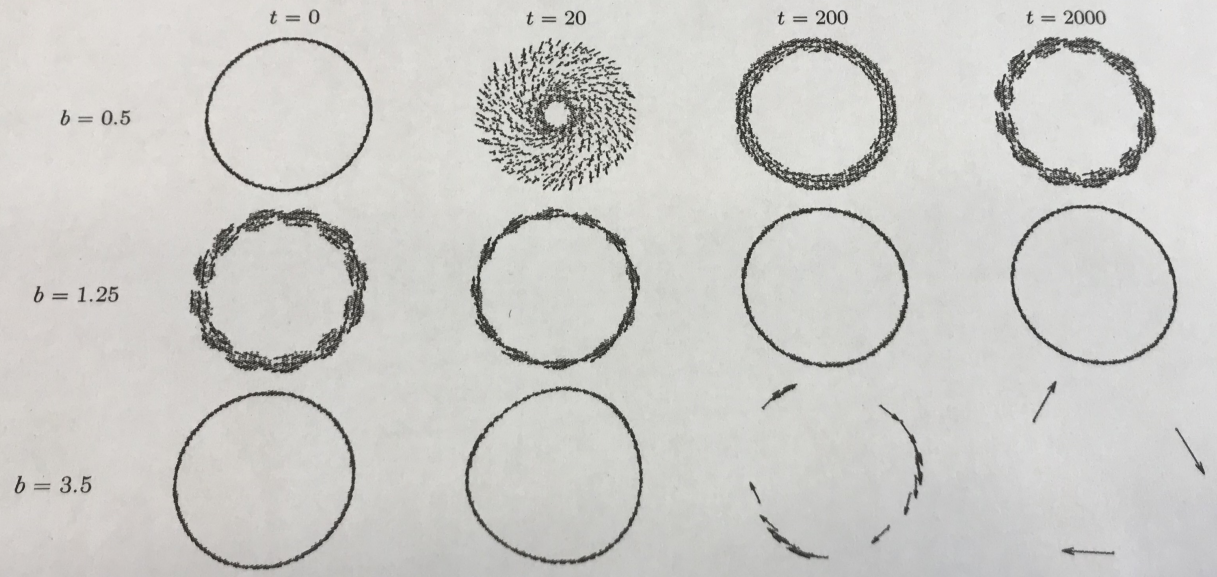


FIG. 9. $N = 1000$ *particles*, $a = 5, |u_0| = 0.5$. *The Figure shows the evolution of a mill ring for increasing values of b, i.e., decreasing repulsion. The evolution of the second and third rows is computed starting from the stable pattern of the previous line.*

Figure 9 we show the evolution of a mill ring solution with b taken equal to $0.5, 1.25$, and 3.5, respectively. The parameter choices are marked as $(*)$ in Figure 7. The

Thank you