Topology of Random Cell Complexes

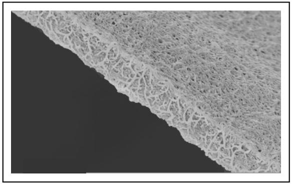
Ben Schweinhart

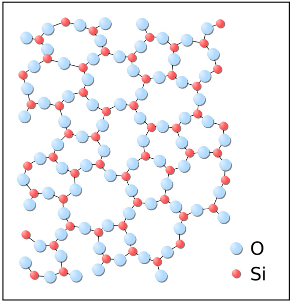
Princeton University

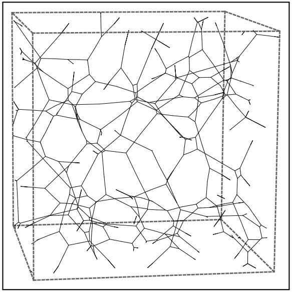
December 10, 2014

Presentation Outline

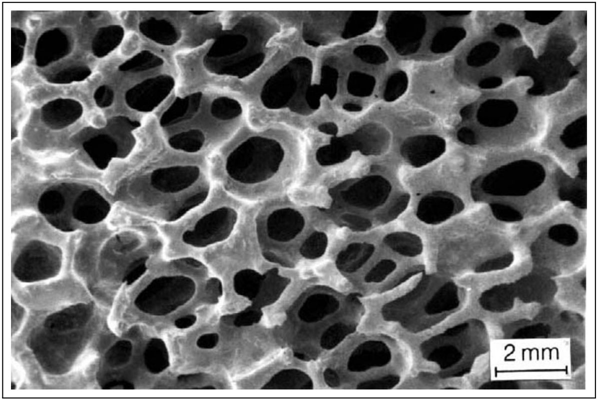
Examples







Open Cell Foams



- Interesting topology
- Very important material in practice, not well understood

Grain Growth

- Show movie
- Steady state for which scale-free properties have converged? Dependence on initial conditions?
- Need metric - a nice one is given by considering the local topology.

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- These systems have interesting topology, but they have not yet been studied using topological methods.*
- Crystallography doesn't apply to disordered materials.
- Plan: First, I will introduce a new, general method to quantify the topology of cell complexes.
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Idea of Swatches

Key Idea

Quantify the local topology of cell complexes by using probability distributions of local configurations.

- Used to define a distance on cell complexes. The distance can be used to, e.g., quantify the variability of cell complexes generated in a particular way, compare two cell complexes (i.e. experimental results with simulations), or test convergence to a steady state
- Applicable to many different physical systems, computable.
- Joint work with Jeremy Mason (Boğaziçi University) and Bob MacPherson (Institute for Advanced Study)
- Paper on the arxiv: Topological Similarity of Random Cell Complexes, and Applications to Dislocation Networks.

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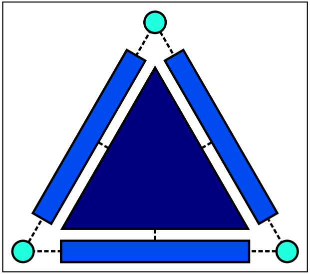
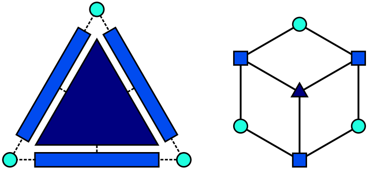
Regular Cell Complexes



Definition

A Regular Cell Complex is a space built inductively by attaching cells in each dimension (points, line segments, disks, etc). Each cell is glued on by homeomorphisms from their boundaries into the existing structure.

Graph Representation



- Represent a regular cell complex by the adjacency graph of the cells, labeled by dimension.
- Captures all topological properties.

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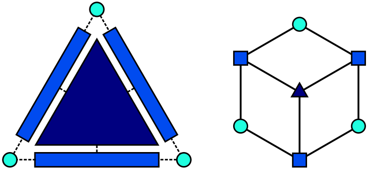

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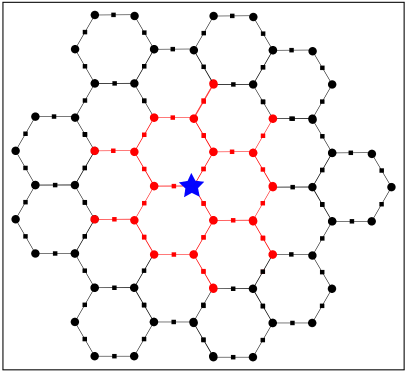
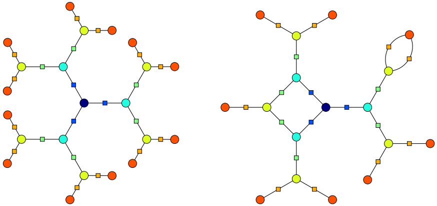


Definition

The swatch at vertex v of radius r is the neighborhood of v of radius r in the graph distance (every edge has length one).

