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Globally consistent three-family Standard Models in F-theory

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Outline

I. F-theory: key ingredients

(non-)Abelian gauge symmetries, matter & Yukawa couplings

[Recent developments: global constraints on gauge symmetry → implications for F-theory ``swampland''] Brief

II. Particle physics model building in F-theory:

Building blocks (via toric techniques)

First globally consistent three family Standard Models

III. Landscape of three family Standard Models:

Globally consistent models via toric techniques Highlight

IV. Outlook: work in progress & open issues

I. Global constrains on gauge symmetry in F-theory

M.C. and Ling Lin, ``The Global Gauge Group Structure of F-theory Compactification with U(1)s,'' JHEP, arXiv:1706.08521 [hep-th]

Review:

M.C. and Ling Lin, arXiv:1809.00012[hep-th], "TASI Lectures on Abelian and Discrete Symmetries in F-theory"

II. First three - family Standard Models:

M.C., Denis Klevers, Damián Kaloni Mayorga Peña, Paul-Konstantin Oehlmann, Jonas Reuter, ``Three-Family Particle Physics Models from Global F-theory Compactifications,'' JHEP, arXiv:1503.02068 [hep-th].

with Z_2 matter parity:

M.C., Ling Lin, Muyang Liu and Paul-Konstantin Oehlmann, ``An F-theory Realization of the Chiral MSSM with Z_2-Parity'', JHEP, arXiv:1807.01320 [hep-th]

III. Landscape of three-family Standard Models

M.C., James Halverson, Ling Lin, Muyang Liu and Jiahua Tian, ``A Quadrillion Standard Models from F-theory,'' arXiv:1903.00009[hep-th]

I. F-theory basic ingredients

Type IIB string perspective

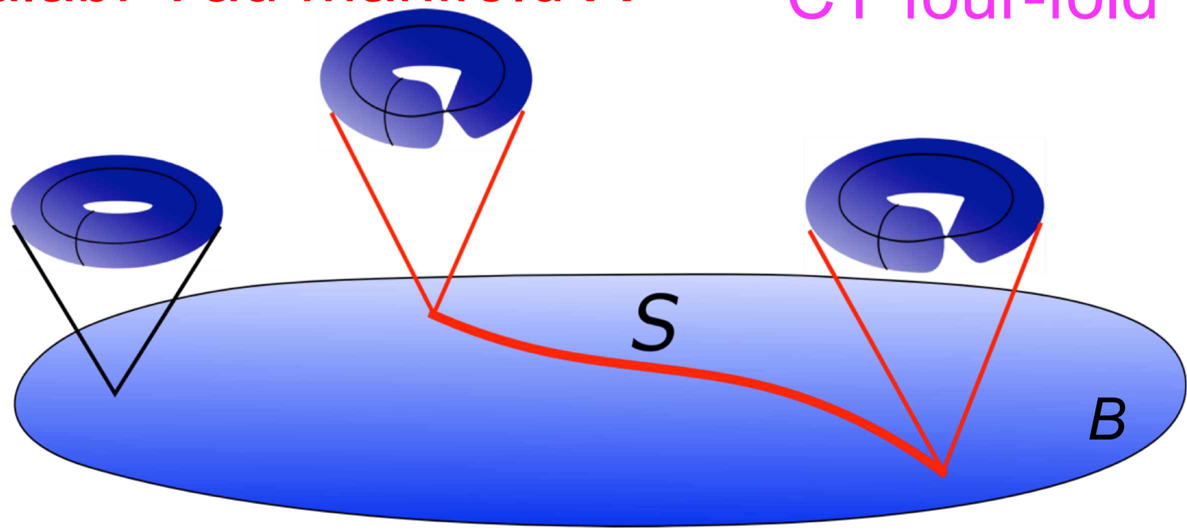
F-theory compactification to 4D

[Vafa'96], [Morrison,Vafa'96],...
c.f., review [Weigand 1806.01854]

Singular elliptically fibered Calabi-Yau manifold X CY four-fold

Modular parameter of two-torus
(elliptic curve)
[SL(2,Z) of Type IIB]

$\tau \equiv C_0 + i g_s^{-1}$



Weierstrass normal form for elliptic fibration of X

$$y^2 = x^3 + f x z^4 + g z^6 \qquad (x, y, z) \simeq (\lambda^2 x, \lambda^3 y, \lambda z)$$

