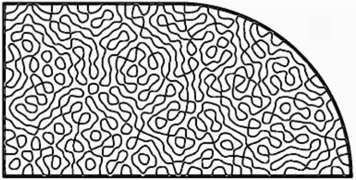
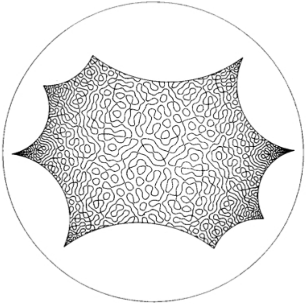
Geometry of nodal sets

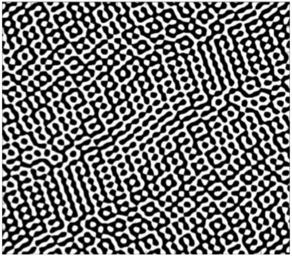
2014.09.29

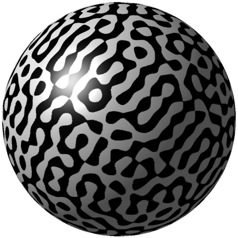
Yaiza Canzani

Pictures of $\varphi_\lambda^{-1}(0)$

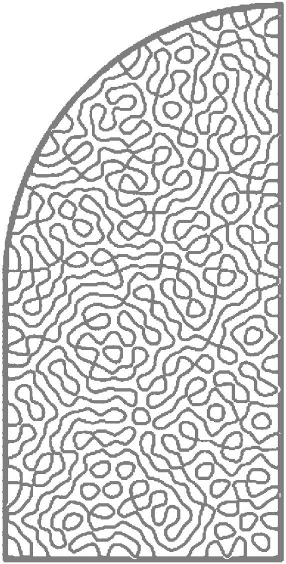






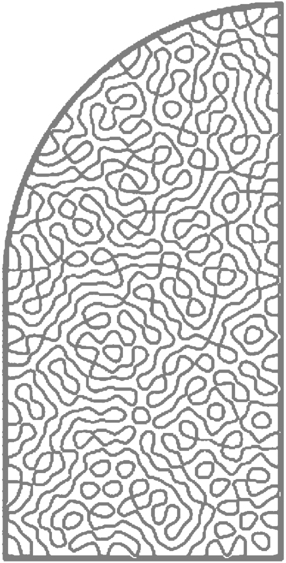


Global geometry of $\varphi_\lambda^{-1}(0)$



As $\lambda \to \infty$

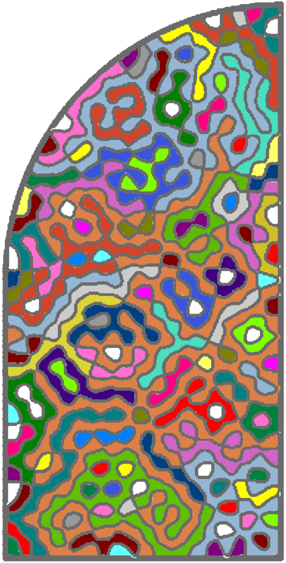
Global geometry of $\varphi_\lambda^{-1}(0)$



As $\lambda \to \infty$

- $c\lambda \leq \text{length}(\varphi_\lambda^{-1}(0)) \leq C\lambda$

Global geometry of $\varphi_\lambda^{-1}(0)$



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- $c\lambda \leq \text{length}(\varphi_\lambda^{-1}(0)) \leq C\lambda$
- $0 \leq \# \{\text{nodal domains of } \varphi_\lambda\} \leq \lambda^2$

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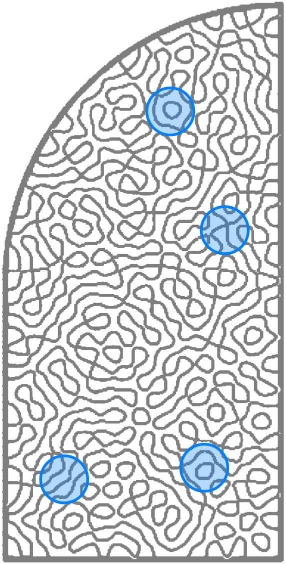

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- $B(x, \frac{C}{\lambda}) \cap \varphi_\lambda^{-1}(0) \neq \emptyset \quad \forall x \in M$