Swatch Types



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- Sub-swatch of a swatch: swatch of smaller radius at the same root (central vertex).

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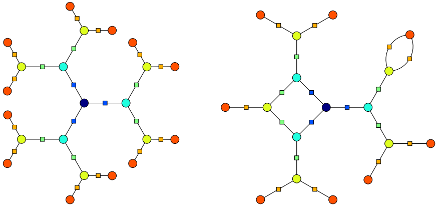

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Cloth

- For each r, consider the probability distribution of swatch types of radius r at the vertices of a cell complex.
- This family of probability distributions is called the cloth of the cell complex.
- Captures all local topological properties of the cell complex.

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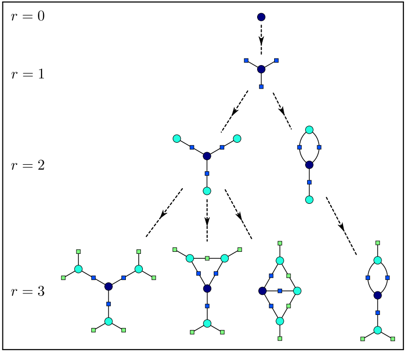
Tree of Swatches



- Connect swatch type of radius r with all subswatch types of radius $r - 1$.
- The cloth is a weighting on this tree, subject to consistency conditions.

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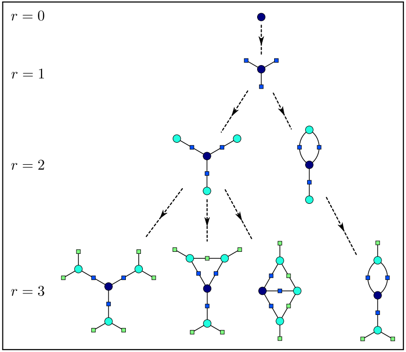

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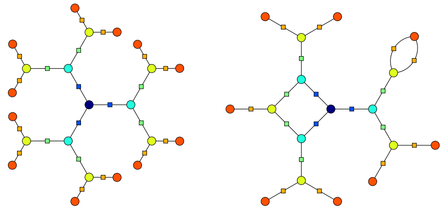

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- $d(S_1, S_2) = \frac{1}{13}$

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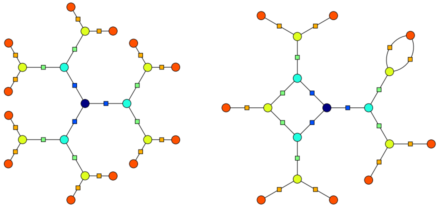

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Distance on Cell Complexes

Definition

If C_1 and C_2 are cell complexes, $d_r(C_1, C_2)$ is the Earth Mover's Distance between the probability distributions at radius r induced by distance on swatches.

- Earth Mover's Distance = the infimum of the costs of transformations between the two probability distributions on swatch types of radius r.
- Cost = amount of probability mass moved between swatch types, weighted by swatch distance.
- Distance can be used to compare different structures, test convergence to steady state, iteratively modify structures to reach a desired state

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Convergence of Cell Complexes

- Limit distance= $d(C_1, C_2) = \lim_{r\to\infty} d_r(C_1, C_2)$.
- A sequence $\{C_i\}$ of cell complexes converges if it is a Cauchy sequence in d.
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Theorem

(Benjamini-Schramm Graph Limit) A convergent sequence of cell complexes gives rise to a limit distribution on the space of countable, connected cell complexes a root specified. The distance can be extended to the space of these distributions.

- Also implies convergence of all local topological properties: those that can be defined in terms of maps from a fixed labeled graph H into the diagram of C_i.
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- Represent network as one-dimensional cell complex.
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Examples

Swatches

A Case Study

Computational Results

Persistent Homology

Knotting

Conclusion

Collaborators

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- Curvature-driven evolution of polygonal curves in $\mathbb{T}^3$.
- The polygonal curves meet each other at vertices of degree three.
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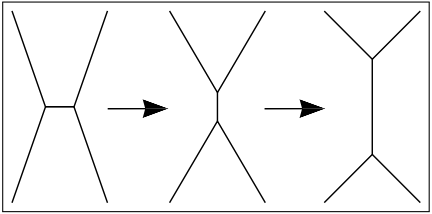
Movie

