Aspects of 6d QFTs

Ken Intriligator
September 15, 2016

Based on work
in collaboration with…

Spectacular Collaborators!!

Clay Córdova

Thomas Dumitrescu

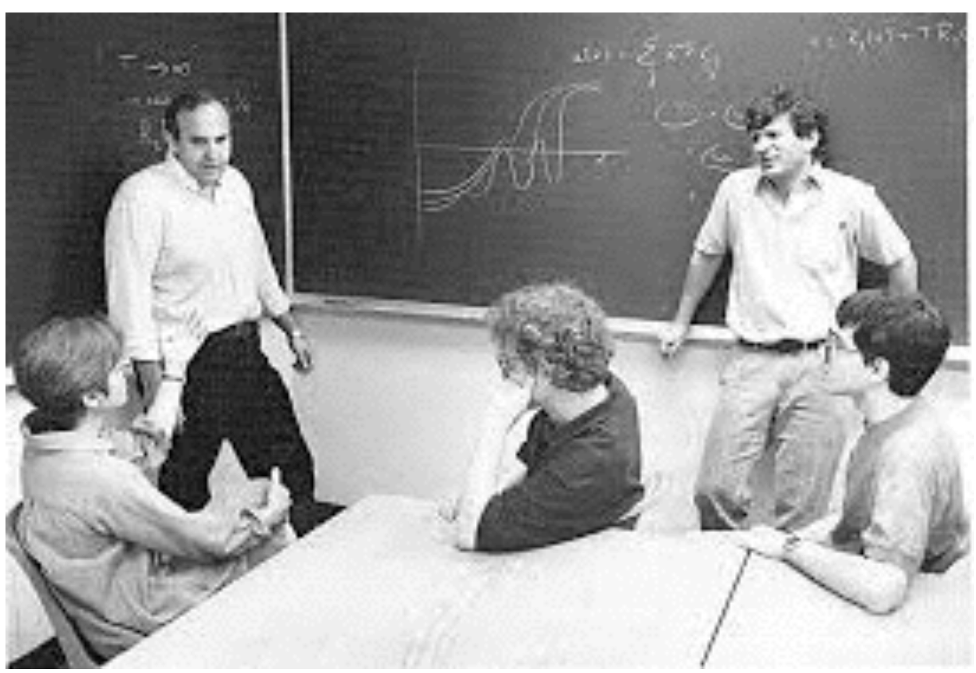
1506.03807, 1602.01217, 1609.nnnnn, and in progress.

I thank them for the wonderful collaboration!

Time out for some nostalgia

𝄞 “It was 25 years ago today….” (… 60-25=35….)

I was starting my last year of grad school, and getting ready to apply for postdoc jobs. My top choice: **Rutgers,** to work with **Nati** (I already knew him from his papers, and from meeting at Harvard and Banff). I am indeed very, very fortunate that it worked out for me! It was great! (Unfortunately, I do not have any photos of my own from that time.)



?, Tom, Dan, Steve, ?

Mike, Tom, Nati, Steve?

“To Seiberg” (verb)

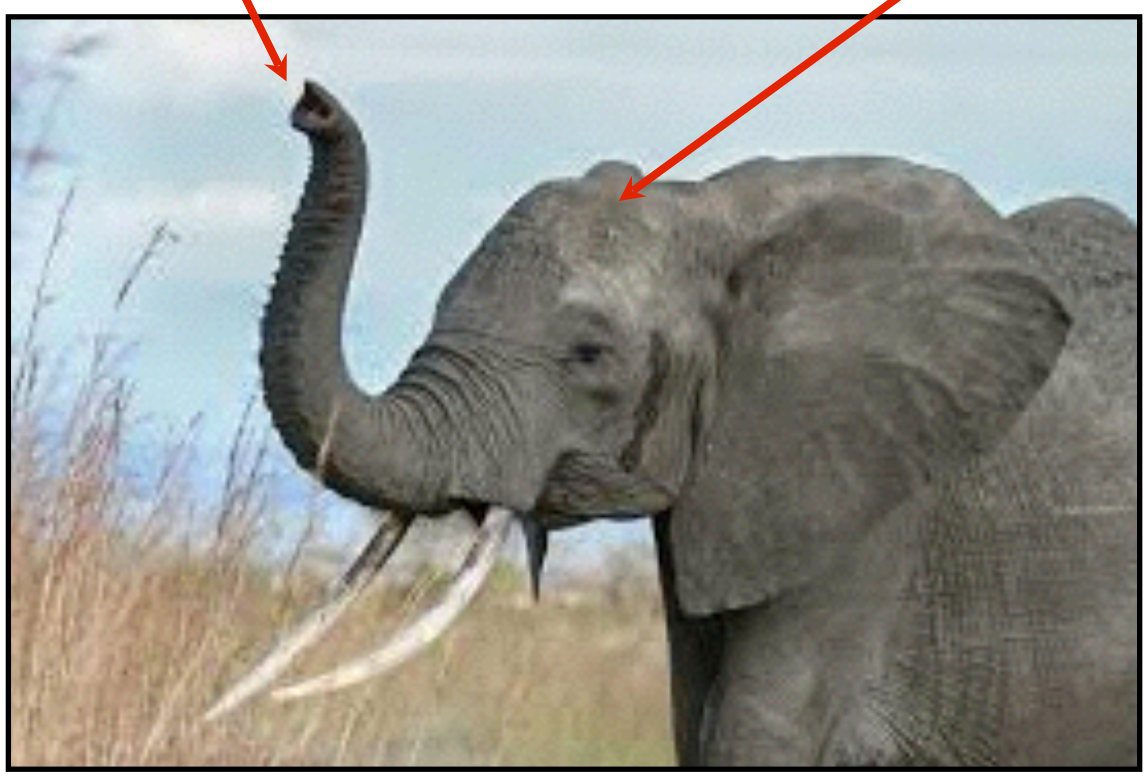
- Identify an interesting question or issue.
- Identify symmetries, distinguish global, gauge, discrete, broken, etc. Make a table.
- Make vague notions precise, find previously unappreciated puzzles and / or subtleties.
- Solve them via new ideas and methods.
- Move to next topic. Repeat.

