The mathematical work of Vladimir Voevodsky

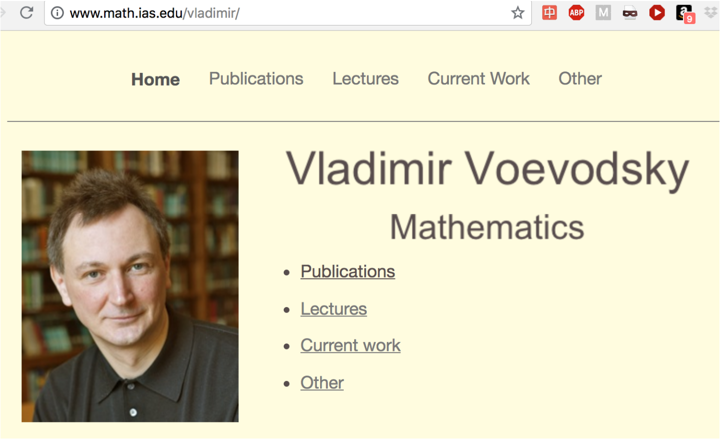
Daniel R. Grayson

University of Illinois at Urbana-Champaign

Institute for Advanced Study

Institute for Advanced Study - Princeton - September 11, 2018

His home page



The memorial web site

www.math.ias.edu/Voevodsky/

Vladimir Voevodsky

Владимир Александрович Воеводский

4 June 1966 – 30 September 2017

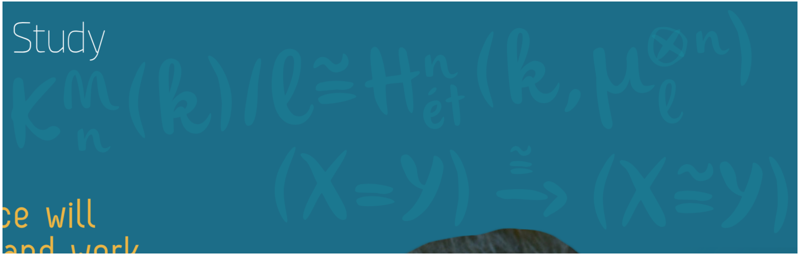
Vladimir Voevodsky was an algebraist with a deep understanding of topology who found novel ways to apply topology to algebraic geometry and to the foundations and formalization of mathematics. His work on foundations was interrupted by his sudden death in September, 2017. This web site commemorates his work and his life and serves as an archive of his works, both complete and incomplete, for those wishing to examine and to extend his work.

The course of his life

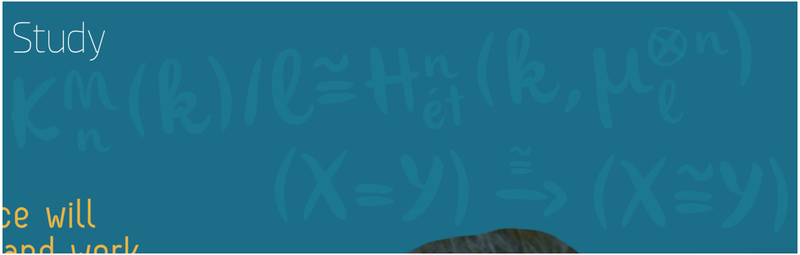
- born 1966, June 4
- BS Moscow State University, 1989, or not
- PhD Harvard, 1992, with David Kazhdan
- Harvard junior fellow, 1993-1996
- Northwestern, 1997-1998
- Harvard University and Max-Planck Institute, Visiting Scholar, 1996-1997
- Institute for Advanced Study, 1998-2017, member and then professor
- Fields Medal, 2002, for motivic cohomology and the Milnor conjecture
- Annals paper with final proof of Bloch-Kato conjecture, 2010
- Univalent Foundations publicized, 2010
- special year on Univalent Foundations, IAS, 2012-2013
- death, 2017, September 30, age 51

The poster for this conference

The formulas in the background



The formulas in the background



Part 1, Bloch-Kato Conjecture:

$$K_n^M(k)/\ell \cong H^n_{\acute{e}t}(k, \mu_\ell^{\otimes n})$$

Part 2, Univalence Axiom:

$$(X = Y) \xrightarrow{\cong} (X \cong Y)$$

Part 1, Bloch-Kato Conjecture:

