The detectability lemma and quantum gap amplification

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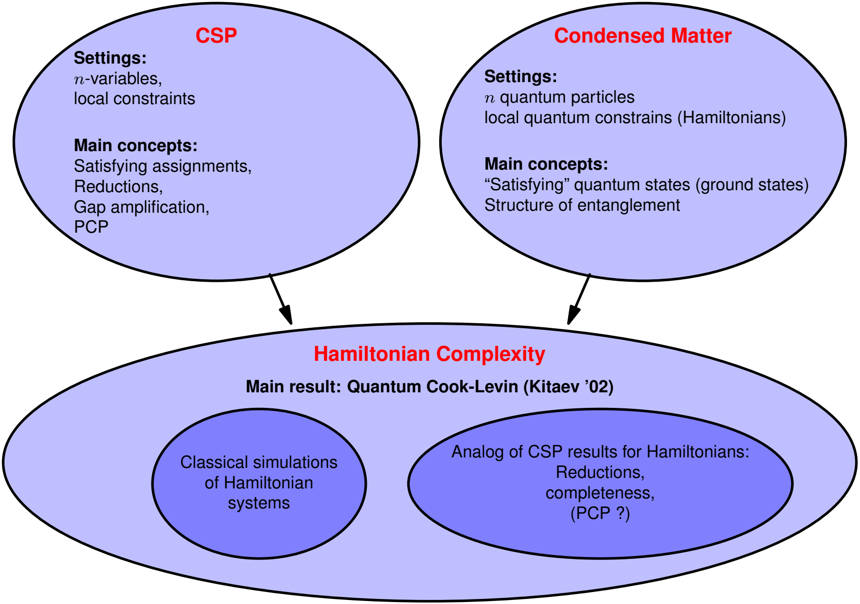
Quantum Hamiltonian complexity



CSP in quantum language

MAX-3-SAT

Settings:

- n bits: $x_1, \ldots x_n$
- M constraints: $f(x_1, \ldots, x_n) = C_1 \wedge \cdots \wedge C_M$
- C_i – 3-local CNF clause

Goal: find the minimal possible # of violations

MAX-3-SAT – in quantum language

Settings:

- Hilbert space of n qubits – 2^n basis states: $|00\cdots00\rangle, |00\cdots01\rangle, \ldots, |11\cdots11\rangle$
- Hamiltonian with M terms: $H = Q_1 + \ldots + Q_M$
- Q_i – 3-local "classical projections"

Goal: approximate the lowest eigenvalue of H: $\lambda(H) = 0$ or $\lambda(H) \geq 1$.

Example:

$C_i = (x_k \vee \bar{x}_\ell \vee x_m)$

Violating when $(x_k, x_\ell, x_m) = (0, 1, 0)$

$$\Longrightarrow \quad Q_i = \begin{array}{cc} & |010\rangle \\ |010\rangle & \begin{pmatrix} 0 & & & & & & & \\ & 0 & & & & 0 & & \\ & & 1 & & & & & \\ & & & 0 & & & & \\ & & & & 0 & & & \\ & 0 & & & & 0 & & \\ & & & & & & 0 & \\ & & & & & & & 0 \end{pmatrix} \end{array} \otimes \mathbb{1}$$

CSP is satisfiable $\Leftrightarrow$ $\lambda(H) = 0$

Moving to eigenvectors and eigenvalues

Local Hamiltonian: $H = Q_1 + Q_2 + \ldots$ $\quad 2^n \times 2^n$ matrix

$C_i = (x_k \vee \bar{x}_\ell \vee x_m)$
Violating when $(x_k, x_\ell, x_m) = (0, 1, 0)$ $\quad \Longrightarrow \quad$

$$Q_i = \begin{array}{cc} & \begin{array}{cccccccc} & & |010\rangle & & & & & \end{array} \\ \begin{array}{c} \\ \\ |010\rangle \\ \\ \\ \\ \\ \\ \end{array} & \begin{pmatrix} 0 & & & & & & & \\ & 0 & & & 0 & & & \\ & & 1 & & & & & \\ & & & 0 & & & & \\ & & & & 0 & & & \\ & 0 & & & & 0 & & \\ & & & & & & 0 & \\ & & & & & & & 0 \end{pmatrix} \end{array} \otimes \mathbb{1}$$

Eigenvectors: $H |x_1, \ldots, x_n\rangle =$ (# violations) $|x_1, \ldots, x_n\rangle$

Eigenvalue of $|x_1, \ldots, x_n\rangle$: # of violations.