Nati: “What is QFT?”

(see his webpage for his talk)

Perturbation theory around free field Lagrangian theories

5d & 6d SCFTs, + deformations, compactifications



5d & 6d SCFTs?!?!
Taken for granted now.
Was originally shocking, a controversial claim….

(unexplored…something crucial for the future?)

CFTs + perturbations

(Above 4d, starting from free theory, added interactions all look IR free. Quoting Duck Soup: “That’s irrelevant!”)

“That’s the answer! There’s whole lot of relevants in the circus!”

...Thanks to Nati....

“It was 20 years ago today....”

hep-th/9608111
RU-96-69

Five Dimensional SUSY Field Theories, Non-trivial Fixed Points and String Dynamics

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We study (non-renormalizable) five dimensional supersymmetric field theories. The theories are parametrized by quark masses and a gauge coupling. We derive the metric on the Coulomb branch exactly. We use stringy considerations to learn about new non-trivial interacting field theories with exceptional global symmetry E_n (E_8, E_7, E_6, $E_5 = Spin(10)$, $E_4 = SU(5)$, $E_3 = SU(3) \times SU(2)$, $E_2 = SU(2) \times U(1)$ and $E_1 = SU(2)$). Their Coulomb branch is $\mathbf{R}^+$ and their Higgs branch is isomorphic to the moduli space of E_n instantons. One of the relevant operators of these theories leads to a flow to $SU(2)$ gauge theories with $N_f = n-1$ flavors. In terms of these $SU(2)$ IR theories this relevant parameter is the inverse gauge coupling constant. Other relevant operators (which become quark masses after flowing to the $SU(2)$ theories) lead to flows between them. Upon further compactifications to four and three dimensions we find new fixed points with exceptional symmetries.

August 1996

hep-th/9609161
RU-96-85

Non-trivial Fixed Points of The Renormalization Group in Six Dimensions

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We start a systematic analysis of supersymmetric field theories in six dimensions. We find necessary conditions for the existence of non-trivial interacting fixed points. String theory provides us with examples of such theories. We conjecture that there are many other examples.

September 1996

Quoting Nati's 6d paper:

"It was 20 years ago today…."

…these theories are often referred to as "tensionless string theories." It is our view that they are conventional interacting local quantum field theories. This possible interpretation was first mentioned in [1,4] but became more clear after one of these theories (in three dimensions) was given a Lagrangian description [9] and the renormalization group flows out of the five dimensional theories appeared consistent with field theory [7]. It is likely that some of these theories do not arise under renormalization group flow from a free field theory at short distance. Therefore, they do not have a continuum Lagrangian description. It is an extremely interesting and challenging problem to find a good presentation of these theories.

[1] E. Witten, Strings '95
[4] N. Seiberg and E. Witten, "Comments on String Dynamics in 6d."
[9] K. Intriligator and N. Seiberg, "Mirror Symmetry in 3d Gauge Theory"
[7] N. Seiberg, "5d Susy Field Theories, Non-Trivial Fixed Points and String Dyn."

Next: 6d ("L")STs

or "N"?

(Some controversy: leaking from brane into the bulk)

hep-th/9705221
RU-97-42

New Theories in Six Dimensions and

Matrix Description of M-theory on T^5 and T^5/Z_2

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We present four infinite series of new quantum theories with super-Poincare symmetry in six dimensions, which are not local quantum field theories. They have string like excitations but the string coupling is of order one. Compactifying these theories on T^5 we find a Matrix theory description of M theory on T^5 and on $T^5/\mathbb{Z}_2$, which is well defined and is manifestly U-duality invariant.

May 1997

hep-th/9710014

SEMICLASSICAL DECAY OF NEAR-EXTREMAL FIVEBRANES

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Abstract

We argue that a near-extremal charge-k type II NS fivebrane can be reliably described in semiclassical string perturbation theory as long as both k and $\frac{\mu}{k}$ are large, where μ is the energy density in string units. For a small value of the asymptotic string coupling g, the dynamics in the throat surrounding the fivebrane reduces to the CGHS model with massive fields. We find that the energy density leaks off the brane in the form of Hawking radiation at a rate of order $\frac{1}{k^{7/2}}$ in string units independently of g to leading order. In the $g \to 0$ limit the radiation persists but never reaches asymptotic infinity because the throat becomes infinitely long.

Then: Juan's paper on AdS/CFT.

OK, back to the talk!

RG flows:

“# d.o.f.”