Topological Moves



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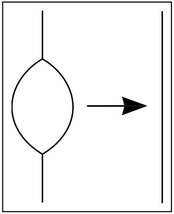

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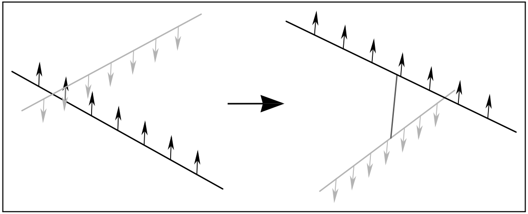

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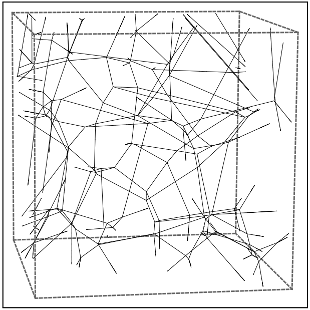
Initial Conditions



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- Random Graph: Place random points on the three-torus. Randomly create edges between pairs that are close enough.
- Both evolve to “steady state”.

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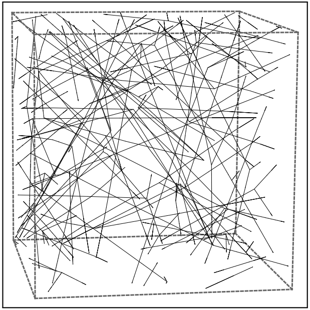

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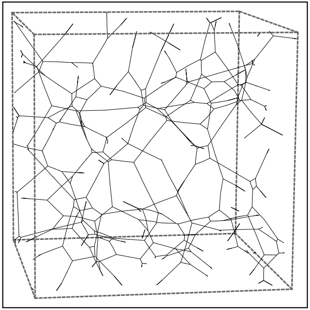

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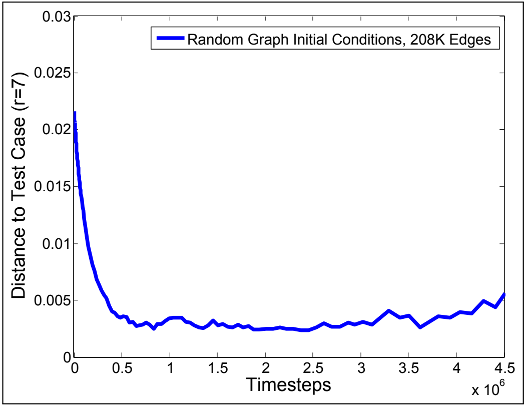

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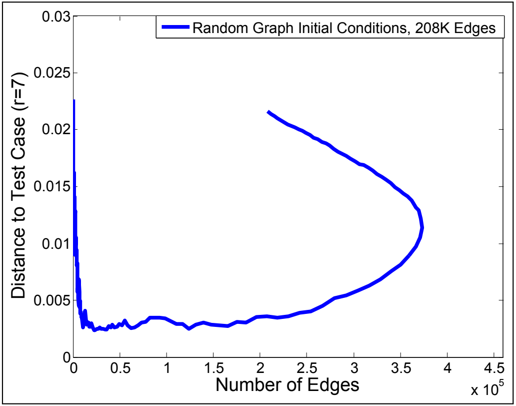

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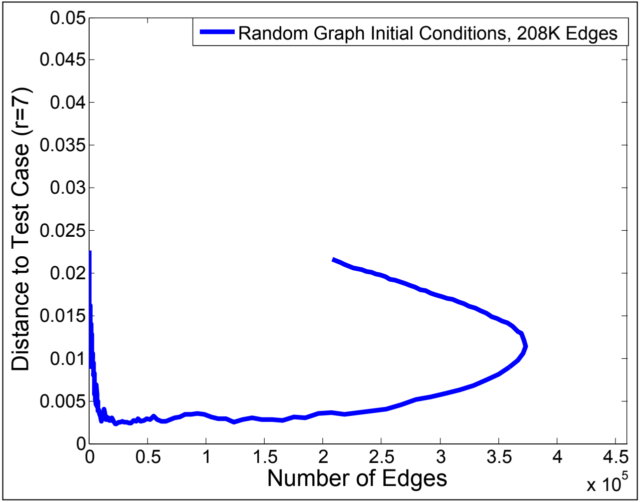
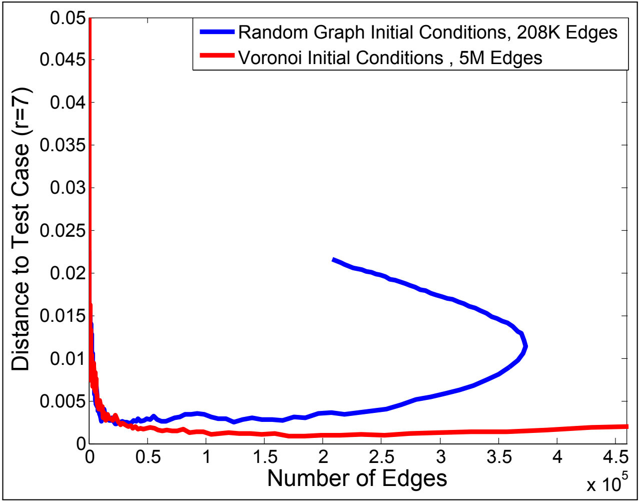