$\lambda(H)$ = Lowest eigenvalue = minimal # of violations.

The problem: Decide whether $\lambda(H) = 0$ or $\lambda(H) \geq 1$

Moving to the general quantum CSP

Quantum CSP (The Local Hamiltonian problem)

$H = \sum_{i=1}^{M} Q_i$ general local projections $\lambda(H) =$ lowest eigenvalue of H

Decide: $\lambda(H) = 0$ or $\lambda(H) \geq \frac{1}{poly(n)}$

Note:

Q_i are no longer diagonal in the standard basis,
and the eigenvectors of H are superpositions:

$$|\psi\rangle = \overbrace{a_1 |0 \cdots 00\rangle + a_2 |0 \cdots 01\rangle + \ldots}^{2^n \text{ terms}}$$

A different view on LH:

$$\lambda(H) = \min_\psi \langle\psi| H |\psi\rangle = \min_\psi \langle\psi| \sum_i Q_i |\psi\rangle = \min_\psi \sum_i \langle\psi| Q_i |\psi\rangle$$

$0 \leq \langle\psi| Q_i |\psi\rangle \leq 1$ – The **energy** of the constraint: how much it is violated.

(compare to 0 or 1 in the classical case)

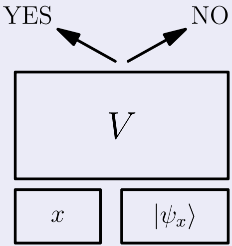
Hamiltonian Complexity - The quantum analog of CSP

Quantum NP (QMA)

Decision problems that can be solved with a quantum witness $|\psi\rangle$ and a polynomial quantum verifier V_x.

$$x \in L : \quad \exists\, |\psi_x\rangle \text{ s.t. } \Pr[V(x, |\psi_x\rangle) = \text{yes}] \geq 2/3 \,,$$

$$x \notin L : \quad \forall\, |\phi\rangle \,, \Pr[V(x, |\phi\rangle) = \text{yes}] \leq 1/3 \,.$$



Quantum Cook-Levin

The Local-Hamiltonian problem is QMA-complete (Kitaev, 98).

- Inclusion is the easy direction
- Hardness is similar to Cook-Levin – but with some quantum twists.

Central results in Hamiltonian Complexity

Many results and techniques can be imported from CSP to Hamiltonian complexity:

- Reductions using Gadgets — Aharonov, van Dam, Kempe, Landau, Lloyd, Regev '04
- 2-Local (even planar) Hamiltonian is QMA complete — Kempe, Kitaev, Regev '04; Oliveira, Terhal '05
- Satisfaction Threshold — Najag '07; Shondhi '09; Bravyi '09
- Lovasz local lemma — Ambainis, Kempe, Sattath '09

But sometimes things are different:

- Local Hamiltonian in 1d is QMA-complete (but 1d CSP is in P). — Aharonov, Gottesman, Irani, Kempe '07

What about Quantum PCP (QPCP) ?

Quantum PCP

Two ways to formulate the QPCP conjecture:

QPCP conjecture I

$\forall L \in \mathrm{QMA}$ there is a quantum verifier that has access to only q random qubits from the witness.

Quantum reduction $\Longleftrightarrow$

QPCP conjecture II

Deciding whether

- $\lambda(H) = 0$ or
- $\lambda(H) \geq c \cdot M$

is QMA-hard.