[z:x:y] - homogeneous coordinates on $\mathbb{P}^2_{[2,3,1]}$

Calabi-Yau conditions: B-3D Kähler

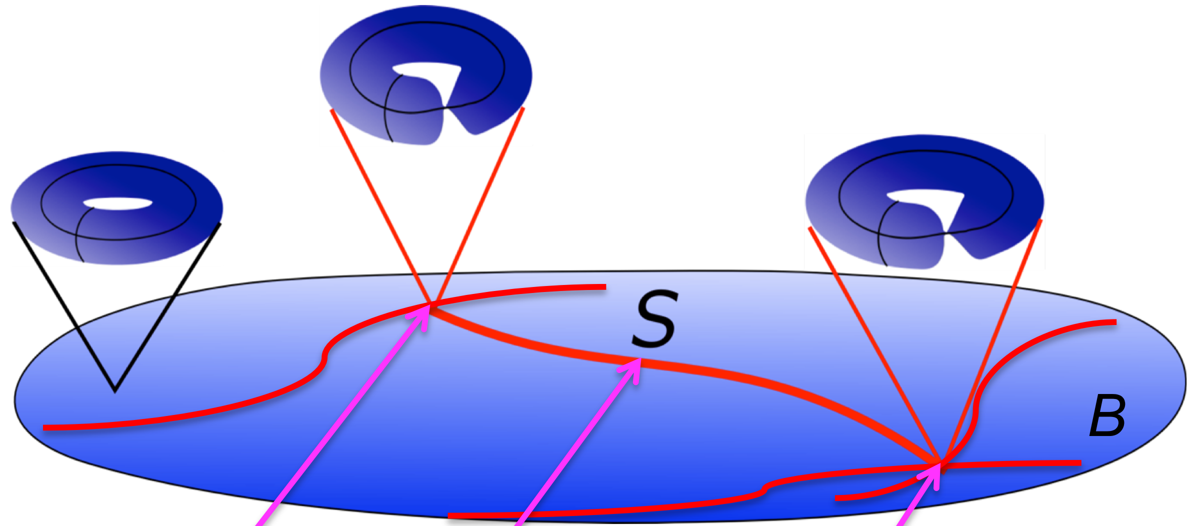
f, g - sections of $\overline{\mathcal{K}}_B^6$ and $\overline{\mathcal{K}}_B^4$ on ***B***

[$\overline{\mathcal{K}}_B$ – anti-canonical bundle on ***B***]

[x:y:z]- sections of specific line-bundles on ***B***

F-theory compactification

Singular torus fibered Calabi-Yau manifold X



Modular parameter of two-torus
(elliptic curve)

$$\tau \equiv C_0 + i g_s^{-1}$$

Weierstrass normal form for elliptic fibration of *X*

$$y^2 = x^3 + f x z^4 + g z^6$$

Matter
(co-dim 2; chirality- G_4-flux)

Yukawa couplings
(co-dim 3)

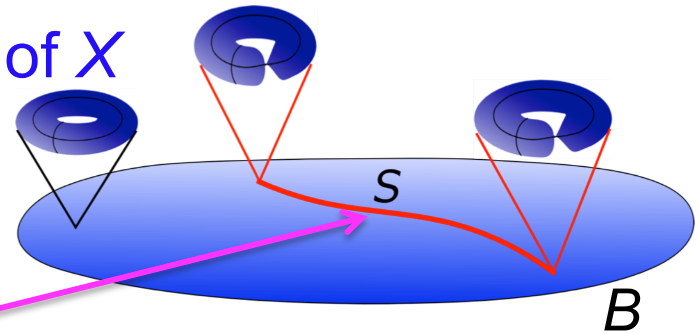
divisor- singular elliptic-fibration, $g_s \rightarrow \infty$
location of (p,q) 7-branes

non-Abelian gauge symmetry
(co-dim 1) – ADE singularities

Non-Abelian Gauge Symmetry

[Kodaira],[Tate], [Vafa], [Morrison,Vafa],...[Esole,Yau],
[Hayashi,Lawrie,Schäfer-Nameki],[Morrison], ...

- Weierstrass normal form for elliptic fibration of *X*



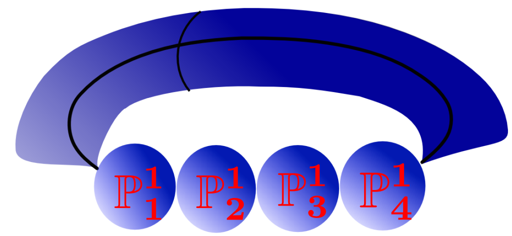
$$y^2 = x^3 + fxz^4 + gz^6$$

- Severity of singularity along divisor S in B

  specified by [$ord_S(f)$,$ord_S(g)$,$ord_S(\Delta)$]

- Resolution: structure of a tree of $\mathbb{P}^1$'s over *S*

  Resolved I_n-singularity ←→ SU(n) Dynkin diagram



n=5

Cartan gauge bosons: supported by (1,1) form $\omega_i \leftrightarrow \mathbb{P}^1_i$ on resolved X

(via M-theory Kaluza-Klein reduction of C_3 potential $C_3 \supset A^i \omega_i$)

Non-Abelian gauge bosons: light M2-brane excitations on $\mathbb{P}^1$'s [Witten]

Deformation: [Grassi, Halverson, Shaneson'14-'15]