UV CFT (+relevant)

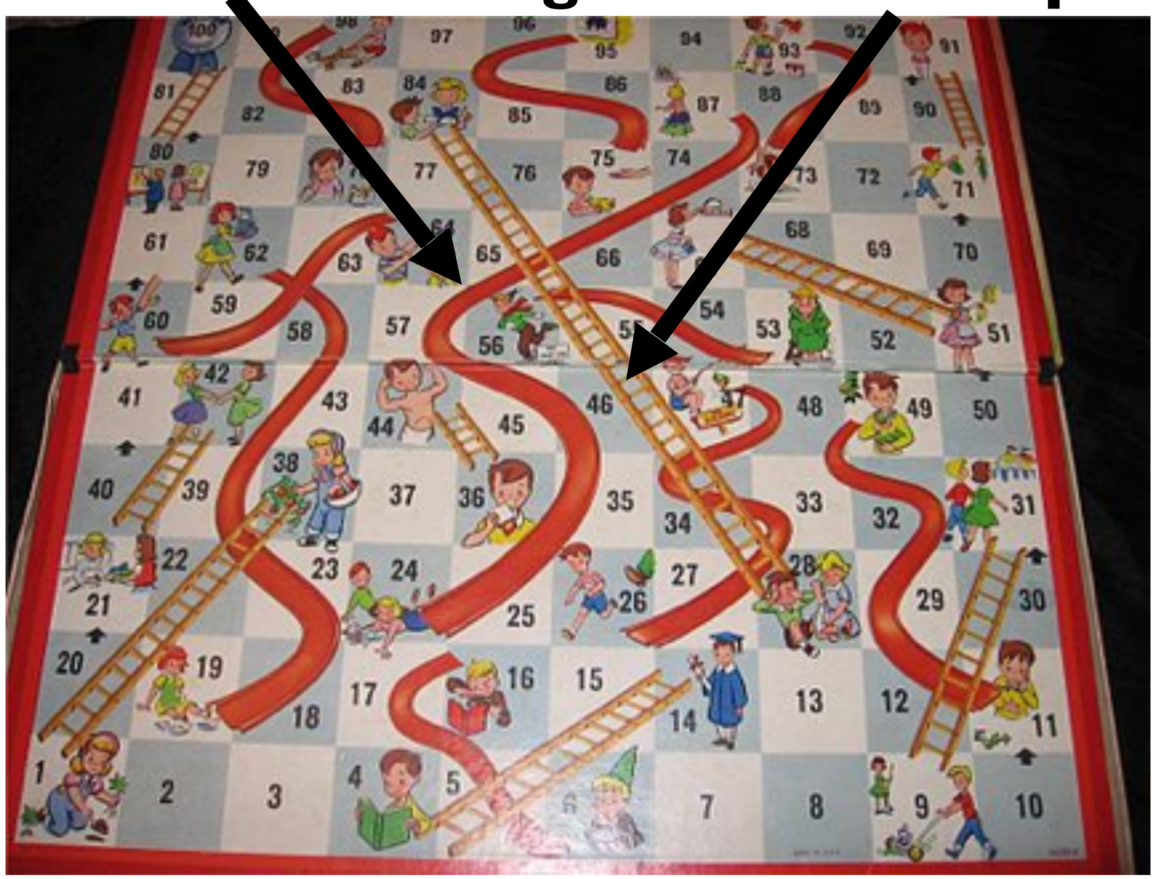
“chutes”

course graining

IR CFT (+irrelevant)

“ladders”

E.g. Higgs mass **E.g. dim 6 BSM ops**



We often have to guess where it goes, check using a-thm, symmetry & anom. matching, indices, etc., Interconnected web: Seiberg duality, many generalizations + interconnections+checks.

Classification of SCFT algebras= super-algebras:

Nahm '78

$d > 6$	no SCFTs can exist	
$d = 6$	$OSp(6,2\|\mathcal{N}) \supset SO(6,2) \times Sp(\mathcal{N})_R$	$(\mathcal{N},0)$ $8\mathcal{N}Qs$
$d = 5$	$F(4) \supset SO(5,2) \times Sp(1)_R$	8Qs
$d = 4$	$Su(2,2\|\mathcal{N} \neq 4) \supset SO(4,2) \times SU(\mathcal{N})_R \times U(1)_R$	$4\mathcal{N}Qs$
$d = 4$	$PSU(2,2\|\mathcal{N} = 4) \supset SO(4,2) \times SU(4)_R$	
$d = 3$	$OSp(4\|\mathcal{N}) \supset SO(3,2) \times SO(\mathcal{N})_R$	$2\mathcal{N}Qs$
$d = 2$	$OSp(2\|\mathcal{N}_L) \times OSp(2\|\mathcal{N}_R)$	$\mathcal{N}_L Qs + \mathcal{N}_R \bar{Q}s$

Unitary SCFTs: operators in unitary reps of the s-algs

Dobrev and Petkova PLB '85 for 4d case.
Shiraz Minwalla's excellent paper '97 for all d=3,4,5,6.

$P_\mu \uparrow \quad K_\mu \downarrow$

Q $\uparrow$ S $\downarrow$

$\{Q,Q\} = 2P_\mu$

$\{S,S\} = 2K_\mu$

descendants

$Q^\ell(\mathcal{O}_\mathcal{R})$

$\mathcal{O}_\mathcal{R}$ super-primary $S(\mathcal{O}_\mathcal{R}) = 0$

$\left|P_{\mu_1}\ldots P_{\mu_\ell}|\mathcal{O}_\mathcal{R}\rangle\right|^2 \overset{!}{\geq} 0$

$\left|Q_1\ldots Q_\ell|\mathcal{O}_\mathcal{R}\rangle\right|^2 \overset{!}{\geq} 0$

give CFT and SCFT unitarity bounds.

Bounds saturated for "short" multiplets. Have null ops, set to 0.

$\{Q,Q\} \sim P_\mu \sim 0$ modulo descendants. Grassmann algebra.