This is, however, a difficult problem

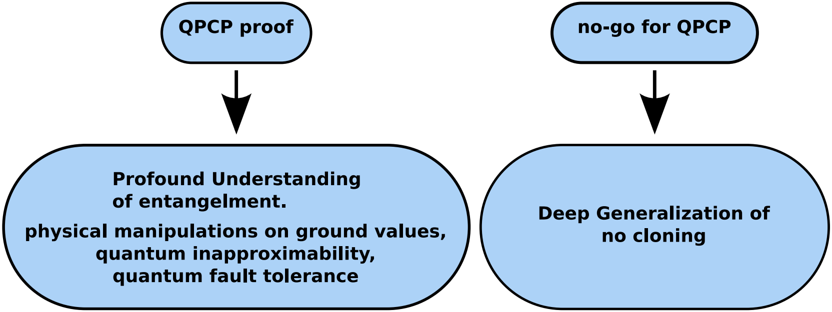
Difficulties in proving QPCP

Main Problem: no cloning

There is no quantum transformation $|\psi\rangle|0\rangle \mapsto |\psi\rangle|\psi\rangle$ for all $|\psi\rangle$

- No cloning was thought to prevent QECCs – but QECCs exist!
- QPCP seems to require even more (QECCs are not locally decodable)

Possible implications of resolving this question:



We focus on a central ingredient in Dinur's PCP proof, *which does not require cloning*:

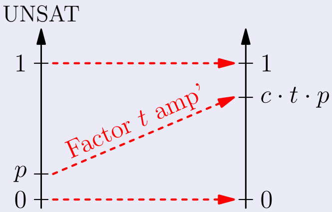
Gap Amplification

What we want to amplify

In the Classical case

$$\mathrm{UNSAT} \stackrel{\mathrm{def}}{=} \frac{\text{min \# violations}}{M}$$

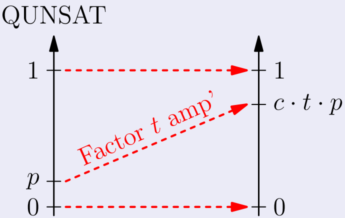
Find an efficient transformation to a new CSP s.t.,



In the Quantum case

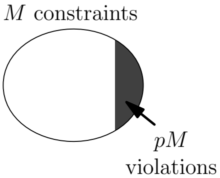
$$\mathrm{QUNSAT} \stackrel{\mathrm{def}}{=} \frac{\lambda(H)}{M}$$

Find an efficient transformation to a new L.H. s.t.,



The probability of detecting a violation in a random constraint

Classically:



p – Probability of picking a violated constraint

Quantumly:

Quantum Measurement

$$|\psi\rangle \xrightarrow{1} \Pi|\psi\rangle \qquad \text{Prob} = \|\Pi\psi\|^2$$

$$|\psi\rangle \xrightarrow{0} (\mathbb{1} - \Pi)|\psi\rangle \qquad \text{Prob} = \|(\mathbb{1} - \Pi)\psi\|^2$$

In our case, when measuring Q_i:

$$\Pr[1] = \|Q_i \,|\psi\rangle\,\|^2 = \langle\psi|\, Q_i \,|\psi\rangle$$

= constraint's energy

The probability of measuring a violation for a **random** i.

$$p = \frac{\text{\# of violations}}{M} \geq \text{UNSAT}$$

$$p = \frac{1}{M}\sum_i \langle\psi|\, Q_i \,|\psi\rangle = \frac{\lambda(H)}{M} \geq \text{QUNSAT}$$

We want to amplify the probability of measuring a violation

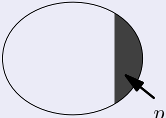
Amplification by repitition

Classically

- Pick t random constraints
- Probability of at least 1 violation:

$$\Pr = 1 - (1-p)^t \simeq t \cdot p$$

- New system size: M^t



Quantumly, this also works:

- Choose t random constraints
- Measure them one after the other
- After every measurement, the system **collapses** into a new state: $|\psi_1\rangle \to |\psi_2\rangle \to \cdots \to |\psi_t\rangle$, but always:

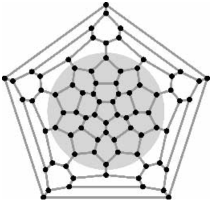
$$\frac{\sum_i \langle\psi_j| Q_i |\psi_j\rangle}{M} \geq \frac{\lambda(H)}{M}$$