$$K_n^M(k)/\ell \cong H^n_{\text{ét}}(k, \mu_\ell^{\otimes n})$$

His early preprints on motivic cohomology

https://faculty.math.illinois.edu/K-theory/

K-theory Preprint Archives

- **864**: September 10, 2007, Motivic Eilenberg-MacLane spaces, by Vladimir Voevodsky.
- **863**: September 10, 2007, Simplicial radditive functors, by Vladimir Voevodsky.
- **639**: June 16, 2003, On motivic cohomology with Z/l-coefficients, by Vladimir Voevodsky.
- **638**: June 16, 2003, Motives over simplicial schemes, by Vladimir Voevodsky.
- **612**: December 19, 2002, On the zero slice of the sphere spectrum, by Vladimir Voevodsky.
- **541**: January 28, 2002, Cancellation theorem, by Vladimir Voevodsky.
- **527**: November 19, 2001, Lectures on motivic cohomology 2000/2001 (written by Pierre Deligne), by Vladimir Voevodsky.
- **502**: July 15, 2001, On 2-torsion in motivic cohomology, by Vladimir Voevodsky.
- **487**: May 19, 2001, Reduced power operations in motivic cohomology, by Vladimir Voevodsky.
- **486**: May 22, 2005, Lectures on Motivic Cohomology, by Vladimir Voevodsky, Carlo Mazza, and Charles Weibel.
- **469**: March 8, 2001, A possible new approach to the motivic spectral sequence for algebraic K-theory, by Vladimir Voevodsky.
- **454**: December 6, 2000, An exact sequence for Milnor's K-theory with applications to quadratic forms, by Dmitry Orlov, Alexander Vishik, and Vladimir Voevodsky.
- **444**: September 6, 2000, Unstable motivic homotopy categories in Nisnevich and cdh-topologies, by Vladimir Voevodsky.
- **443**: September 6, 2000, Homotopy theory of simplicial sheaves in completely decomposable topologies, by Vladimir Voevodsky.
- **392**: March 11, 2000, Open problems in the motivic stable homotopy theory, I, by Vladimir Voevodsky.
- **378**: December 17, 1999, Motivic cohomology are isomorphic to higher Chow groups, by Vladimir Voevodsky.
- **368**: October 8, 1999, Cycles, Transfers and Motivic Homology Theories., by Vladimir Voevodsky, Eric. M. Friedlander, and Andrei Suslin.
- **341**: April 14, 1999, Bloch-Kato conjecture and motivic cohomology with finite coefficients, by Andrei Suslin and Vladimir Voevodsky.
- **305**: October 6, 1998, A^1-homotopy theory of schemes, by Fabien Morel and Vladimir Voevodsky.
- **170**: December 20, 1996, The Milnor Conjecture, by Vladimir Voevodsky.
- **83**: September 26, 1995, Bloch-Kato conjecture and motivic cohomology with finite coefficients, by A. Suslin and V. Voevodsky.
- **76**: June 17, 1995, Bloch-Kato conjecture for Z/2-coefficients and algebraic Morava K-theories, by Vladimir Voevodsky.
- **75**: June 17, 1995, Bivariant cycle cohomology, by E. M. Friedlander and Vladimir Voevodsky.
- **74**: June 17, 1995, Triangulated categories of motives over a field, by Vladimir Voevodsky.
- **41**: November 17, 1994, Nilpotence theorem for cycles algebraically equivalent to zero, by Vladimir Voevodsky.
- **35**: November 4, 1994, Relative cycles and Chow sheaves, by Andrei Suslin and Vladimir Voevodsky.
- **34**: November 4, 1994, Homology of schemes, II, by Vladimir Voevodsky.
- **33**: November 4, 1994, A letter to Beilinson, December 6, 1992, by Vladimir Voevodsky.
- **32**: November 4, 1994, Singular homology of abstract algebraic varieties, by Andrei Suslin and Vladimir Voevodsky.
- **31**: November 4, 1994, Homology of schemes, I, by Vladimir Voevodsky.

The Milnor Conjecture.

V. Voevodsky[1]

December 1996

1 Introduction.

The goal of this paper is to prove the following conjecture:

Conjecture 1.1 (Milnor) *Let k be a field of characteristic not equal to 2. Then the norm residue homomorphisms $K_n^M(k)/2 \to H_{et}^n(k, \mathbf{Z}/2)$ are isomorphisms for all $n \geq 0$.*

Annals of Mathematics **174** (2011), 401–438
doi: 10.4007/annals.2011.174.1.11

On motivic cohomology with $\mathbf{Z}/l$-coefficients

By Vladimir Voevodsky

Abstract

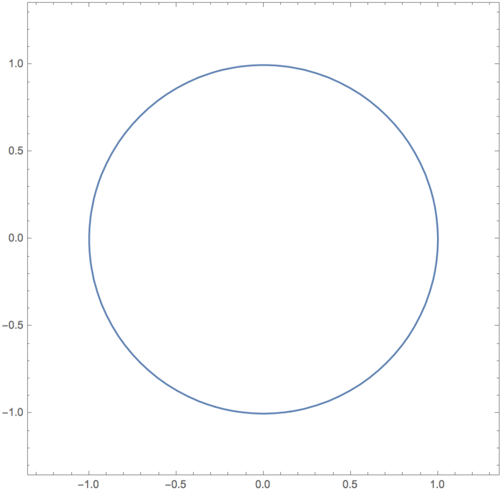
In this paper we prove the conjecture of Bloch and Kato which relates Milnor's K-theory of a field with its Galois cohomology as well as the related comparisons results for motivic cohomology with finite coefficients in the Nisnevich and étale topologies.

Introduction to algebraic geometry

$$x^2 + y^2 = 1$$

In[42]:= `ContourPlot[x^2 + y^2 == 1, {x, -1.3, 1.3}, {y, -1.3, 1.3}]`

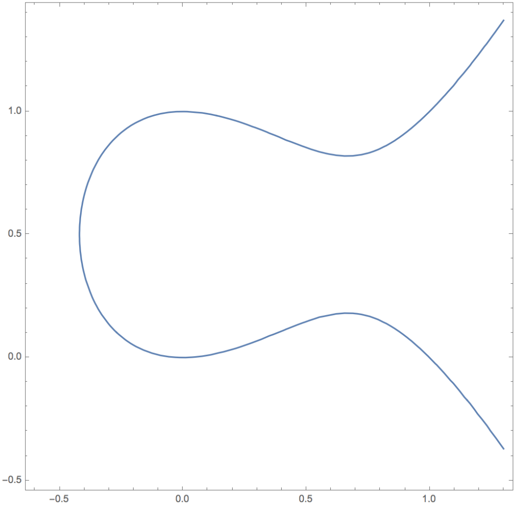
Out[42]=



$$y^2 - y = x^3 - x^2$$

```
In[11]:= ContourPlot[y^2 - y == x^3 - x^2, {x, -0.6, 1.3}, {y, -0.5, 1.4}]
```