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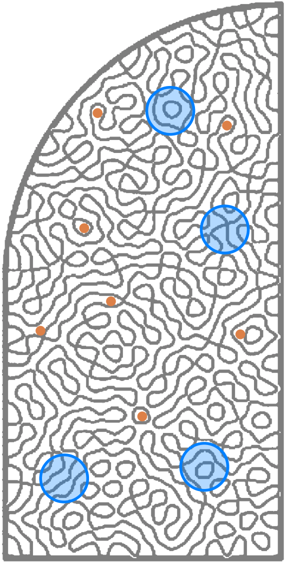

As $\lambda \to \infty$

- $c\lambda \leq \text{length}(\varphi_\lambda^{-1}(0)) \leq C\lambda$
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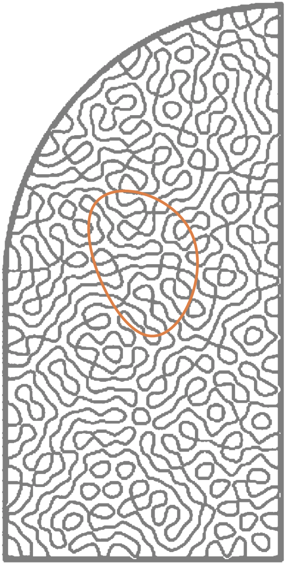
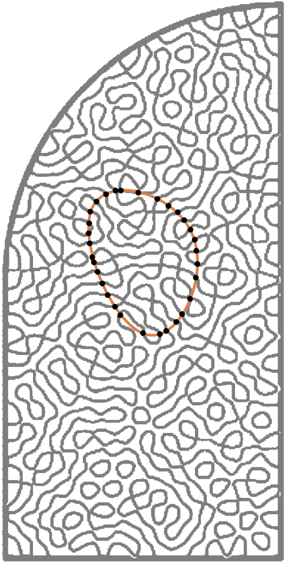
- $B(x, \frac{C}{\lambda}) \cap \varphi_\lambda^{-1}(0) \neq \emptyset \quad \forall x \in M$
- $\frac{c}{\lambda} \leq \text{inrad}\,(\varphi_\lambda^{-1}(0))^c \leq \frac{C}{\lambda}$

Local geometry of $\varphi_\lambda^{-1}(0)$



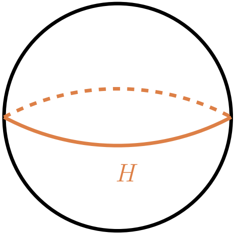
Local geometry of $\varphi_\lambda^{-1}(0)$



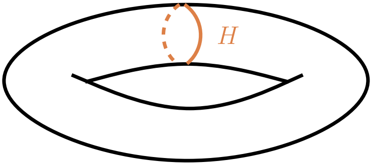
As $\lambda \to \infty$

$$\#\left\{\varphi_\lambda^{-1}(0) \cap H\right\} \leq \ ?$$

There are BAD curves!



$\varphi_\lambda =$ zonal harmonic



$\varphi_\lambda(x, y) = \sin(\lambda x)\sin(\lambda y)$

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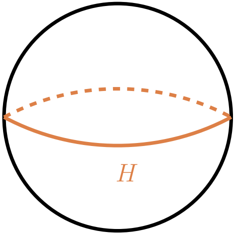

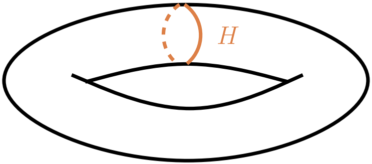

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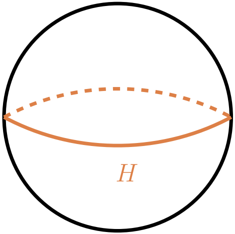
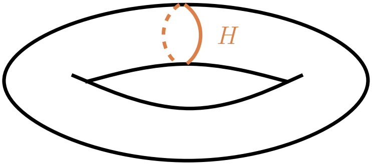


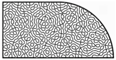


$\varphi_\lambda =$ zonal harmonic

$\varphi_\lambda(x, y) = \sin(\lambda x)\sin(\lambda y)$

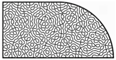
$$\varphi_\lambda|_H = 0 \quad \Rightarrow \quad \#\left\{\varphi_\lambda^{-1}(0) \cap H\right\} = \infty$$