- Probability of measuring at least one violation:

$$\Pr \geq 1 - (1 - \lambda(H)/M)^2 \simeq t \cdot \lambda(H)/M$$

Amplification with expanders

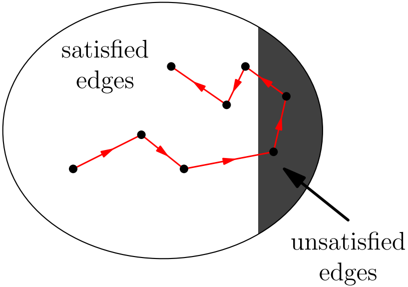
Ajtai et al, Impagliazzo & Zuckerman – RP & BPP amp'

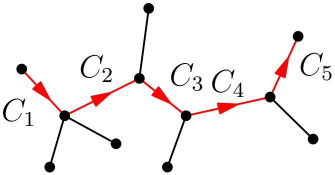


Expander graphs

Taking a t-walk on a d-regular expander graph is almost like picking t random edges.

Advantage: The new system is much smaller: Md^t (instead of M^t)



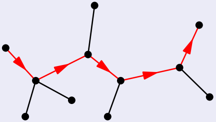


$$C_{\mathbf{e}} = C_1 \wedge C_2 \wedge C_3 \wedge C_4 \wedge C_5$$

Quantum Conspiration?

Conspiration Theory

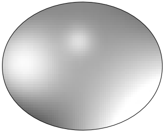
- Classical correlations decay exponentially fast on an expander, but is this also true for quantum correlations?
- How do we know that the expander is “random enough”? – Can the system trick us by collapsing adversely ?



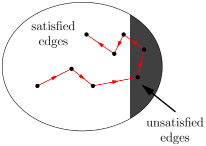
Quantum World

Classical World

- Classically: well-defined partition.
- Quantumly: no assignment $\Rightarrow$ no partition.



Energy distribution



- Is there a distribution of assignments? For two constraints A, B, $\Pr(A = 1, B = 0)$ is not well defined:

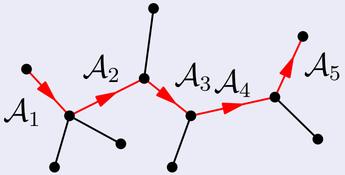
$$\Pr\left[(A{=}1) \to (B{=}0)\right] = \|(I - B)A\,|\psi\rangle\,\|^2 \neq \|A(I - B)\,|\psi\rangle\,\|^2 = \Pr\left[(B{=}0) \to (A{=}1)\right]$$

A and B do not commute $\Rightarrow$ no distribution over assignments

Our Results

Quantum Gap Amplification

Consider a quantum CSP problem on a d-regular expander graph $G = (V, E)$ with a second largest eigvenvalue $0 < \lambda < 1$. The edges are projections and the vertices are qudits of dimension W.



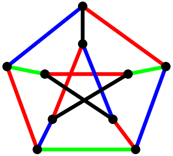
$$\mathcal{A}_{\vec{e}} = \mathcal{A}_1 \cap \mathcal{A}_2 \cap \mathcal{A}_3 \cap \mathcal{A}_4 \cap \mathcal{A}_5$$

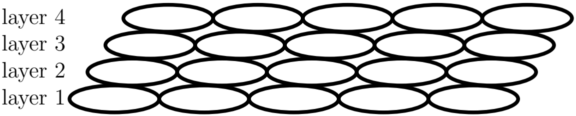
Let G^t be the hyper graph of t-walks derived from G, and for each such t-walk define a projection into the **intersection** of all accepting subspaces along the walk.

Then:

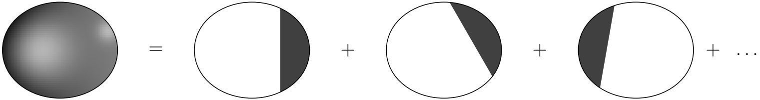
$$\mathrm{QUNSAT}(G^t) \geq c(\lambda) K(q, d) \min \left(t \cdot \mathrm{QUNSAT}(G), 1 \right)$$

Layers





- Typically, constraints can be arranged in a finite number g of layers.
- In a **fixed** layer, all projections commute and can be measured simultaneously – there exists an underlying joint probability distribution.
- With respect to one layer, we have a distribution of classical systems:



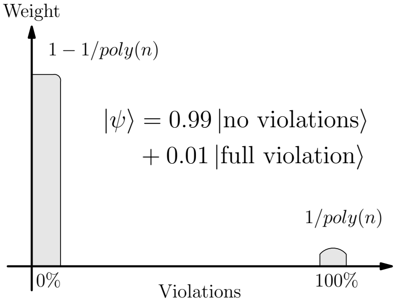
We can use the classical PCP result on each member.