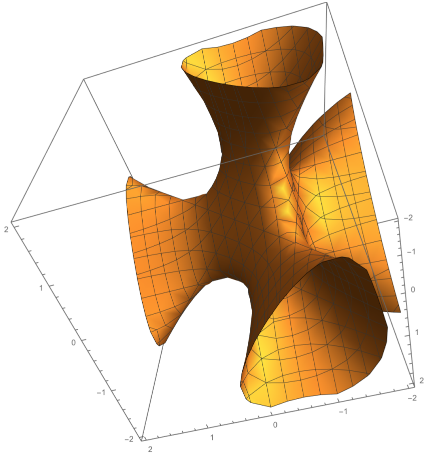
Out[11]=



$$x^3 + y^3 + z^3 + 1 = (x + y + z + 1)^3$$

In[27]:= `ContourPlot3D[x^3 + y^3 + z^3 + 1 == (x + y + z + 1)^3, {x, -2, 2}, {y, -2, 2}, {z, -2, 2}]`

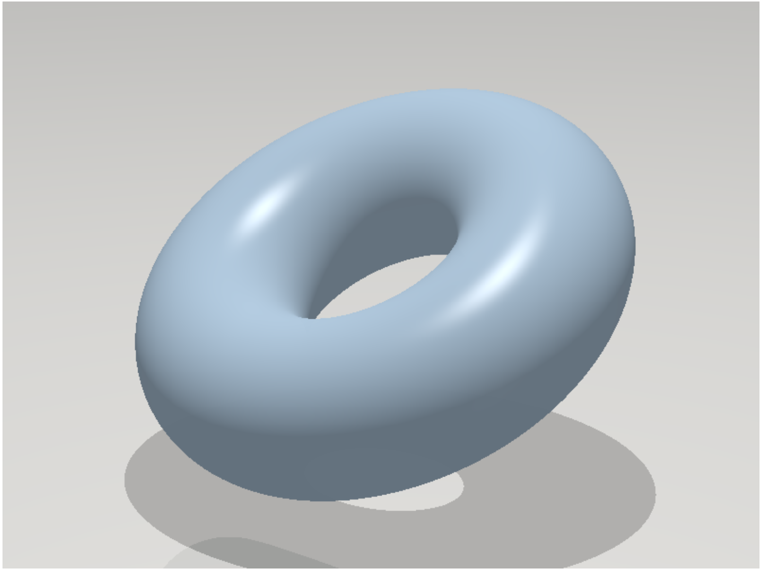
Out[27]=

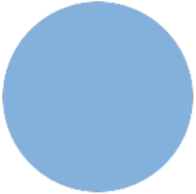


Introduction to topology

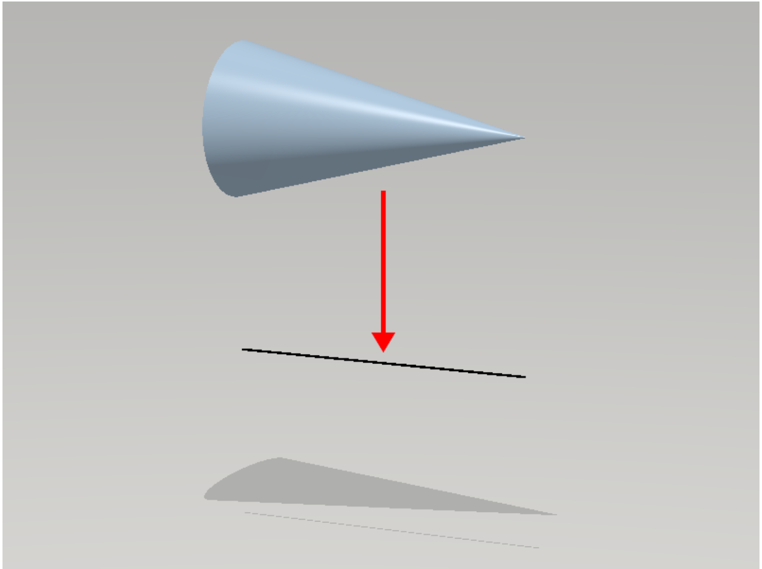
The animations on this slide and on subsequent slides can be viewed with Acrobat Reader.