U(1)'s- Abelian Gauge Symmetry & Mordell-Weil group

rational sections of elliptic fibr. ⟺ rational points of elliptic curve

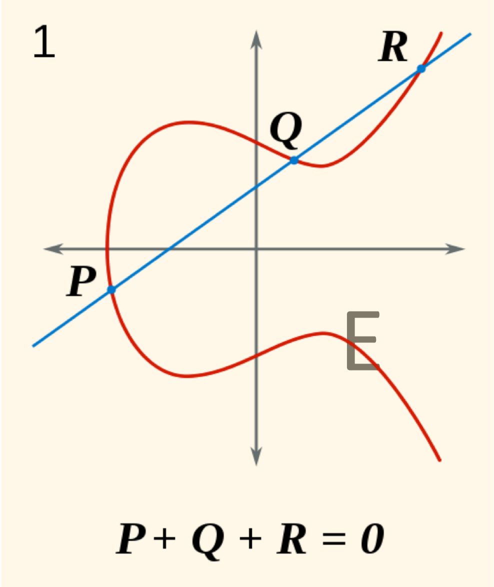
Rational point Q on elliptic curve E with zero point P

- is solution (x_Q, y_Q, z_Q) in field K of Weierstrass form

$$y^2 = x^3 + fxz^4 + gz^6$$

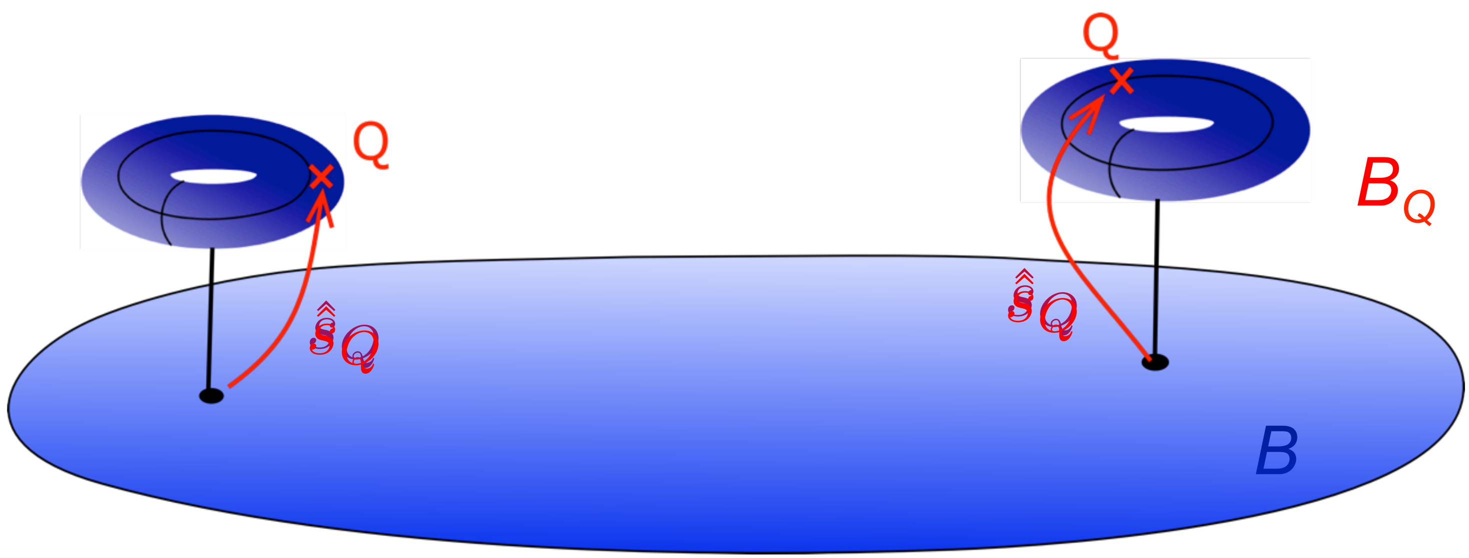
- Rational points form group (addition) on E