- There are only g layers $\Rightarrow$ there must be a layer with a constant energy.

But there's a problem . . .

The problem (yet another conspiration)

- It is trivial to satisfy all constraints in **one fixed layer**.
- But not simultaneously in all layers (because $\lambda(H) > 0$).
- What if the violation/satisfation distribution in every layer conspires to be:



No amplification:

- In the "no violations" part there is no amplification because all constraints are satisfied.
- In the "full violations" part there is no amplification because we have reached the maximum.

We must somehow rule out this possibility if we want to show amplification

The detectability lemma

The detectability lemma ($\ell = 0$ case)

Settings:

$\Rightarrow$ $\Pi^{(j)}$ is the projection into the satisfying subspace of the j'th layer

$\Rightarrow$ Minimal energy is $\epsilon_0 \stackrel{\text{def}}{=} \lambda(H) > 0$

$\Rightarrow$ Projections are taken from a **fixed** set.

Then:

$$\|\Pi^{(1)} \cdots \Pi^{(g)} |\psi\rangle \|^2 \leq \frac{1}{\epsilon_0/c + 1} \simeq 1 - \epsilon_0/c$$

where c is constant **independent of** n.

In a sequential measurent of all layers, the probability of not detecting any violations is bounded away from 1 by a constant.

Corollary

There must be at least one layer j with a constant projection on the violating subspace

$$\|(I - \Pi^{(j)}) |\psi\rangle \| \geq c'$$

General outline of the proof

- Define

$$|\Omega\rangle \overset{\text{def}}{=} \Pi^{(1)} \cdots \Pi^{(g)} |\psi\rangle$$

  We wish to show $\|\Omega\|^2 \leq \frac{1}{\epsilon_0/c+1}$.

- Note:

$$\|\Omega\|^2 \epsilon_0 \leq \langle\Omega| H |\Omega\rangle$$

- We prove:

$$\boxed{\langle\Omega| H |\Omega\rangle \leq c\big(1 - \|\Omega\|^2\big)}$$

$$\Downarrow$$

$$\|\Omega\|^2 \epsilon_0 \leq c\big(1 - \|\Omega\|^2\big)$$

$$\Downarrow$$

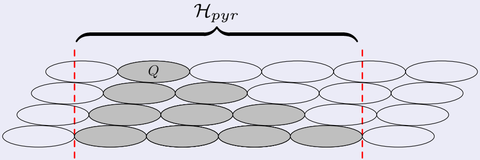
$$\|\Omega\|^2 \leq \frac{1}{\epsilon_0/c + 1}$$

- In the commuting case, $\langle\Omega| H |\Omega\rangle = 0$ (trivial)
- In the general case, we show that energy contribution due to non-commuteness is exponentially decreasing.
- We must find a way to quantify non-commuteness.

The XY decomposition I – Pyramids

Pyramids

- For every projection Q, we define a set of projections $\mathrm{Pyr}[Q]$
- H_{pyr} - The supporting Hilbert space.



The XY decomposition

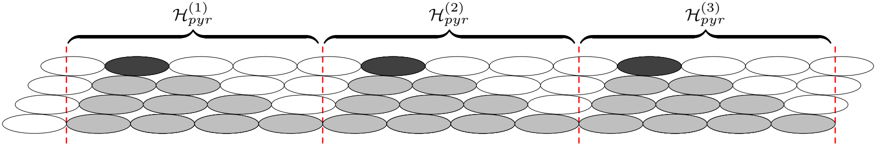
$$\begin{aligned} H_{pyr} &= X \oplus Y \\ X &= \text{common eigenvectors} \\ Y &= \text{The rest} \\ P_X, P_Y &= \text{Projections to } X, Y \end{aligned}$$

Observations:

- P_X, P_Y commute with all $Q' \in \mathrm{Pyr}[Q]$
- $\|P_Y \left(\prod_{Q' \in \mathrm{Pyr}[Q]} Q'\right) P_Y\| \leq \theta < 1$
- If the projections are drawn from a fixed set, $\theta < 1$ is constant!