- First Case: Random Graph Initial Conditions

Convergence Results, II



- Second Case: Large Voronoi

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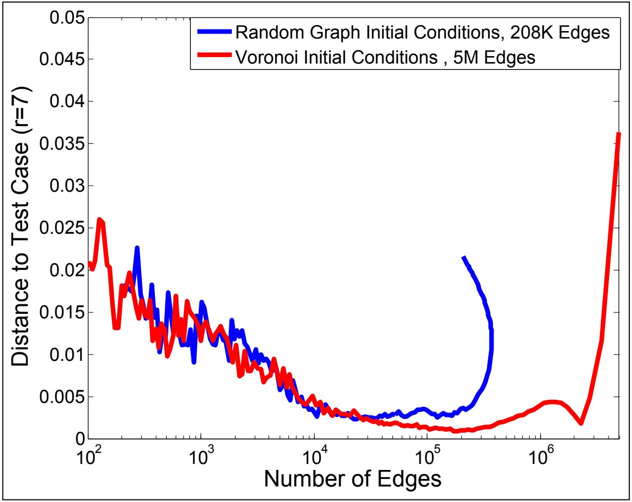

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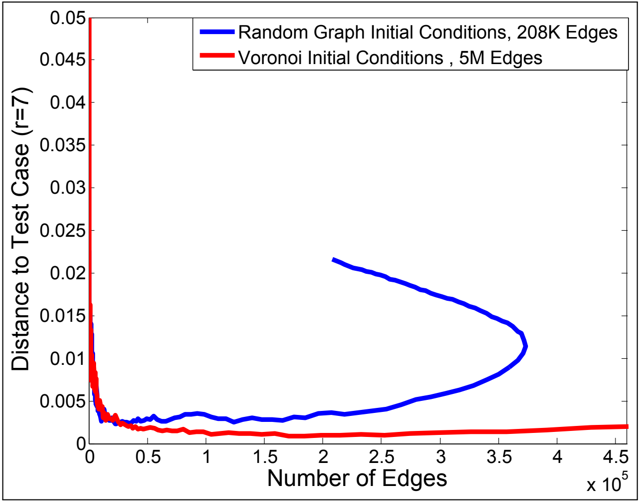

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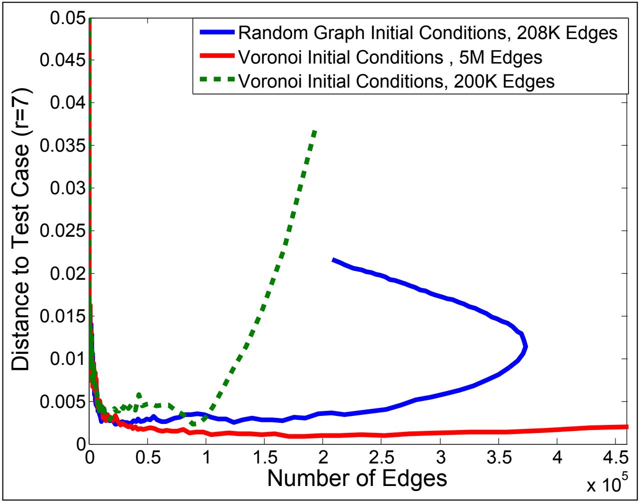