➡ Mordell-Weil group of rational points



U(1)'s-Abelian Symmetry &Mordell-Weil Group

Point Q induces a rational section $\hat{s}_Q : B \to X$ of elliptic fibration

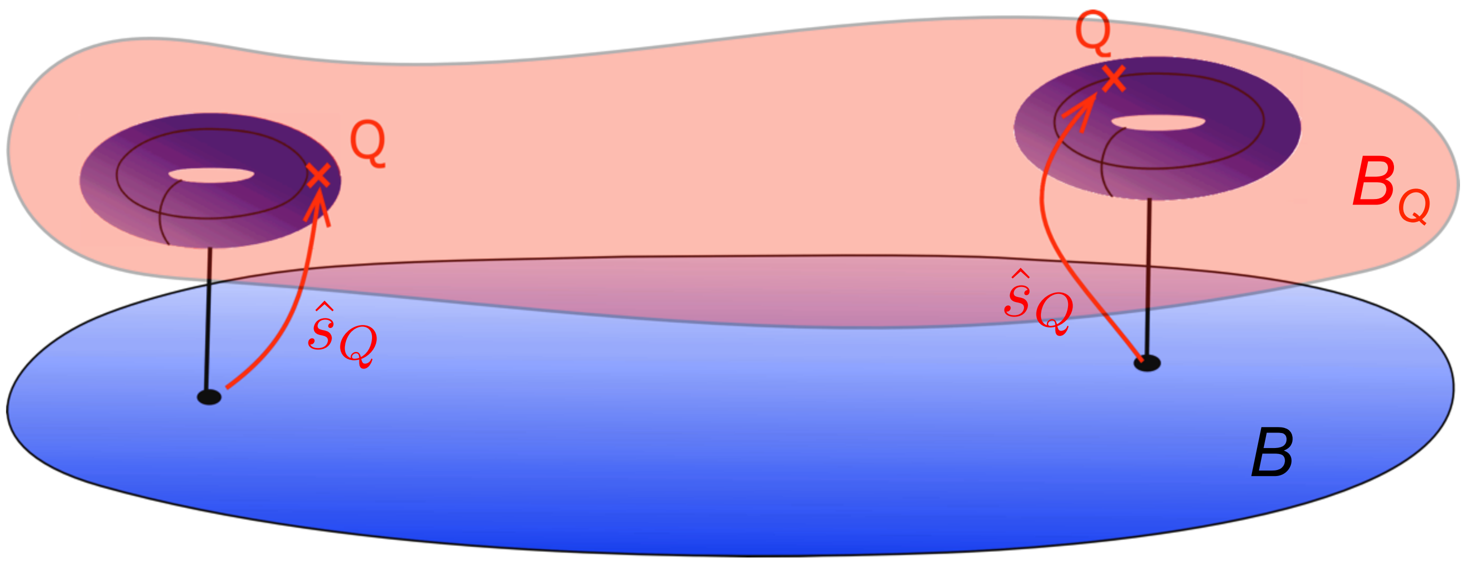


➡ $\hat{s}_Q$ gives rise to a second copy of *B* in *X*:

new divisor B_Q in *X*

U(1)'s-Abelian Symmetry &Mordell-Weil Group

Point Q E induces a rational section $\hat{s}_Q : B \to X$ of elliptic fibration



➡ $\hat{s}_Q$ gives rise to a second copy of B in X:

new divisor B_Q in X

Shioda map of $\hat{s}_Q$, complementary to $\mathcal{B}_{\mathcal{P}}$ - zero section & $\mathcal{E}_i$ - Cartan divisors:

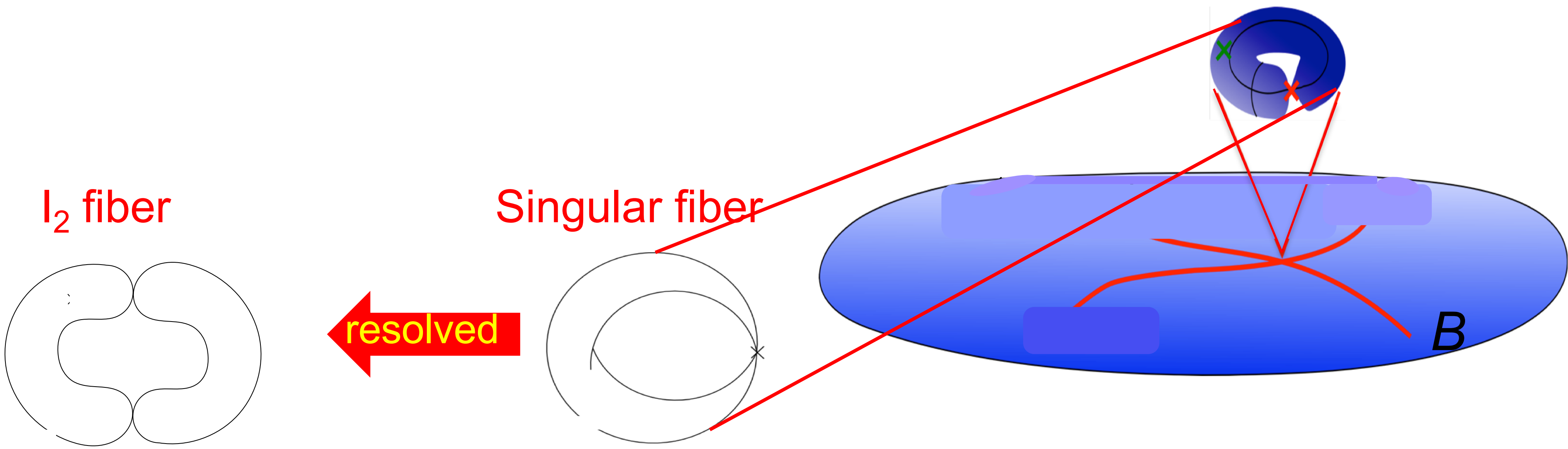
$$\sigma\left(\hat{s}_Q\right) = B_Q - B_P - \sum_i l_i E_i + \cdots$$

[M.C., Lin, 1706.08521] – a bit more later

Implications for global constraints on gauge symmetry & F-theory ``swampland''