Introduction to homotopy theory

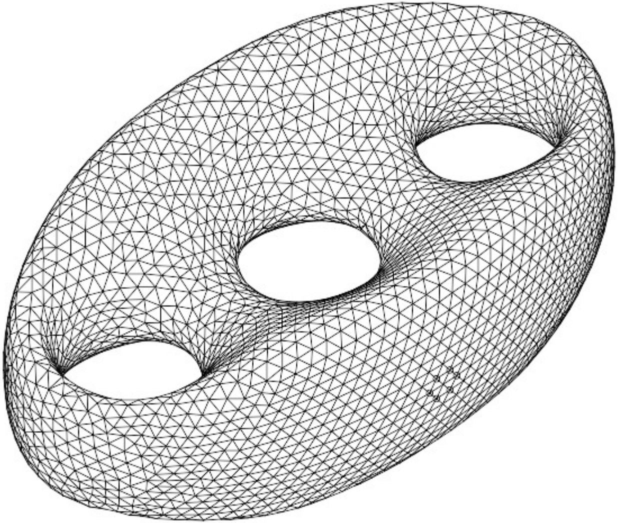




A fibration



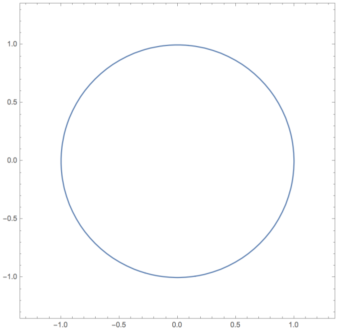
Topology in combinatorial style



Mixing topology with algebraic geometry

```
In[42]:= ContourPlot[x^2 + y^2 == 1, {x, -1.3, 1.3}, {y, -1.3, 1.3}]
```

Out[42]=

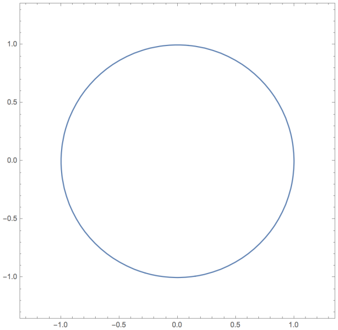


*

Mixing topology with algebraic geometry

In[42]:=
```
ContourPlot[x^2 + y^2 == 1, {x, -1.3, 1.3}, {y, -1.3, 1.3}]
```

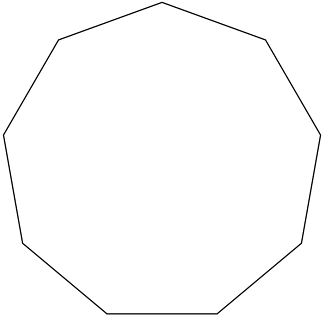
Out[42]=



*

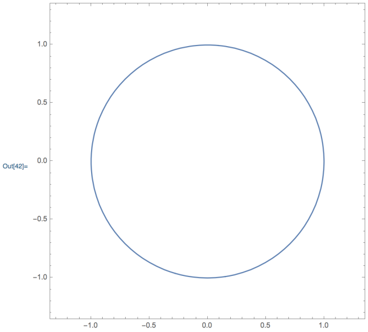
In[39]:=
```
Graphics[{FaceForm[Transparent], EdgeForm[Thick], RegularPolygon[9]}]
```

Out[39]=



Mixing more topology with algebraic geometry

```
In[42]:= ContourPlot[x^2 + y^2 == 1, {x, -1.3, 1.3}, {y, -1.3, 1.3}]
```

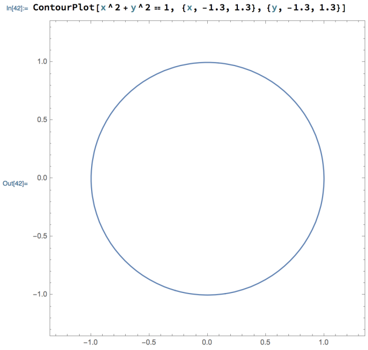


Zariski topology
étale topology
Nisnevich topology

From his thesis

The most important cause why the topologies usually used are not satisfactory for our purpose is that there are all *too weak*. Let me explain what I mean. The only cause why we are to use the topologies and sheaves in the homotopy theory is the absence of the direct limits in the category of schemes which we need to define such objects as a cone of a morphism, suspension or realization of a simplicial scheme. From the other hand there are several situations when the direct limits in the category of schemes exist. The most important examples are the symmetric powers and the objects like $\partial\Delta^n$ or ∂I^n which can be considered as the direct limits of the suitable diagrams of affine spaces. Therefore it is natural to try to find the topology such that the direct limits of such kind would be representable by the corresponding direct limits of sheaves. (Note that the functor which takes an object of the category to the corresponding representable sheaf of sets preserves inverse limits but not in general direct ones.)

Mixing more topology with algebraic geometry



Zariski topology
étale topology
Nisnevich topology
h-topology
qfh-topology
cdh-topology

From the obituary in *Nature*

In Voevodsky's motivic homotopy theory, familiar classical geometry was replaced by homotopy theory – a branch of topology in which a line may shrink all the way down to a point. He abandoned the idea that maps between geometric objects could be defined locally and then glued together, a concept that Grothendieck considered to be fundamental. A colleague commented that if mathematics were music, then Voevodsky would be a musician who invented his own key to play in.

The Fields Medal is awarded, 2002

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Tuesday, 20 August, 2002, 17:27 GMT 18:27 UK

Prize for 'big picture' mathematicians

AFP

By Richard Black
BBC Science Correspondent

Mathematicians from France and the United States have been awarded the world's top maths prize, the Fields Medal.

The medal is given every four years, and is regarded as the maths equivalent of the Nobel Prizes.