Level $\quad Q^{\wedge\ell}(\mathcal{O}_\mathcal{R}) \quad \ell = 0\ldots\ell_{max} \leq N_Q$

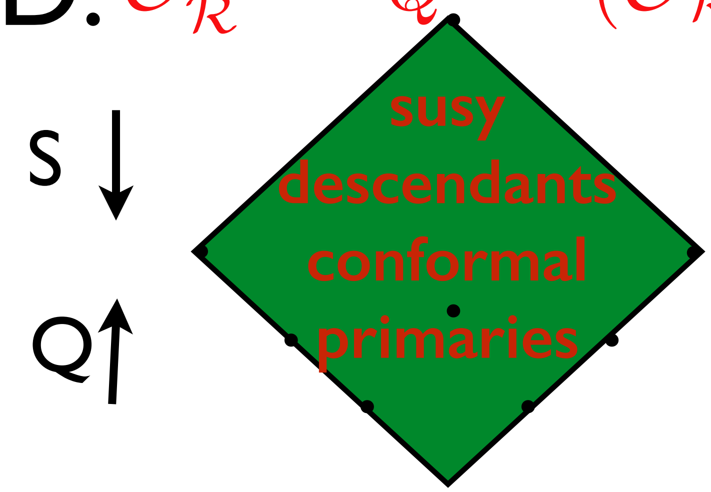
Classify SCFT multiplets and all susy deformations

$Q(\mathcal{O}_{\mathcal{R}}^{\text{top}}) \sim 0$

Cordova, Dumitrescu, KI

$\{Q, Q\} \sim P \sim 0$

D: $\mathcal{O}_{\mathcal{R}}^{\text{top}} = Q^{\wedge N_Q}(\mathcal{O}_{\mathcal{R}})$

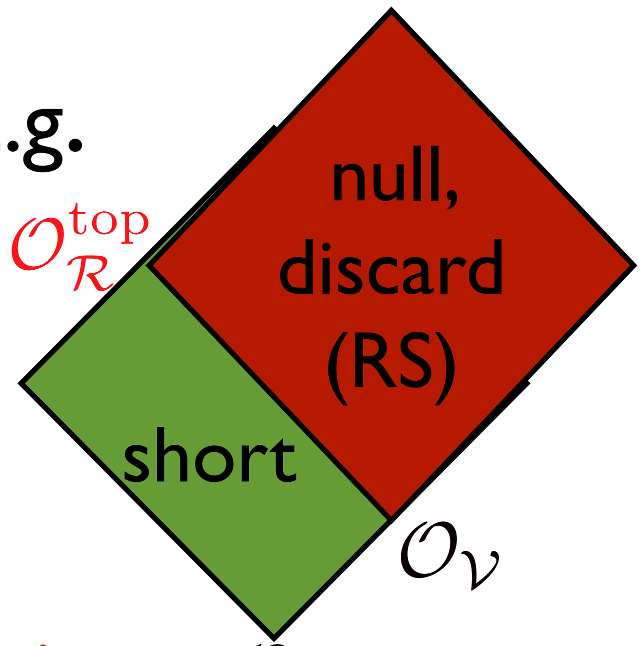


primary: $\mathcal{O}_{\mathcal{R}}$

Generic long =
"walk in the park"

E.g.

F: $\mathcal{O}_{\mathcal{R}}^{\text{top}}$



primary: $\mathcal{O}_{\mathcal{R}}$

Generic short =
"proceed with caution"

Non-Generic Short
(small R-symm quant #s)

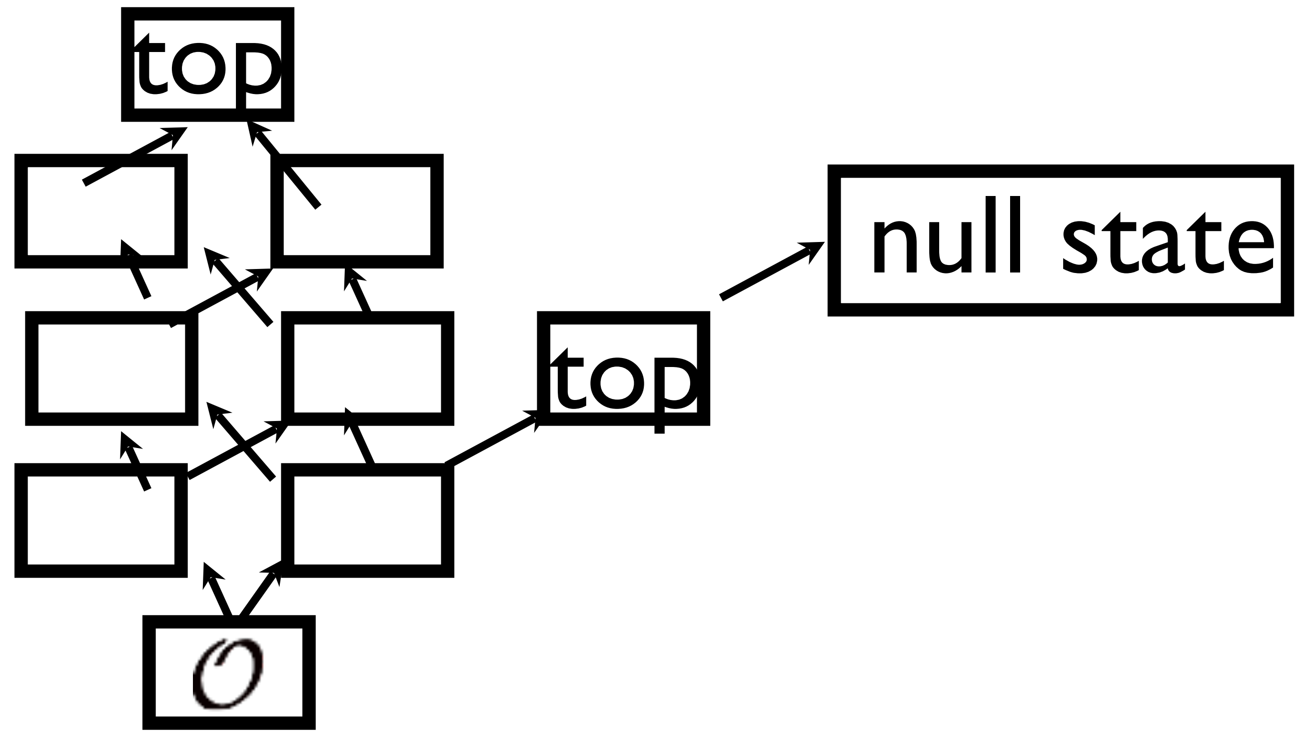
= a **ZOO** of sporadic cases.
E.g. Dolan + Osborn for some 4d N=2,4 cases.
We analyzed algorithms to eliminate only nulls; many problems. Non-trivial. We conjecture and test a general algorithm (to appear).