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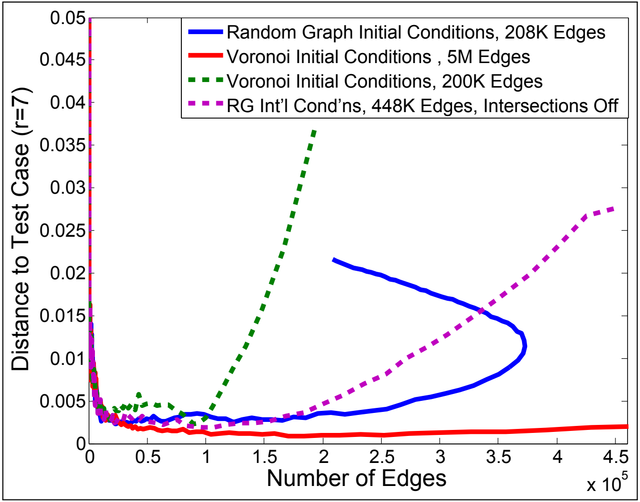
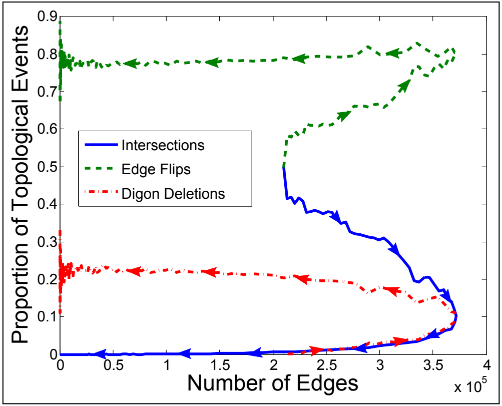


- Fourth Case: Random Graph with Intersections Disabled

On Intersections



- Plot preponderance of different topological changes as simulation proceeds.
- Intersections computationally expensive; can disregard for better steady state statistics.

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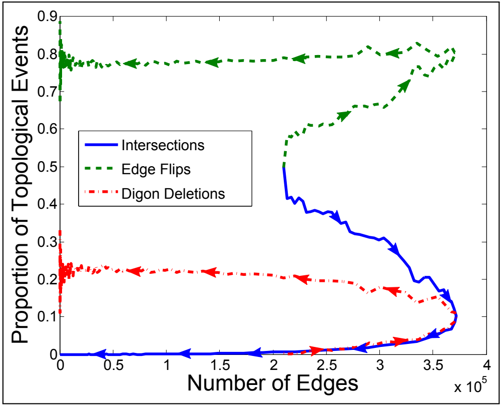

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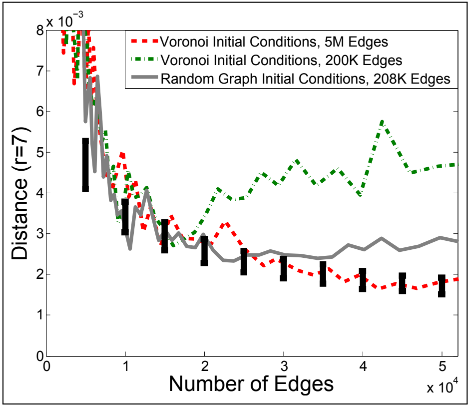
- Intersections computationally expensive; can disregard for better steady state statistics.

Error Estimation



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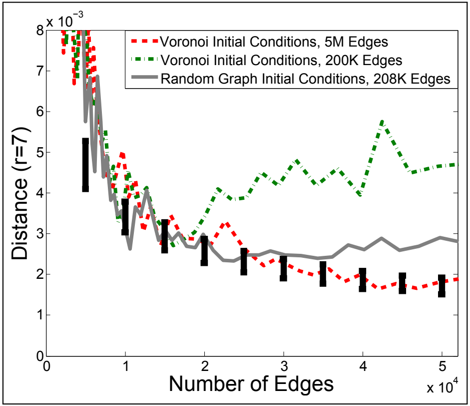

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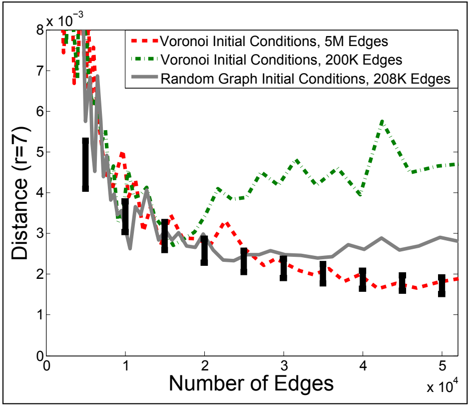

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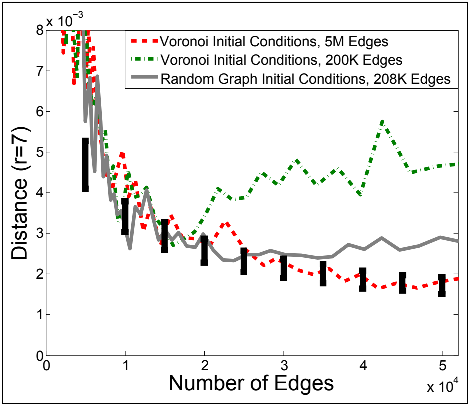