See also:

- 09 Aug 02 | South Asia
  Indians claim maths discovery
- 23 Jul 02 | Science/Nature
  How random is pi?
- 14 Mar 02 | Science/Nature
  British physicist wins religious prize
- 19 Nov 99 | Science/Nature
  Mathematicians crack big puzzle

Internet links:

- International Congress of Mathematicians
- Institute des Hautes Etudes Scientifiques
- Institute for Advanced Study

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Top Science/Nature stories now:

- Date for first Australians
- Fifth closest star discovered

From the laudatory article in the conference proceedings

ICM 2002 · Vol. I · 99–103

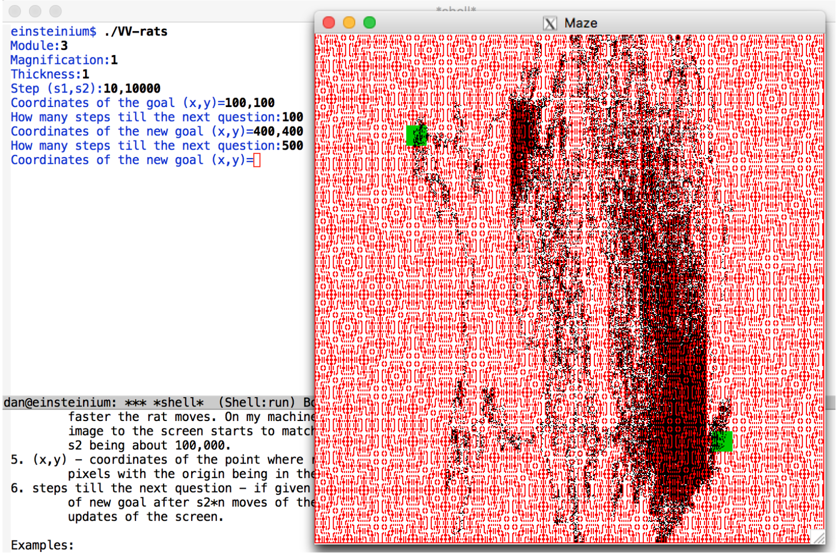
The Work of Vladimir Voevodsky

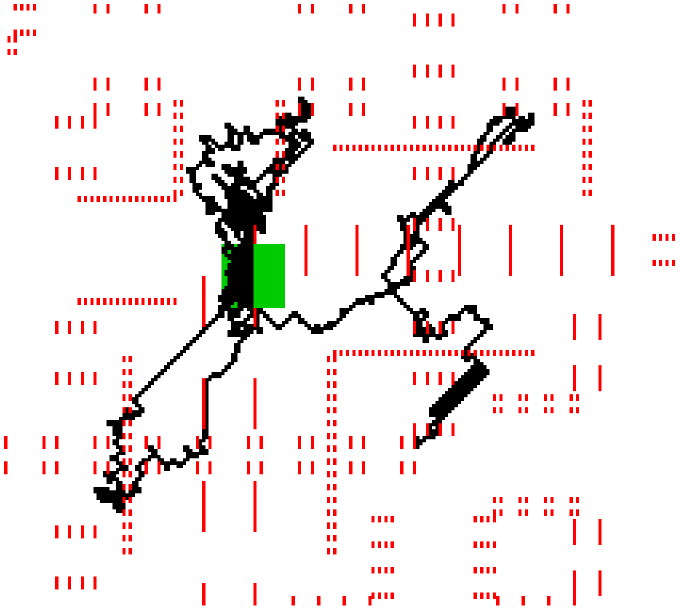
Christophe Soulé*

Vladimir Voevodsky is an amazing mathematician. He has demonstrated an exceptional talent for creating new abstract theories, about which he proved highly nontrivial theorems. He was able to use these theories to solve several of the main long standing problems in algebraic K-theory. The field is completely different after his work. He opened large new avenues and, to use the same word as Laumon, he is leading us closer to the world of *motives* that Grothendieck was dreaming about in the sixties.

Part 1.5, an interlude

Artificial intelligence, 1997





Nature photography

Nature photography

Nature photography

Mathematical biology, 2008

Singletons

Vladimir Voevodsky

Started January 4, 2008

Contents

From an interview with Roman Mikhailov, 2012

Secure | https://baaltii1.livejournal.com/198675.html

BAALTII1

RECENT

САП

Интервью Владимира Воеводского (часть 1)

В результате я выбрал, как сейчас понимаю неправильно, проблему восстановления истории популяций по их современной генетической композиции. Я провозился с этой задачей в общей сложности около двух лет и в конце концов, уже в 2009 году, понял, что то, что я придумывал, бесполезно. В моей жизни, пока, это была, пожалуй, самая большая научная неудача. Очень много работы было вложено в проект, который полностью провалился. Какая-то польза, конечно, все-таки, была - я выучил много из теории вероятности, которую знал плохо, а также узнал много нового про демографию и демографическую историю.

Part 2, Univalence Axiom:

$$(X = Y) \xrightarrow{\cong} (X \cong Y)$$

From a public lecture, March 26, 2014

In 1999/2000, again at the IAS, I was giving a series of lectures, and Pierre Deligne was taking notes and checking every step of my arguments. Only then did I discover that the proof of a key lemma in "*Cohomological Theory*" contained a mistake and that the lemma, as stated, could not be salvaged.

Fortunately, I was able to prove a weaker and more complicated lemma which turned out to be sufficient for all applications. A corrected sequence of arguments was published in 2006.