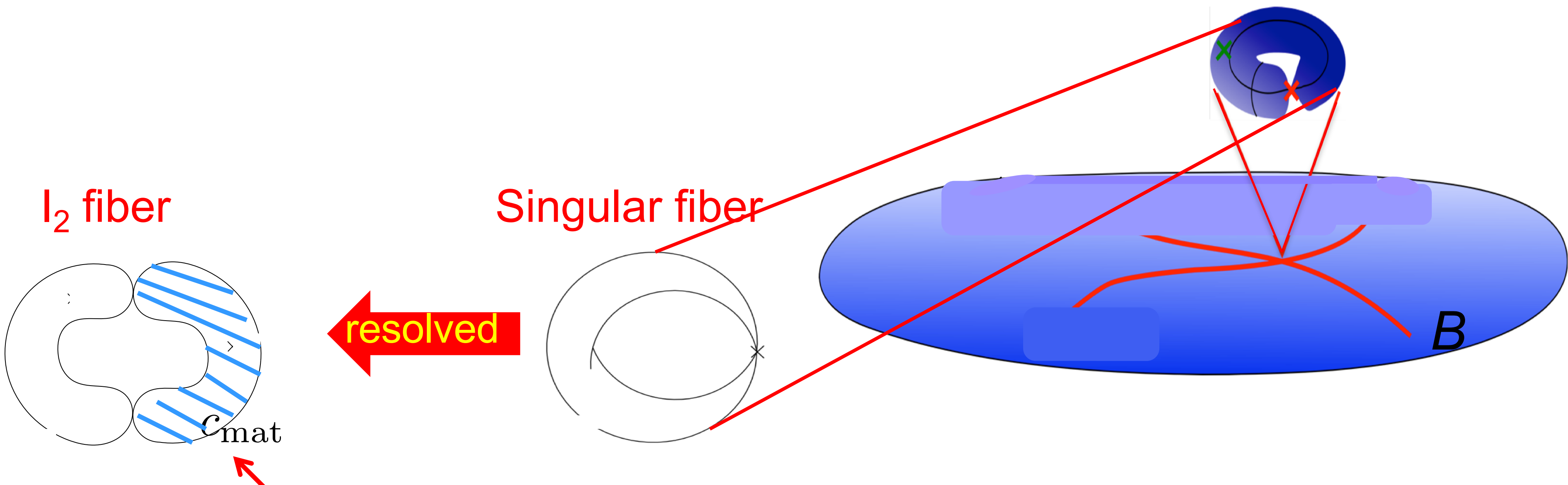
Matter

Singularity at codimension-two in *B*:



Matter

Singularity at codimension-two in B:



w/isolated (M2-matter) curve wrapping $\mathbb{P}^1$ → charged matter

(determine rep. via intersection theory)

III. Particle physics in F-theory

Globally consistent models

Initial focus: F-theory with SU(5) grand unification

[10 10 5 coupling,...] [Donagi,Wijnholt'08][Beasley,Heckman,Vafa'08]...

Model Constructions:

Local [Donagi,Wijnholt'09-10]...[Marsano,Schäfer-Nameki,Saulina'09-11]...

Review: [Heckman]

Global

[Blumehagen,Grimm,Jurke,Weigand'09][M.C., Garcia-Etxebarria,Halverson'10]...
[Marsano,Schäfer-Nameki'11-12]...[Clemens,Marsano,Pantev,Raby,Tseng '12]...

Also SO(10) …[Buchmüller, Dierigl, Oehlmann, Rühle'17]

Other particle physics models:

Standard Model building blocks (via toric techniques)

[Lin,Weigand'14] SM x U(1) [1604.04292]

First global 3-family Standard, Pati-Salam, Trinification models
[M.C., Klevers, Peña, Oehlmann, Reuter, 1503.02068]

Global 3-family Standard Model with Z_2 matter parity
[M.C., Lin, Liu, Oehlmann, 1807.01320]

Construction of elliptically fibered Calabi-Yau manifold

i. Elliptic curve E

Examples of constructions via toric techniques:

E_{F_i} as a Calabi-Yau hypersurface in the two-dimensional toric variety $\mathbb{P}_{F_i}$,
[generalized weighted projective spaces, associated with16 reflexive polytopes F_i]:

c.f., [Klevers, Pena, Oehlmann, Piragua, Reuter '14]