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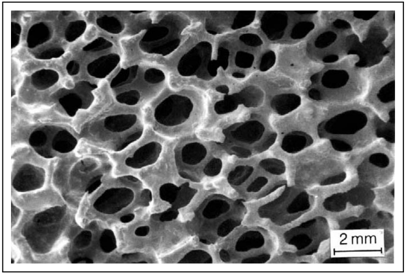
Presentation Outline

2D Persistent Homology of Objects in 3-space



- Consider ϵ-neighborhoods of $X \subset \mathbb{R}^3$, X_ϵ.
- As ϵ increases, holes form and disappear.
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- Look at figures in Mathematica

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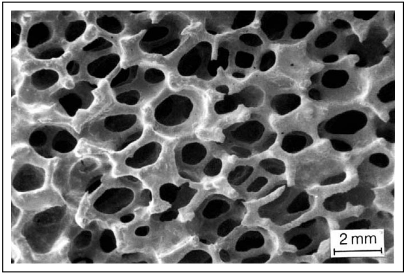

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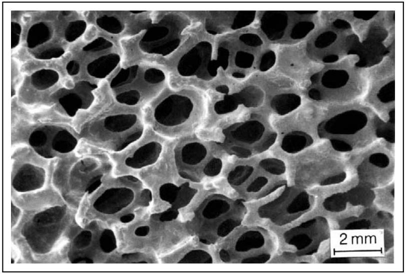

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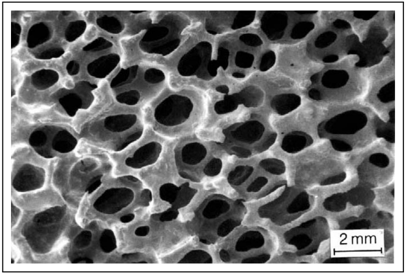

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Persistent Homology, ct'd

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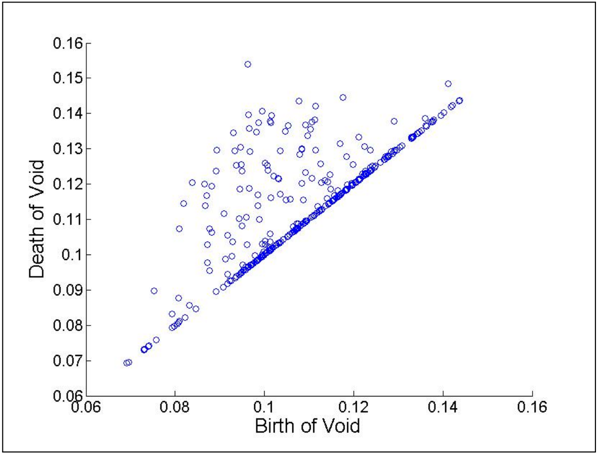
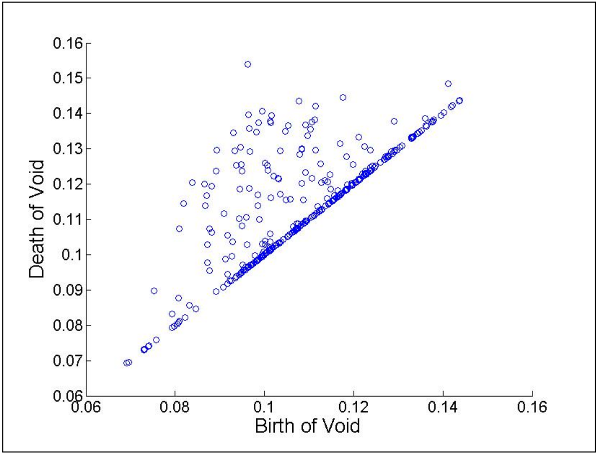


- Plot time when hole appears vs time when it disappears
- Points on x=y line are noise

Persistent Homology, Continued



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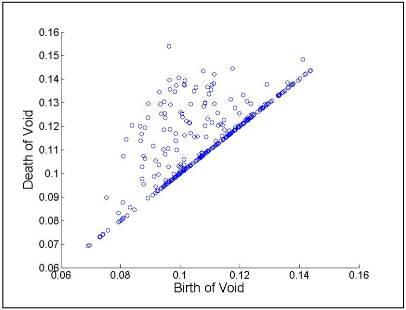
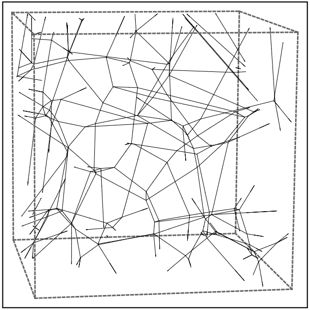
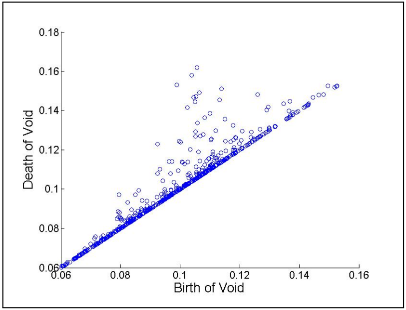
1-skeleton of Voronoi Decomposition