A comparison

1994 version:

Proposition 4.22 *Let W be a smooth semi-local scheme over a field k and $W = U \cup V$ be an open covering of W. Then for any pretheory (F, ϕ, A) over k such that A is the category of abelien groups the following sequence is exact:*

$$0 \longrightarrow F(W) \longrightarrow F(U) \oplus F(V) \longrightarrow F(U \cap V) \longrightarrow 0.$$

2006 version:

LEMMA 22.10. *Suppose that F is a homotopy invariant presheaf with transfers. Then for any open covering $S = U_0 \cup V$ there is an open $U \subset U_0$ such that $S = U \cup V$ and the sequence $F(MV(Q))$ is exact, where $Q = Q(S, U, V)$:*

$$0 \to F(S) \to F(U) \oplus F(V) \to F(U \cap V) \to 0.$$

From a public lecture, March 26, 2014

This story got me scared. Starting from 1993 multiple groups of mathematicians studied the "*Cohomological Theory*" paper at seminars and used it in their work and none of them noticed the mistake.

And it clearly was not an accident. ***A technical argument by a trusted author, which is hard to check and looks similar to arguments known to be correct, is hardly ever checked in detail.***

An email from 2002

```
Date:  Tue, 10 Sep 2002 09:15:21 -0400 (EDT)
From:  Vladimir Voevodsky <vladimir@ias.edu>
To:  dan@math.uiuc.edu

...

Vladimir.

PS I am thinking again about the applications of computers to pure
math.  Do you know of anyone working in this area?  I mean mostly
some kind of a computer language to describe mathematical structures,
their properties and proofs in such a way that ultimately one may
have mathematical knowledge archived and logically verified in a
fixed format.
```

Notes on homotopy λ-calculus

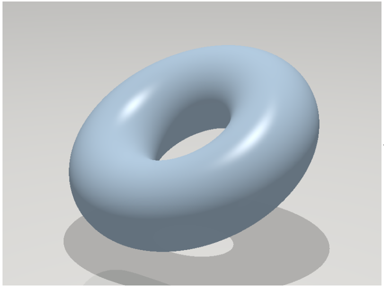
Vladimir Voevodsky

Started Jan. 18, Feb. 11, 2006

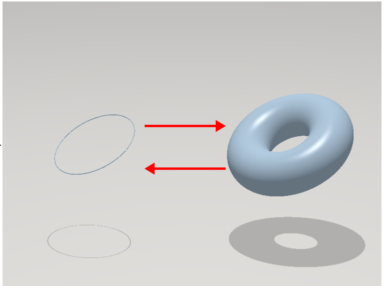
Contents

Univalence Axiom: $(X = Y) \xrightarrow{\cong} (X \cong Y)$

$\ldots$ as viewed in topology:



$\xrightarrow{\cong}$



Date: Mon, 01 May 2006 10:10:30 CDT
To: Peter May <may@math.uchicago.edu>
From: "A. Bousfield" <bous@uic.edu>
Subject: Re: Simplicial question

Dear Peter,

I think that the answer to Voevodsky's basic question is "yes," and I'll try to sketch a proof.

Since the Kan complexes X and Y are homotopy equivalent, they share the same minimal complex M, and we have trivial fibrations X -> M and Y -> M by Quillen's main lemma in "The geometric realization of a Kan fibration ." Thus X + Y -> M + M is also a trivial fibration where "+" gives the disjoint union. We claim that the composition of X + Y -> M + M with the inclusion M + M >-> M x Delta^1 may be factored as the composition of an inclusion X + Y >-> E with a trivial fibration E -> M x Delta^1 such that the counterimage of M + M is X + Y. We may then obtain the desired fibration

E -> M x Delta^1 -> Delta^1

whose fiber over 0 is X and whose fiber over 1 is Y.

We have used a case of:

Claim. The composition of a trivial fibration A -> B with an inclusion B -> C may be factored as the composition of an inclusion A >-> E with a trivial fibration E -> C such that the counterimage of B is A.

...

His first lecture about univalent foundations

The equivalence axiom and univalent models of type theory.

(Talk at CMU on February 4, 2010)

By Vladimir Voevodsky

Abstract

I will show how to define, in any type system with dependent sums, products and Martin-Lof identity types, the notion of a homotopy equivalence between two types and how to formulate the Equivalence Axiom which provides a natural way to assert that "two homotopy equivalent types are equal". I will then sketch a construction of a model of one of the standard Martin-Lof type theories which satisfies the equivalence axiom and the excluded middle thus proving that M.L. type theory with excluded middle and equivalence axiom is at least as consistent as ZFC theory.

Models which satisfy the equivalence axiom are called univalent. This is a totally new class of models and I will argue that the semantics which they provide leads to the first satisfactory approach to type-theoretic formalization of mathematics.

Univalent Foundations

https://en.wikipedia.org/wiki/Univalent_foundations

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Univalent foundations

From Wikipedia, the free encyclopedia

Univalent foundations are an approach to the foundations of mathematics in which mathematical structures are built out of objects called *types*. Types in the univalent foundations do not correspond exactly to anything in set-theoretic foundations, but they may be thought of as spaces, with equal types corresponding to homotopy equivalent spaces and with equal elements of a type corresponding to points of a space connected by a path. Univalent foundations are inspired both by the old Platonic ideas of Hermann Grassmann and Georg Cantor and by the "categorical" mathematics in the style of Alexander Grothendieck. It departs from the use of predicate logic as the underlying formal deduction system, replacing it, at the moment, by a version of the Martin-Löf type theory. The development of the univalent foundations is closely related with the development of homotopy type theory.