The XY decomposition II – Ponzis

A Ponzi is a set of constraints from a fixed layer, whose pyramids don't intersect



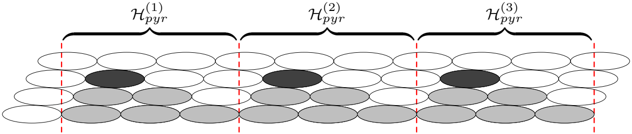
- A constant number of Ponzi's is needed to cover all constraints
- E_{ponz} – The energy operator of a Ponzi.
- The entire Hilbert space is decomposed in terms of XY sectors ν:

$$\text{An XY sector: } \nu = (X, X, Y, X, Y, \ldots)$$
$$\text{Projection onto } \nu\text{: } P_\nu = P_X \otimes P_X \otimes P_Y \otimes P_X \cdots$$
$$|\nu| = \text{No. of Y spaces}$$
$$|\Omega\rangle = \sum_\nu P_\nu |\Omega\rangle \stackrel{\text{def}}{=} \sum_\nu |\Omega_\nu\rangle$$

- E_{ponz} commutes with the decomp':

$$\langle\Omega| E_{ponz} |\Omega\rangle = \sum_\nu \langle\Omega_\nu| E_{ponz} |\Omega_\nu\rangle$$

The energy of Ponzi



$\nu = (X, X, Y, X, Y, \ldots)$

$|\nu| =$ No. of Y spaces

$|\Omega\rangle \stackrel{\text{def}}{=} \sum_\nu |\Omega_\nu\rangle$

$$\Pi^{(1)} \cdots \Pi^{(g)} = D \cdot R \qquad |\Omega\rangle = D \cdot R\,|\psi\rangle \stackrel{\text{def}}{=} D\,|\phi\rangle$$

$$|\Omega_\nu\rangle = P_\nu D\,|\phi\rangle = P_\nu D P_\nu\,|\phi\rangle = (P_\nu D P_\nu)\,|\phi_\nu\rangle$$

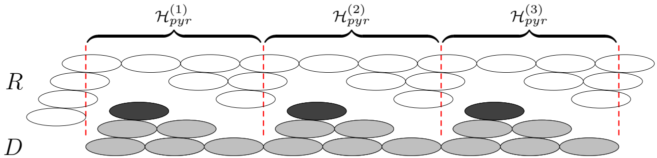
Exponential decay: $\|P_\nu D P_\nu\| \le \theta^{|\nu|}$

- Only the Y pyramids contribute to $\langle\Omega_\nu|\, E_{ponz}\, |\Omega_\nu\rangle$, but their norm decays exponentially!

$$\langle\Omega_\nu|\, E_{ponz}\, |\Omega_\nu\rangle \le |\nu|\theta^{|\nu|}\|\phi_\nu\|^2$$

- We obtain an upper bound: $\langle\Omega|\, E_{ponz}\, |\Omega\rangle \le \sum_{s=0}^{\infty} s\theta^s$.

The energy of Ponzi



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Open questions

- The quantum amplification lemma is trivial to prove when all the projections commute (one does not need the detectability lemma). Still it is not clear:
  - if the quantum PCP can be proved for commuting Hamiltonians
  - what is the complexity of this special class? is it NP? QMA ?
- Can the XY decomposition or the detectability lemma be used elsewhere to handle the non-commuteness of the LH problem ?
- Are there any interesting implications of the detectability lemma and the XY decomposition to solid-state physics?
- Currently, the detectability lemma allows us an RP type of amplification (one-sided errors). Can it be generalized to prove a BPP type of amplification (two-sided errors) ?
- What is the right definition of quantum PCP? (one-sided errors/two sided errors?)
- Can we prove a quantum PCP with exponential witness ?
- If there is no quantum PCP theorem, then what is the complexity of approximating $\mathrm{QUNSAT}(H)$ up to a constant? It must be NP-hard - but is it inside NP?

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