Unitary constraints on bottom primary and nulls.
We then find the op. dim. constraints on the top components.
As we increase d or N, fewer or none relevant deformations.

In zoo: cases (in d=3) with mid-level susy top compts

(Find two and multi-headed animals in the multiplet zoo)

Q $\uparrow$ S $\downarrow$



E.g. 3d $\mathcal{N} \geq 4$ $T_{\mu\nu}$ multiplet: the stress-tensor is at top, at level 4.

Another top, at level 2, Lorentz scalar. Gives susy-preserving "universal mass term" relevant deformations. First found in 3d N=8 (KI '98, Bena & Warner '04; Lin & Maldacena '05). Special to 3d. Indeed, they give a deformed susy algebra that is special to 3d (non-central extension).

We give a complete classification for d=3,4,5,6

Cordova, Dumitrescu, KI

Not exactly taming the SCFT zoo (or freeing the SCFTs … sorry, old reference … will be lost on many). But we give a detailed tour and full picture of the many multiplets.

We give a complete classification of all susy-preserving deformations. They can be the start (if relevant) or end (if irrelevant) of susy RG flows between SCFTs, analyzed near the UV or IR SCFT fixed points.

E.g. d=4, N=3 SCFTs (all irrelevant)

Primary $\mathcal{O}$	Deformation $\delta\mathscr{L}$	Comments
$B_1\overline{B}_1 \left\{\begin{matrix}(R_1+4,0\,;2R_1+8)\\ \Delta_{\mathcal{O}}=4+R_1\end{matrix}\right\}$	$Q^4\overline{Q}^2\mathcal{O}\in\left\{\begin{matrix}(R_1,0\,;2R_1+6)\\ \Delta=7+R_1\end{matrix}\right\}$	F-Term $(*)$
$B_1\overline{B}_1 \left\{\begin{matrix}(0,R_2+4\,;-2R_2-8)\\ \Delta_{\mathcal{O}}=4+R_2\end{matrix}\right\}$	$Q^2\overline{Q}^4\mathcal{O}\in\left\{\begin{matrix}(0,R_2\,;-2R_2-6)\\ \Delta=7+R_2\end{matrix}\right\}$	F-Term $(*)$
$B_1B_1 \left\{\begin{matrix}(R_1+2,R_2+2\,;2(R_1-R_2))\\ \Delta_{\mathcal{O}}=4+R_1+R_2\end{matrix}\right\}$	$Q^4Q^4\mathcal{O}\in\left\{\begin{matrix}(R_1,R_2\,;2(R_1-R_2))\\ \Delta=8+R_1+R_2\end{matrix}\right\}$	–
$L\overline{B}_1 \left\{\begin{matrix}(0,0\,;r+6)\ ,\ r>0\\ \Delta_{\mathcal{O}}=1+\frac{1}{6}r\end{matrix}\right\}$	$Q^6\mathcal{O}\in\left\{\begin{matrix}(0,0\,;r)\ ,\ r>0\\ \Delta=4+\frac{1}{6}r>4\end{matrix}\right\}$	F-term $(\star)$
$B_1\overline{L} \left\{\begin{matrix}(0,0\,;r-6)\ ,\ r<0\\ \Delta_{\mathcal{O}}=1-\frac{1}{6}r\end{matrix}\right\}$	$\overline{Q}^6\mathcal{O}\in\left\{\begin{matrix}(0,0\,;r)\ ,\ r<0\\ \Delta=4-\frac{1}{6}r>4\end{matrix}\right\}$	F-Term $(\star)$
$L\overline{B}_1 \left\{\begin{matrix}(R_1+2,0\,;r+4)\ ,\ r>2R_1+6\\ \Delta_{\mathcal{O}}=2+\frac{2}{3}R_1+\frac{1}{6}r\end{matrix}\right\}$	$Q^6\overline{Q}^2\mathcal{O}\in\left\{\begin{matrix}(R_1,0\,;r)\ ,\ r>2R_1+6\\ \Delta=6+\frac{2}{3}R_1+\frac{1}{6}r>7+R_1\end{matrix}\right\}$	$(\dagger)$
$B_1\overline{L} \left\{\begin{matrix}(0,R_2+2\,;r-4)\ ,\ r<-2R_2-6\\ \Delta_{\mathcal{O}}=2+\frac{2}{3}R_2-\frac{1}{6}r\end{matrix}\right\}$	$Q^2\overline{Q}^6\mathcal{O}\in\left\{\begin{matrix}(0,R_2\,;r)\ ,\ r<-2R_2-6\\ \Delta=6+\frac{2}{3}R_2-\frac{1}{6}r>7+R_2\end{matrix}\right\}$	$(\dagger)$
$L\overline{B}_1 \left\{\begin{matrix}(R_1,R_2+2\,;r+2)\ ,\ r>2(R_1-R_2)\\ \Delta_{\mathcal{O}}=3+\frac{2}{3}(R_1+2R_2)+\frac{1}{6}r\end{matrix}\right\}$	$Q^6\overline{Q}^4\mathcal{O}\in\left\{\begin{matrix}(R_1,R_2\,;r)\ ,\ r>2(R_1-R_2)\\ \Delta=8+\frac{2}{3}(R_1+2R_2)+\frac{1}{6}r>8+R_1+R_2\end{matrix}\right\}$	$(\ddagger)$
$B_1\overline{L} \left\{\begin{matrix}(R_1+2,R_2\,;r-2)\ ,\ r<2(R_1-R_2)\\ \Delta_{\mathcal{O}}=3+\frac{2}{3}(2R_1+R_2)-\frac{1}{6}r\end{matrix}\right\}$	$Q^4\overline{Q}^6\mathcal{O}\in\left\{\begin{matrix}(R_1,R_2\,;r)\ ,\ r<2(R_1-R_2)\\ \Delta=8+\frac{2}{3}(2R_1+R_2)-\frac{1}{6}r>8+R_1+R_2\end{matrix}\right\}$	$(\ddagger)$
$L\overline{L} \left\{\begin{matrix}(R_1,R_2\,;r)\\ \Delta_{\mathcal{O}}>2+\max\left\{\begin{matrix}\frac{2}{3}(2R_1+R_2)-\frac{1}{6}r\\ \frac{2}{3}(R_1+2R_2)+\frac{1}{6}r\end{matrix}\right\}\end{matrix}\right\}$	$Q^6\overline{Q}^6\mathcal{O}\in\left\{\begin{matrix}(R_1,R_2\,;r)\\ \Delta>8+\max\left\{\begin{matrix}\frac{2}{3}(2R_1+R_2)-\frac{1}{6}r\\ \frac{2}{3}(R_1+2R_2)+\frac{1}{6}r\end{matrix}\right\}\end{matrix}\right\}$	D-Term