The univalence axiom, in his *Foundations*

GitHub, Inc. [US] | https://github.com/UniMath/Foundations/blob/master/Proof_of_Extensionality/funextfun.v

```
(** ** Univalence axiom. *)


Definition eqweqmap { T1 T2 : UU } ( e: paths T1 T2 ) : weq T1 T2 .
Proof. intros. destruct e . apply idweq. Defined.

Axiom univalenceaxiom :  forall T1 T2 : UU ,  isweq ( @eqweqmap T1 T2 ).

```

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```

A function $f : X \to Y$ is an *equivalence* if, for each y in Y, there is just one x in X with $f(x) = y$.

The univalence axiom, in his *Foundations*

GitHub, Inc. [US] https://github.com/UniMath/Foundations/blob/master/Proof_of_Extensionality/funextfun.v

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A function $f : X \to Y$ is an *equivalence* if, for each y in Y, there is just one x in X with $f(x) = y$.

The notation for a function being an equivalence is $f : X \xrightarrow{\cong} Y$.
The notation for the type of all equivalences between X and Y is $X \cong Y$.

The univalence axiom, in his *Foundations*

GitHub, Inc. [US] https://github.com/UniMath/Foundations/blob/master/Proof_of_Extensionality/funextfun.v

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Univalence Axiom: $(X = Y) \xrightarrow{\cong} (X \cong Y)$

The notion of h-level, in his *Foundations*

GitHub, Inc. [US] https://github.com/UniMath/Foundations/blob/master/Generalities/uu0.v

```
(** *** h-levels of types *)


Fixpoint isofhlevel (n:nat) (X:UU): UU:=
match n with
O => iscontr X |
S m => forall x:X, forall x':X, (isofhlevel m (paths x x'))
end.
```

The notion of h-level

h-level	*type T*	*elements r, s*	*identity type $r = s$*
0	*true*		
1	*proposition*	proofs p, q	$p = q$ is true

The notion of h-level

h-level	*type* T	*elements* r, s	*identity type* $r = s$
0	*true*		
1	*proposition*	proofs p, q	$p = q$ is true
2	*set*	elements x, y	$x = y$ is a proposition

The notion of h-level

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2	*set*	elements x, y	$x = y$ is a proposition
3	*a type of h-level 3*	elements a, b	$a = b$ is a set
4	*a type of h-level 4*	elements a, b	$a = b$ is a type of h-level 3
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Feasibility of the encoding, in his *Foundations*

- functions whose corresponding values are all equal, are equal
- the type of functions from one set to another is a set
- a subtype of a set is a set (call it a *subset*)
- the type of all subsets* of a set is a set;
- whether a type is of h-level n, is a proposition
- equivalences have inverse functions that are equivalences
- whether a function is an equivalence, is a proposition
- the type of natural numbers and the finite types are sets
- equivalent types have the same h-level
- propositions that imply each other are equivalent
- subsets defined by equivalent predicates are equal

How to encode the notion of "group"

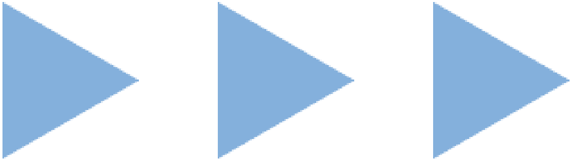
Let U be a *universe*.

A *group* in U is a sequence $(G, e, i, m, \lambda, \rho, \lambda', \rho', \alpha, \iota)$, where

- G is a type of U
- $e : G$
- $i : G \to G$
- $m : G \times G \to G$
- λ is a proof that for every $a : G$, $m(e, a) = a$
- ρ is a proof that for every $a : G$, $m(a, e) = a$
- λ' is a proof that for every $a : G$, $m(i(a), a) = e$
- ρ' is a proof that for every $a : G$, $m(a, i(a)) = e$
- α is a proof that for every $a, b, c : G$, $m(m(a, b), c) = m(a, m(b, c))$
- ι is a proof that G is a set

An example of a type of h-level 3

The type of all triangles (with unlabeled vertices).



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Monday, September 24, 2012 (All day) to Thursday, August 15, 2013 (All day)

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The HoTT Book

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Univalent Foundations of Mathematics
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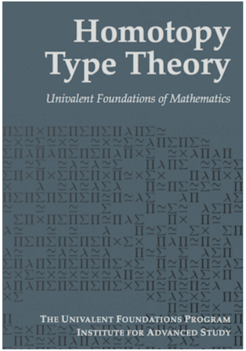

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Univalent Mathematics

This Coq library aims to formalize a substantial body of mathematics using the univalent point of view.

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Contents

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Some scientific articles describing the contents of the UniMath library are listed in the wiki.

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Vladimir's final lecture

-och-akti/logic-in-stockholm-2/logic-colloquium-201/logic-colloquium-2017-august-14-20-1.7176...

Special session on category theory and type theory in honor of Per Martin-Löf on his 75th birthday

Dates: August 17–19, 2017

Speakers:

- Thierry Coquand (Göteborg University)
- Richard Garner (Macquarie University, Sydney)
- André Joyal (University of Quebec, Montreal)
- Vladimir Voevodsky (Institute for Advanced Study, Princeton)

Vladimir's final project

MODELS, INTERPRETATIONS AND THE INITIALLITY CONJECTURES

VLADIMIR VOEVODSKY

ABSTRACT. Work on proving consistency of the intensional Martin-Löf type theory with a sequence of univalent universes ("MLTT+UA") led to the understanding that in type theory we do not know how to construct an interpretation of syntax from a model of inference rules. That is, we now have the concept of a model of inference rules and the concept of an interpretation of the syntax and a conjecture that implies that the former always defines the latter. This conjecture, stated as the statement that the term model is an initial object in the category of all models of a given kind, is called the Initiallity Conjecture. In my talk I will outline the various parts of this new vision of the theory of syntax and semantics of dependent type theories.