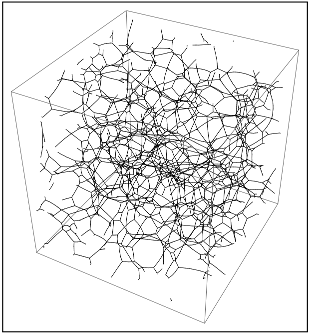




- Persistent Homology recovers the cells.

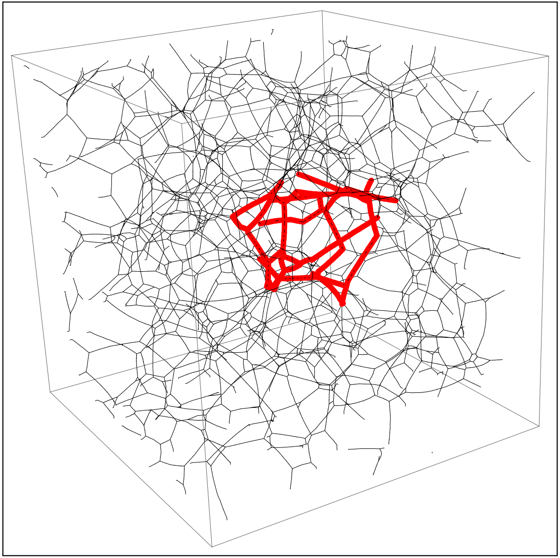
Steady State of CDE Simulation



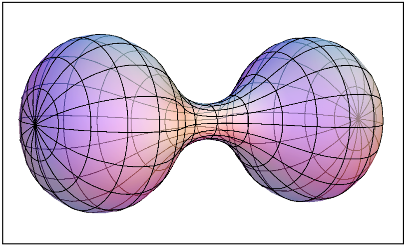


- More surprising voids!

Minimal Cycles



Persistence Tree



- Usual interpretation of Persistent Homology represents this as two points for two voids
- Correct structure: Persistence Tree
- Adjacency of minimal cycles?

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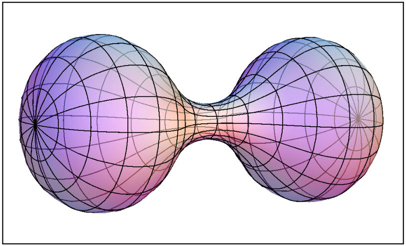

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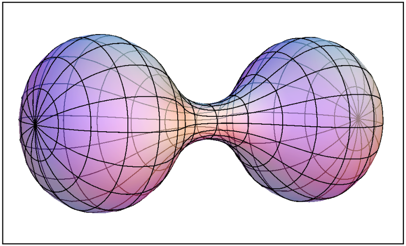

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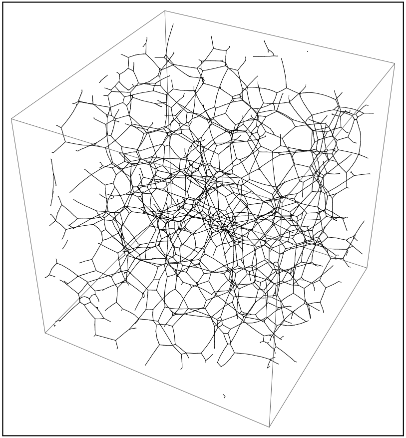
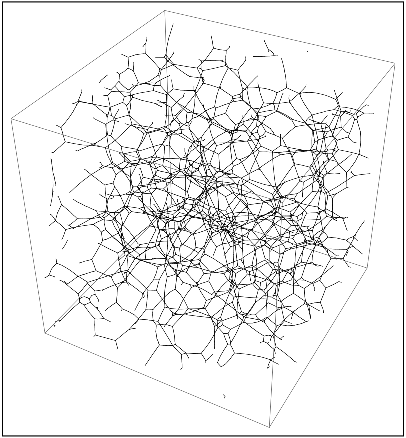
Presentation Outline

Unknotted Networks



- Many embedded graphs from physical systems appear to be unknotted.
- How to measure this?