Table 25: Deformations of four-dimensional $\mathcal{N}=3$ SCFTs. The $\mathfrak{su}(3)_R$ Dynkin labels $R_1, R_2\in\mathbb{Z}_{\geq 0}$ and the $\mathfrak{u}(1)_R$ charge $r\in\mathbb{R}$ denote the R-symmetry representation of the deformation.

6d SCFTs and QFTs

- We show there are no susy relevant operator deformations of 6d SCFTs only via going out along the moduli space. Then theory flows to new low-energy thy +(irrelevant ops).
- Spontaneous conformal breaking: low-energy theory contains the massless dilaton w/ irrelevant interactions.
- Global symmetries have 6d analog of 't Hooft anomaly matching conditions. The 't Hooft anomalies can often be exactly computed, e.g. by inflow or other tricks.
- Anomaly matching for broken symmetries (via NG bosons) requires certain interactions in the low energy thy, like the WZW term but totally different in the details for 6d vs 4d.
- 6d a-theorem? Unclear w/o susy. We proved it for susy flows on the Coulomb branch. Higgs branch is to appear.

Anomaly polynomial

Alvarez-Gaume, Witten; Alvarez-Gaume+Ginsparg.

$$\mathcal{I}_{d+2} = \mathcal{I}_{d+2}^{\text{gauge}} + \mathcal{I}_{d+2}^{\text{gravity+global}} + \mathcal{I}_{d+2}^{\text{mixed}}$$

Must cancel, restricts G & matter. Nati's paper!

``'t Hooft anomalies'' Const. on RG flows. Matching. Useful!

We'll discuss these too. Seem to have been mostly neglected to date.

E.g. 6d: N=(2,0):

$$\mathcal{I}_g = r_g \mathcal{I}_{u(1)} + \frac{k_g}{24} p_2(F_{SO(5)_R})$$

A,D,E group G

Free (2,0) tensor mult

Interaction part

Duff, Liu, Minasian; Witten; Freed, Harvey, Minasian, Moore

N M5s+inflow: $k_{su(N)} = N^3 - N$ Harvey, Minasian, Moore

Other methods: $k_g = h_g^\vee d_g$ KI; Yi; Ohmori, Shimizu, Tachikawa, Yonekura. See also Ki-Myeong Lee et. al.

Exact info about mysterious SCFTs (and "L"STs).

Longstanding hunch

e.g. Harvey Minasian, Moore '98

Supersymmetric multiplet of anomalies: should be able to relate conformal anomaly a to 't Hooft-type anomalies for the superconformal R-symmetry in 6d, as in 2d and 4d.

$T^{\mu\nu} \leftrightarrow J_R^{\mu,a}$ Stress-tensor supermultiplet

$g_{\mu\nu} \leftrightarrow A^a_{R,\mu}$ Source: bkgrd SUGRA supermultiplet

$T^{\mu\nu}$ $T^{\rho\sigma}$

$\boxed{\text{a?}}$

$T^{\kappa\lambda}$ $T^{\zeta\psi}$

susy? $\longleftrightarrow$

$J^{\mu,a}, T^{\mu\nu}$ $J^{\rho,a}, T^{\rho\sigma}$

$\boxed{I_8}$

$J^{\kappa,a}, T^{\kappa\lambda}$ $J^{\zeta,a}, T^{\zeta\psi}$

4-point fn with too many indices. Hard to get a (and to compute).