1. Introduction

The first few steps in all approaches to the set-theoretic semantics of dependent type theories remain insufficiently understood. The constructions which have been worked out in detail in the case of a few particular type systems by dedicated authors are being extended to the wide variety of type systems under consideration today by analogy. This is not acceptable in mathematics. Instead we should be able to obtain the required results for new type systems by *specialization* of general theorems and constructions formulated for abstract objects the instances of which combine together to produce a given type system.

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C-system of a module over a Jf-relative monad

Vladimir Voevodsky

(Submitted on 1 Feb 2016)

Let F be the category with the set of objects $\mathbf{N}$ and morphisms being the functions between the standard finite sets of the corresponding cardinalities. Let $Jf : F \to Sets$ be the obvious functor from this category to the category of sets. In this paper we construct, for any relative monad $\mathbf{RR}$ on Jf and a left module $\mathbf{LM}$ over $\mathbf{RR}$, a C-system $C(\mathbf{RR}, \mathbf{LM})$ and explicitly compute the action of the B-system operations on its B-sets.
In the following paper it is used to provide a rigorous mathematical approach to the construction of the C-systems underlying the term models of a wide class of dependent type theories.
This paper is a result of evolution of arXiv:1407.3394. However this paper is much more detailed and contains a lot of material that is not contained in arXiv:1407.3394. It also does not cover some material that is covered in arXiv:1407.3394.

Topology in another combinatorial style

In[134]:=
```
n = 30; d = 2 Pi / n;
g[i_, j_] := 3 * {Cos[i], Sin[i], 0} + {Cos[i] Cos[j], Sin[i] Cos[j], Sin[j]};
f[i_, j_] := Polygon[ {g[i, j], g[i + d, j], g[i + d, j + d], g[i, j + d]}];
Graphics3D[Table[f[i, j], {i, 0, 2 Pi (n - 1) / n, d}, {j, 0, 2 Pi (n - 1) / n, d}]]
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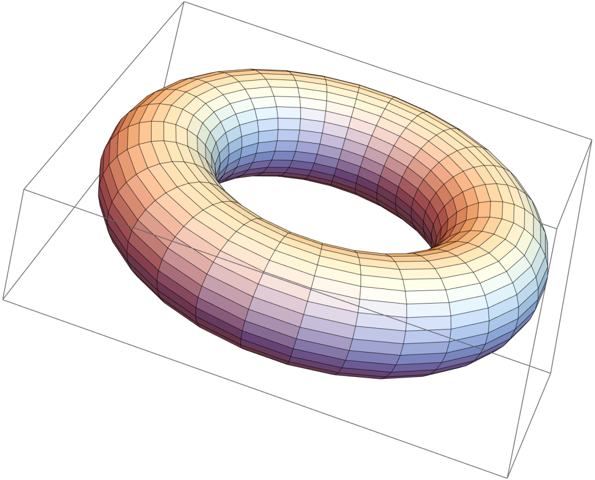
Out[136]=



Cubical Type Theory: a constructive interpretation of the univalence axiom*

Cyril Cohen[1], Thierry Coquand[2], Simon Huber[2], and Anders Mörtberg[3]

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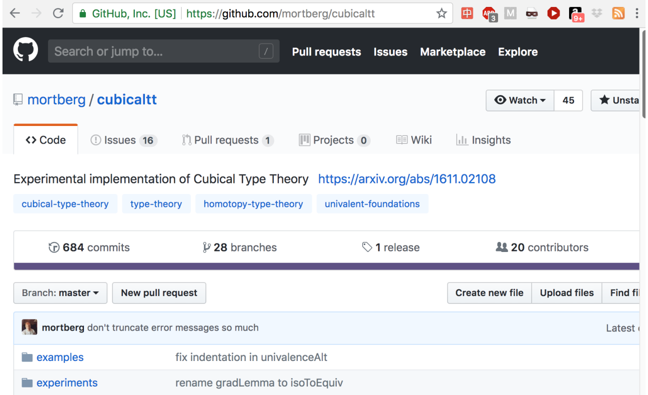
Abstract

This paper presents a type theory in which it is possible to directly manipulate n-dimensional cubes (points, lines, squares, cubes, etc.) based on an interpretation of dependent type theory in a cubical set model. This enables new ways to reason about identity types, for instance, function extensionality is directly provable in the system. Further, Voevodsky's univalence axiom is provable in this system. We also explain an extension with some higher inductive types like the circle and propositional truncation. Finally we provide semantics for this cubical type theory in a constructive meta-theory.

A cubical proof assistant



From the obituary in *Nature*

Motivic homotopy theory is blossoming, despite Voevodsky's change of focus about ten years ago. Many dedicated researchers continue to find new ways to apply his fundamental ideas to algebra, geometry and topology. Similarly, Univalent Foundations is destined to remain a vibrant area of research. Formalizing Voevodsky's work on motives in the Univalent Foundations would close the circle in a fitting way and fulfil one of his dreams.

Fossil hunting at a latitude of 78.3