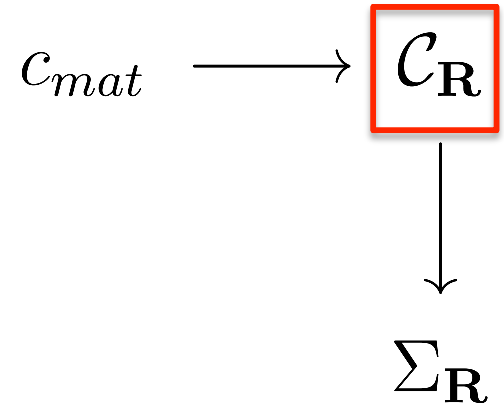
$$E_{F_i} = \{p_{F_i} = 0\} \text{ in } \mathbb{P}_{F_i}$$

ii. Elliptically fibered Calabi-Yau space: X_{F_i}

Impose Calabi-Yau condition:
coordinates in $\mathbb{P}_{F_i}$ and coeffs. of E_{F_i} lifted to
sections of specific line-bundles on B

$$E_{F_i} \subset \mathbb{P}_{F_i} \longrightarrow X_{F_i}$$
$$\downarrow$$
$$B$$

iii. Chiral index for D=4 matter:



$$\chi(\mathbf{R}) = \int_{\mathcal{C}_{\mathbf{R}}} G_4$$

a) construct G_4 flux by computing $H_V^{(2,2)}(\hat{X})$
[so-called vertical fluxes – do not induce Gukov-Vafa-Witten potential]

b) determine matter surface $\mathcal{C}_{\mathbf{R}}$ (via resultant techniques)

iv. Global consistency – D3 tadpole cancellation:

$$\frac{\chi(X)}{24} = n_{\mathrm{D3}} + \frac{1}{2}\int_X G_4 \wedge G_4$$

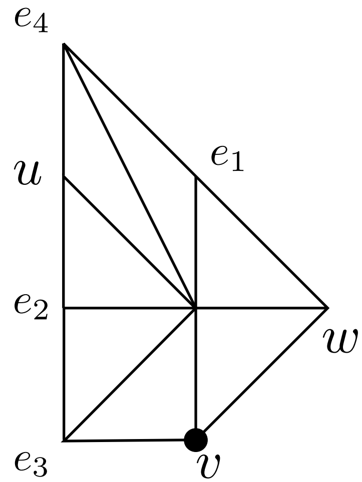
a) satisfied for integer and positive n_{D3}

b) constraint on integer valued flux G_4 $G_4 + \frac{1}{2}c_2(X) \in H^4(\mathbb{Z}, \hat{X})$

Standard Model

[M.C., Klevers, Peña, Oehlmann, Reuter,1503.02068]

F_{11} polytope



P_{F11}

$\mathbb{P}^2$[u:v:w] with four non-generic blow-ups [e_1:e_2:e_3:e_4]

Elliptic curve:

$$p_{F_{11}} = s_1 e_1^2 e_2^2 e_3 e_4^4 u^3 + s_2 e_1 e_2^2 e_3^2 e_4^2 u^2 v + s_3 e_2^2 e_3^2 u v^2 + s_5 e_1^2 e_2 e_4^3 u^2 w + s_6 e_1 e_2 e_3 e_4 u v w + s_9 e_1 v w^2$$

hypersurface constraint in P_{F11}

Construction of Calabi-Yau four-fold

Construction of Calabi-Yau four-fold

$$E_{F_{11}} \subset \mathbb{P}_{F_{11}} \longrightarrow X_{F_{11}} \downarrow B$$

Coordinates and s_i → sections of line-bundles of the base B

[Toric techniques via Stanley-Reisner ideal] →

Section	Line Bundle
u	$\mathcal{O}(H - E_1 - E_2 - E_4 + \mathcal{S}_9 + [K_B])$
v	$\mathcal{O}(H - E_2 - E_3 + \mathcal{S}_9 - \mathcal{S}_7)$
w	$\mathcal{O}(H - E_1)$
e_1	$\mathcal{O}(E_1 - E_4)$
e_2	$\mathcal{O}(E_2 - E_3)$
e_3	$\mathcal{O}(E_3)$
e_4	$\mathcal{O}(E_4)$

section	Line Bundle
s_1	$\mathcal{O}_B(3[K_B^{-1}] - \mathcal{S}_7 - \mathcal{S}_9)$
s_2	$\mathcal{O}_B(2[K_B^{-1}] - \mathcal{S}_9)$
s_3	$\mathcal{O}_B([K_B^{-1}] + \mathcal{S}_7 - \mathcal{S}_9)$
s_4	$\mathcal{O}_B(2\mathcal{S}_7 - \mathcal{S}_9)$
s_5	$\mathcal{O}_B(2[K_B^{-1}] - \mathcal{S}_7)$
s_6	K_B^{-1}
s_7	$\mathcal{O}_B(\mathcal{S}_7)$
s_8	$\mathcal{O}_B([K_B^{-1}] + \mathcal{S}_9 - \mathcal{S}_7)$
s_9	$\mathcal{O}_B(\mathcal{S}_9)$
s_{10}	$\mathcal{O}_B(2\mathcal{S}_9 - \mathcal{S}_7)$

H - hyperplane divisor;

K_B^{-1} -anti-canonical divisor $\overline{\mathcal{K}}$

Fibration depends only on additional S_7 and S_9 divisor classes

Construction of Calabi-Yau four-fold → Divisors

$$E_{F_{11}} \subset \mathbb{P}_{F_{11}} \longrightarrow X_{F_{11}} \longrightarrow B$$