Easier to isolate anomaly term, and enjoys anomaly matching

On the moduli space

Need to supersymmetrize the dilaton LEEFT

Spontaneous conf'l symm breaking: dilaton has derivative interactions to give Δa anom matching Schwimmer, Theisen; Komargodski, Schwimmer

6d case: $$\mathcal{L}_{\text{dilaton}} = \frac{1}{2}(\partial\varphi)^2 - b\frac{(\partial\varphi)^4}{\varphi^3} + \Delta a\frac{(\partial\varphi)^6}{\varphi^6}$$ (schematic; derivative order shown)

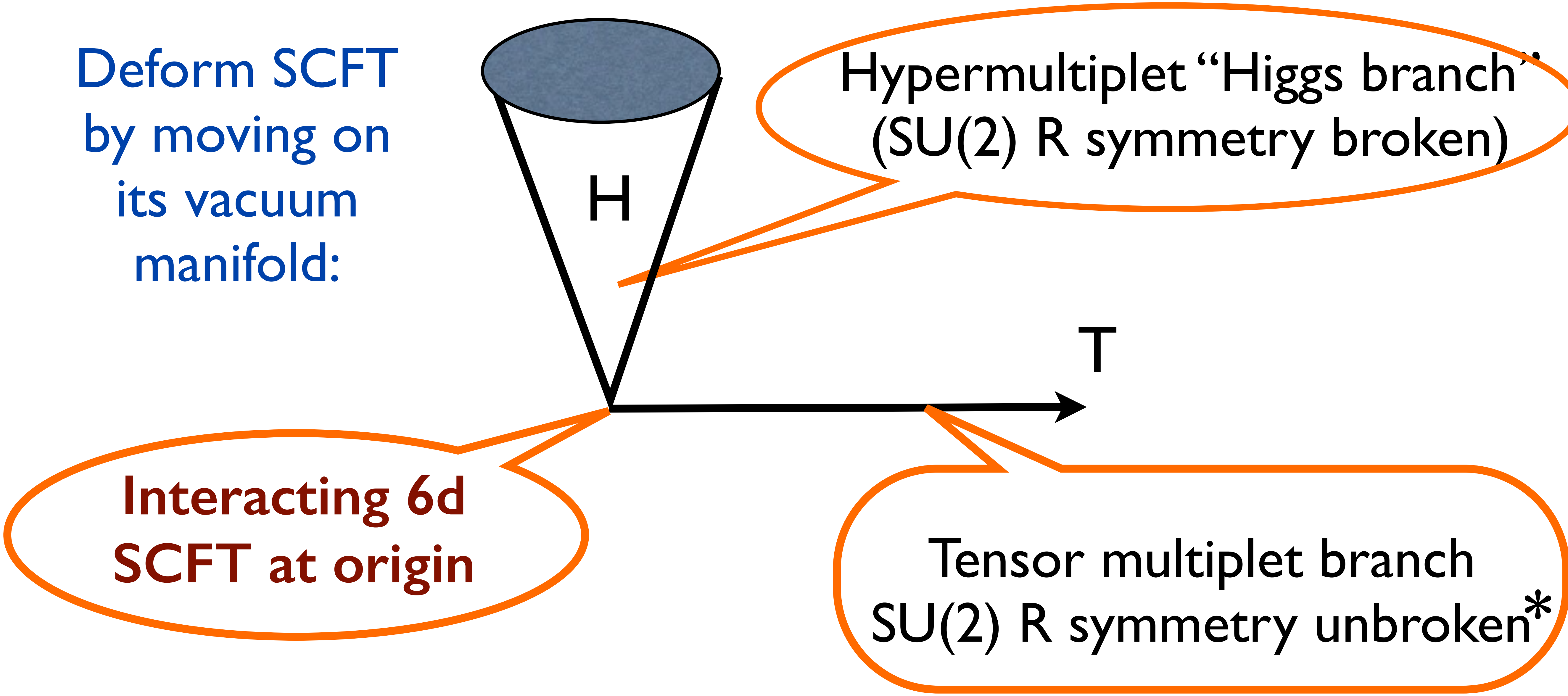
Maxfield, Sethi; Elvang, Freedman, Hung, Kiermaier, Myers, Theisen. b>0, but what is it good for? Interpretation? Clue: noticed that for N=(2,0) on Coulomb branch:

$$\Delta a \sim b^2$$

M&S: via (2,0) susy; EFHKMT: some scatt. amplitudes then, fits with AdS/CFT.

Cordova, Dumitrescu, Yin: proved it using (2,0) methods. Our parallel work proves for general (1,0) theories on Coulomb branch.

6d (1,0) susy moduli



* Easier case. Just dilaton, no NG bosons. Dilaton = tensor multiplet.

(1,0) 't Hooft anomalies

$$\mathcal{I}_8^{\text{gravity+global}} \supset \alpha c_2(R)^2 + \beta c_2(R) p_1(T) + \gamma p_1^2(T) + \delta p_2(T)$$

Exactly computed for many (1,0) SCFTs

Ohmori, Shimizu, Tachikawa; & +Yonekura (OST, OSTY)

E.g. for theory of N small E8 instantons:

$$\mathcal{E}_N : \ (\alpha, \beta, \gamma, \delta) = (N(N^2+6N+3), -\frac{N}{2}(6N+5), \frac{7}{8}N, -\frac{N}{2})$$

(Aside: "E-string theory" = confusing name: both QFT and "L"ST versions.)

N=(1,0) tensor branch anomaly matching:

KI; Ohmori, Shimizu, Tachikawa, Yonekura

$$\Delta \mathcal{I}_8 \equiv \mathcal{I}_8^{\text{origin}} - \mathcal{I}_8^{\text{tensor branch}} \sim X_4 \wedge X_4$$

must be a perfect square, match I_8 via X_4 sourcing B:

$$\mathcal{L}_{GSWS} = -iB \wedge X_4$$

$$X_4 \equiv 16\pi^2 (x c_2(R) + y p_1(T))$$

x, y = integer coefficients

6d (1,0) tensor LEEFT

Cordova, Dumitrescu, KI

$$\mathcal{L}_{\text{dilaton}} = \frac{1}{2}(\partial\varphi)^2 - b\frac{(\partial\varphi)^4}{\varphi^3} + \Delta a\frac{(\partial\varphi)^6}{\varphi^6}$$

Our deformation classification implies that b=D-term and

$$\Delta a = \frac{98304\pi^3}{7}b^2 > 0$$

Proves the 6d a-theorem for susy tensor branch flows.

b-term susy-completes to terms in $X_4 = \sqrt{\Delta I_8}$

b=(y-x)/2

$$X_4 \equiv 16\pi^2(xc_2(R) + yp_1(T))$$

By recycling a 6d SUGRA analysis from Bergshoeff, Salam, Sezgin '86 (!).