Unknotted Networks



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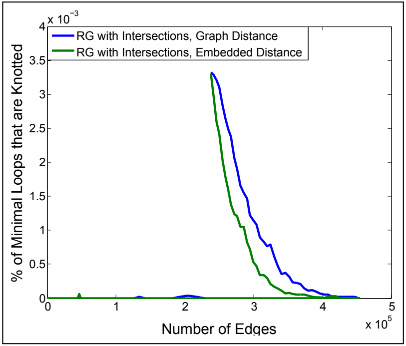

- Look at shortest cycle containing each edge. Is it knotted?
- Start with knotted network, track # of knotted minimal cycles as system evolves.
- Curvature-driven evolution appears to unknot networks.

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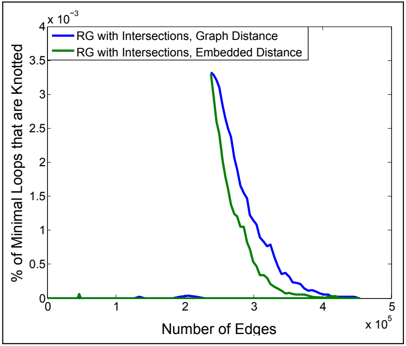

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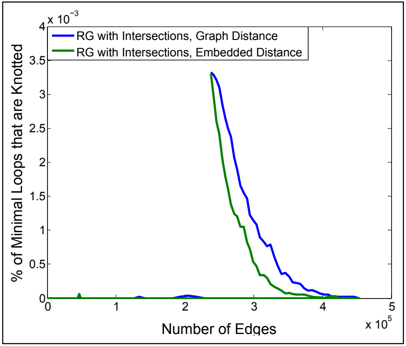

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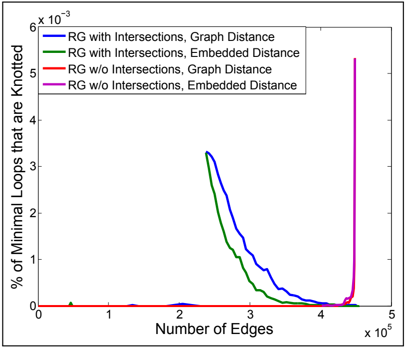

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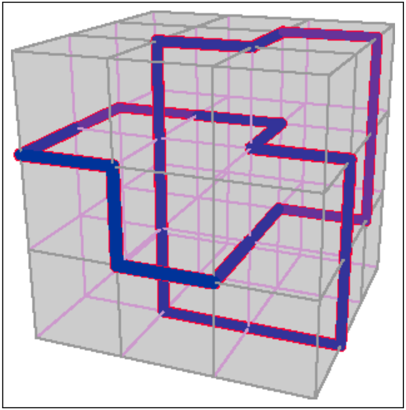
- Network unknots even faster if intersections turned off.

Unknottedness



- Unknotted networks can contain knots.

Defining Unknottedness

Definition

An embedded graph is unknotted if its complement has the homotopy type of a graph.

- Physical interpretation: equivalent to existence of dual network (important for, i.e., open cell foams).
- I'm working on several theoretical questions surrounding this definition.
- Is it equivalent to π_1 of the complement being free?

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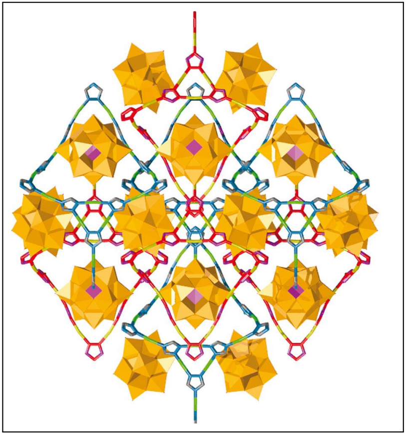
Knotted Materials



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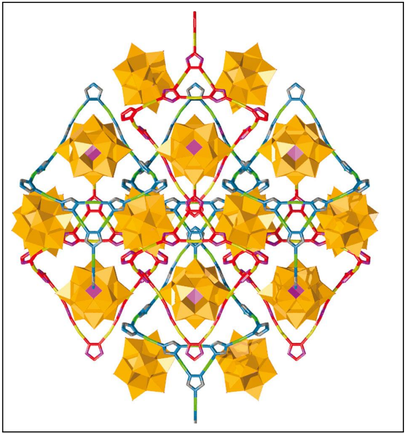

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Conclusion

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- Robert MacPherson (Institute for Advanced Study) - PhD Advisor
- Jeremy Mason (Boğaziçi University) - Swatches