Over the locus $s_3 = 0$ → fiber degenerates to I_2 - fiber → SU(2)

Over the locus $s_9 = 0$ → fiber degenerates to I_3 – fiber → SU(3)

Cartan divisors of these gauge groups:

$$E_1^{SU(2)} = [e_1]\,, \quad E_1^{SU(3)} = [e_2] \quad E_2^{SU(3)} = [u]$$

Two rational sections:

$[u : v : w : e_1 : e_2 : e_3 : e_4]$

$\hat{s}_0 = X_{F_{11}} \cap \{v = 0\}$: $[1 : 0 : s_1 : 1 : 1 : -s_5 : 1]$ - zero section

$\hat{s}_1 = X_{F_{11}} \cap \{e_4 = 0\}$: $[s_9 : 1 : 1 : -s_3 : 1 : 1 : 0]$ - section associated with U(1)

Standard Model gauge symmetry: SU(3) x SU(2) x U(1)

Standard Model Gauge Symmetry & Matter Reps.

gauge algebra $\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$

[M.C., Lin, 1706.08521]

Shioda map:

C-central element

$$\sigma(\hat{s}_1) = S_1 - S_0 + \frac{1}{2} E_1^{\mathfrak{su}(2)} + \frac{1}{3}(2\, E_1^{\mathfrak{su}(3)} + E_2^{\mathfrak{su}(3)})$$

Matter:

$$(\mathbf{3}, \mathbf{2})_{\frac{1}{6}}\,, \quad (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}\,, \quad (\overline{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}}\,, \quad (\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}\,, \quad (\mathbf{1}, \mathbf{1})_1$$

Compatible with the Z_6 global constraint

↓

Construct G_4 for chiral index & D3-tadpole constraint

Standard Model:

Hyperplane divisor class

$H = 4\overline{\mathcal{K}}$

Base B = $\mathbb{P}^3$ Divisors in the base:

$$\mathcal{S}_7 = n_7 H$$
$$\mathcal{S}_9 = n_9 H$$

$$n_7, n_9 \in \mathbb{Z}$$

Solutions (#(families);n_{D3}) for allowed (n_7,n_9):

$n_7 \backslash n_9$	1	2	3	4	5	6	7
7	–	$(27;16)$	–	–			
6	–	$(12;81)$	$(21;42)$	–	–		
5	–	–	$(12;57)$	$(30;8)$	–	$(3;46)$	
4	$(42;4)$	–	$(30;32)$	–	–	–	–
3	–	$(21;72)$	–	–	–	$(15;30)$	
2	$(45;16)$	$(24;79)$	$(21;66)$	$(24;44)$	$(3;64)$		
1	–	–	–	–			
0	–	–	$(12;112)$				
-1	$(36;91)$	$(33;74)$					
-2	–						

Further features:

Higgs doublets - vector pairs
[C_3 w/ G_4=dC_3=0, encoded in the intermediate Jacobian of Y;
higher genus matter curves → hard to calculate]
c.f., [Bies, Mayrhofer, Weigand, 1706.04616]

Vector exotics [ditto; but for quark doublets g=1 → n≤ 1]

Yukawa Couplings (co-dimension 3 singularities)
expected to be generically there for all gauge invariant couplings → could lead to R-parity violation couplings

Three Family Standard Model with Z_2 matter symmetry
[M.C., Lin, Liu, Oehlmann,1807.01320]

Polytope F_2 (biquadric in $\mathbb{P}^1$ x$\mathbb{P}^1$; fibration with bi-section → Z_2;
further specialize fibration to obtain SM

Skip details

III. Landscape of Standard Models

Toric analysis

[M.C., J. Halverson, L. Lin, M. Liu and J. Tian, arXiv:1903.0009]

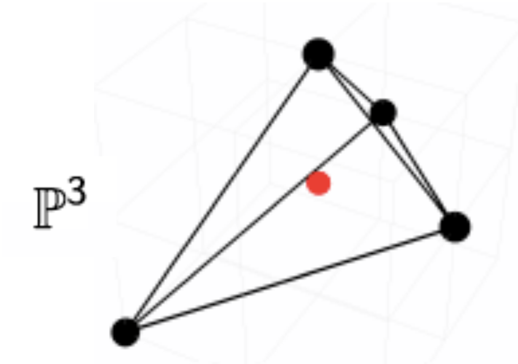
a) Take the same toric elliptic fibration as before:

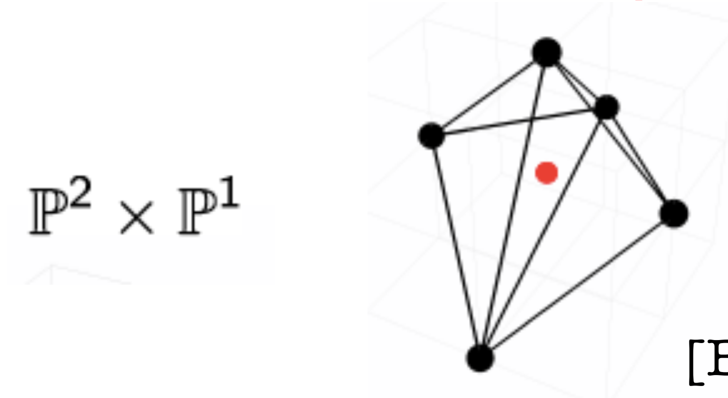
hyperplane constraint in 2D reflexive polytope F_{11} (dP_4):

Gauge symmetry: $$\frac{SU(3) \times SU(2) \times U(1)}{\mathbb{Z}_6}$$ Global Gauge Symmetry

b) Take bases B, associated with 3D reflexive polytopes.