Upshot: $\boxed{a^{\text{origin}} = \frac{16}{7}(\alpha - \beta + \gamma) + \frac{6}{7}\delta}$

So exact 't Hooft anomaly coefficients give the exact conformal anomaly, useful! E.g. using this and **OST** for the anomalies:

$$a(\mathcal{E}_N) = \frac{64}{7}N^3 + \frac{144}{7}N^2 + \frac{99}{7}N$$

(N M5s @ M9 Horava-Witten wall.)

Nati's 6d QFTs, & similar

E.g. consider: 6d, N=(1,0) gauge + matter hypers + tensor theory. Must cancel the irreducible gauge anomaly (for groups with quartic Casimir) by choice of the matter. Can eliminate the remaining, reducible gauge anomaly by a GSWS-type anomaly cancellation, using the tensor multiplet (if the reducible anom coeff has correct sign). Gauge thy instantons are strings, charged under the B-field.

$$\mathcal{L} \sim -\sqrt{c}(\phi F^2 + B \wedge F \wedge F + \dots)$$

NB: this is *not* a standard Lagrangian deformation of free fields by a relevant operator. Coefficient is quantized. Gauge theory without these couplings flows instead to an IR-free, not conformally invariant theory.

a-theorem, and sign, for theories with gauge fields

A free gauge field not conformal for d>4. It is unitary, but it can be regarded as a subsector of a non-unitary CFT.
El-Showk, Nakayama, Rychkov

Applying our formula to a free (1,0) vector multiplet gives

$$\boxed{a^{\text{origin}} = \frac{16}{7}(\alpha - \beta + \gamma) + \frac{6}{7}\delta} \rightarrow \quad a(\text{vector}) = -\frac{251}{210}$$

negative…same value later found for non-unitary, higher derivative (1,0) SCFT version by Beccaria & Tseytlin

We claim that unitary SCFTs satisfy the a-theorem and have a>0 even if they have vector multiplets, as in Nati's theories and generalizations. First, we noticed and resolved a small puzzle…

Mixed Anomalies

$$\mathcal{I}_{d+2} = \mathcal{I}_{d+2}^{\text{gauge}} + \mathcal{I}_{d+2}^{\text{gravity+global}} + \mathcal{I}_{d+2}^{\text{mixed}}$$

Must cancel, restricts G & matter.

``'t Hooft anomalies''

Mostly neglected to date. Can be non-zero in gen'l.

Consider e.g. 4d: $\mathcal{I}_6^{\text{mixed}} \sim c_1(F_1)c_2(F_2)$

E.g. non-zero for QED, $F_1 = U(1)_{\text{gauge}}$ $F_2 = SU(N_f)_L$

For 6d, e.g.: $\mathcal{I}_8^{\text{mixed}} \sim c_2(F_{\text{gauge}})(p_1(T) + c_2(F_{\text{global}}))$

Non-zero e.g. for Sp(N) gauge theory [Witten] of N small SO(32) instantons = 6d (1,0) non-CFT (IR free), UV completes to (L/N)ST. Mixed anomaly is non-zero. Background gauge field instanton configs are thus not gauge invariant. $\partial_\mu j_1^\mu \sim \star(F_1 \wedge F_2 \wedge F_2)$

No Mixed Anomalies for 6d SCFTs

For gauge + tensor theories, the tensor multiplet is already used to cancel the reducible gauge anomalies. It could also be used to cancel the mixed anomalies. Should this be done, and why? Puzzle.

The excellent paper of **Ohmori, Shimizu, Tachikawa, Yonekura** states, but without justification, that **all** gauge anomalies must be cancelled, including mixed ones. The cancellation mechanism then affects their 't Hooft anomalies, upon completing the square.

We **prove** it by our SCFT operator multiplet classification: no such multiplet for a conserved 2-form current operator in the mixed anomaly eqn. $\partial_\mu j_1^\mu \sim F_1^{\mu\nu} \mathcal{O}_{\mu\nu}$ so no mixed anomaly. The non-zero mixed anomaly from the gauge and matter fields must be cancelled by tensor! We note that this is crucial to ensure positivity of a, and the 6d a-thm for these SCFTs.

a, for 6d SCFTs with gauge flds:

E.g. SU(N) gauge group, 2N flavors, 1 tensor + anomaly cancellation for reducible gauge + mixed gauge + R-symmetry anomalies. Use $a^{\text{origin}} = \frac{16}{7}(\alpha - \beta + \gamma) + \frac{6}{7}\delta$

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$$a_{SCFT} = (N^2-1)(-\frac{251}{210}) + 2N^2(\frac{11}{210}) + \frac{199}{210} + \frac{96}{7}N^2 > 0.$$

Can likewise verify that other generalizations have positive a. Also show (to appear) that Higgs branch flows satisfy the 6d a-theorem.

Conclude

- QFT is vast, full of interconnections. SUSY QFTs, SCFTs, and RG flows continue to be rich, testing grounds for exploring QFT.
- Here’s to many years of Nati’s path-leading and inspirational contributions, past and future.
- Here’s to many years of friendship.
- Happy 60 (“the new 40”), Nati!