Known



- $\Omega \subset \mathbb{R}^2$ (Toth-Zelditch)
  If H is good $\Rightarrow$ $(**)$

Known



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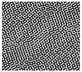
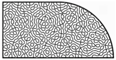


- $\mathbb{T}^2$ (Bourgain-Rudnick)
  If H is positively curved $\Rightarrow$ H is good and $(**)$

Known



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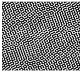

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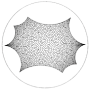
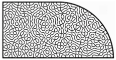


- Hyperbolic compact surface (Jung)
  If H is a geodesic circle $\Rightarrow$ H is good and $(**)$

Known



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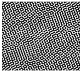

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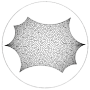
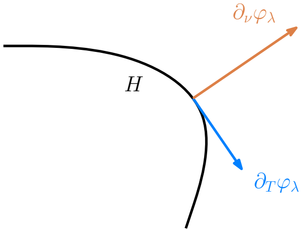


- Hyperbolic compact surface (Jung)
  If H is a geodesic circle $\Rightarrow$ H is good and $(**)$

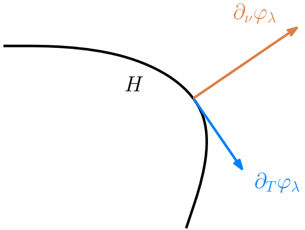
- (M, g) compact C^ω- surface, no boundary (C-Toth)
  If H is good $\Rightarrow$ $(**)$

When is a curve GOOD?



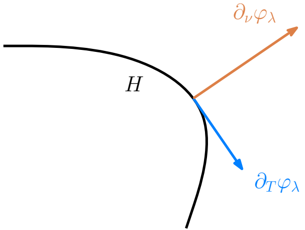
When is a curve GOOD?

- $\|\varphi_\lambda\|_{L^2(H)} \geq c\|\lambda\, \partial_T \varphi_\lambda\|_{L^2(H)}$



When is a curve GOOD?

- $\|\varphi_\lambda\|_{L^2(H)} \geq c\|\lambda\, \partial_T \varphi_\lambda\|_{L^2(H)} \underbrace{\geq e^{-C\lambda}}_{\text{we want}}$



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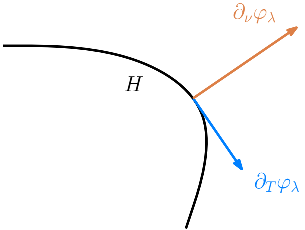

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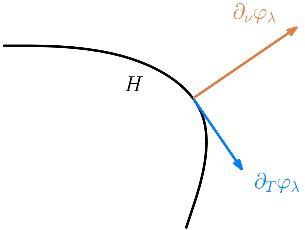

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  $\ldots \geq C$ (Quantum Ergodic case)

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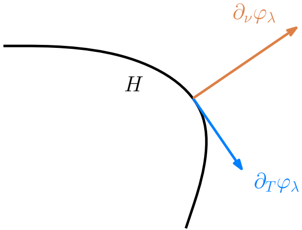

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- **Want to prove:**

  If $\|\lambda\, \partial_\nu \varphi_\lambda\|_{L^2(H)}$ is "large"

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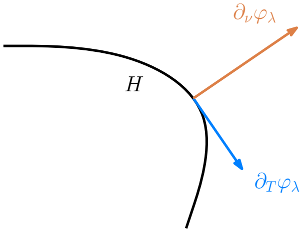

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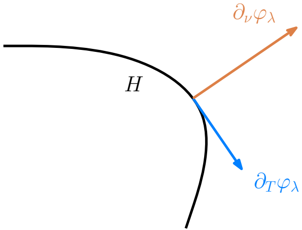

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- **Want to prove:**

  If $\underbrace{\|\lambda\, \partial_\nu \varphi_\lambda\|_{L^2(H)} \text{ is "large"}}_{\geq C \text{ if QE}} \Rightarrow \|\lambda\, \partial_T \varphi_\lambda\|_{L^2(H)} \geq e^{-C\lambda}$