E.g.,





[Batyrev; Kreuzer-Skarke]

For each reflexive polytope, different bases B are associated with different fine-star-regular triangulations of a chosen polytope.
[Triangulations determine intersections of divisors.]
Triangulations grow exponentially with the complexity of a polytope.

c) Specific choice of divisors: $S_{7,9} = \overline{\mathcal{K}}$

[anti-canonical divisor of the base B – fixed by the polytope]

SU (3) and SU (2) divisors S_9 and S_3 with class $\overline{\mathcal{K}}$ →

$$g_{3,2}^2 = 2/\mathrm{vol}(\overline{\mathcal{K}})$$

U(1) - (height-pairing) divisor volume $5\overline{\mathcal{K}}/6$ →

(accounting for a factor of 2 mismatch w/ Cartan generators)

$$\frac{5}{3} g_Y^2 = \frac{2}{\mathrm{vol}(\overline{\mathcal{K}})}$$

⬇

Standard Model with gauge coupling unification

$$g_3^2 = g_2^2 = 5/3 g_Y^2$$

Fibration connected to SU(4)xSU(2)xSU(2) Pati-Salam

[but did not find manifest SO(10) GUT]

c.f., [M.C., Klevers, Peña, Oehlmann, Reuter, 1503.02068]

d) Remaining conditions:

iii. 3-families of quarks and leptons

iv. D3-tadpole constraints

G_4 in terms of (1,1)-forms, Poincaré dual to divisor classes:

$$G_4(a,\,\omega) = a\,G_4^a + \pi^*\omega \wedge \sigma$$

$$G_4^a = [e_4] \wedge ([e_4] + \pi^*[s_6]) + \frac{[e_1] \wedge \pi^*[s_9]}{2} + \frac{\pi^*[s_3] \wedge ([e_2] + 2[u])}{3}$$

$$\omega \in H^{1,1}(B_3)$$

Chirality, D3 tadpole and G_4 integrality expressed in terms of intersection numbers of divisors in the base →

Geometric conditions:

In the case $S_{7,9} = \overline{\mathcal{K}}$ and 3-families it reduces to:

$$\chi(\mathbf{R}) = -\frac{a}{5}\int_{B_3} \overline{\mathcal{K}} \wedge \overline{\mathcal{K}} \wedge \overline{\mathcal{K}} =: -\frac{a}{5}\overline{\mathcal{K}}^3$$

$$n_{\mathrm{D3}} = 12 + \frac{5}{8}\overline{\mathcal{K}}^3 - \frac{45}{2\overline{\mathcal{K}}^3} \in \mathbb{Z}_{\geq 0}$$

Depends only on polytope and not on triangulation!

Landscape count:

$$12 + \frac{5}{8}\overline{\mathcal{K}}^3 - \frac{45}{2\overline{\mathcal{K}}^3} \in \mathbb{Z}_{\geq 0}$$

- Out of 4319 3D reflective polytopes → 708 satisfy the constraint.
  (many of them with large number of lattice points)

- Triangulation of polytopes can be handled combinatorially.
  (each corresponds to a different bases B)
  It can be implemented on computer, e.g., in SageMath:
  i) for 237 polytopes w/ <15 lattice points →414310 MSSM models
  ii) for 471 polytopes w/ ≥ 15 lattice points →
  exponentially growing computation time

c.f., [Halverson, Tian, 1610.08864]

- Provide bound: counting via fine-regular triangulation of each facets → estimate on regular fine-star-triangulation:

$$7.667 \times 10^{13} \lesssim N_{\rm SM}^{\rm toric} \lesssim 1.622 \times 10^{16}$$

(dominated by $\mathcal{P}_8$ polytope)

Outlook

Work in progress:

Number of vector pairs:

Higgs doublets on matter curves w/ $g = 1 + 9/2\overline{K}^3 > 0$

→ technically difficult

→ number of vector exotics

work in progress, M.C., Bies, Lin, Liu

Continuous Data:

Yukawa Couplings [ratios and overall magnitudes]

work in progress, M.C., Lin, Liu, Zoccarato, Zhang

Expected to have R-parity suppressed couplings as it is a Higgs from Pati-Salam.

Outstanding issues:

moduli stabilization,…supersymmetry breaking,…

Further studies

Thank you!