Bipartite Perfect Matching is in quasi-NC

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Joint work with Rohit Gurjar and Thomas Thierauf (University of Aalen, Germany).
http://eccc.hpi-web.de/report/2015/177/.

Matching

$G = (V, E)$ is a graph with n nodes and m edges.

Definition

A matching in G is a set $M \subseteq E$ such that each $v \in V$ is incident to at most one $e \in M$.

For a perfect matching (p.m.): substitute "exactly" for "at most" above.
The perfect matching decision problem, PM, asks whether a given graph has a p.m.
The search problem, SEARCH-PM, asks for a p.m. in a graph if it exists.
Matchings and perfect matchings have been widely studied in combinatorics and complexity theory.

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NC is the class of problems with uniform polynomial size circuits with polylogarithmic depth.
For polylog-depth circuits solving PM, nothing better than exponential size was known.

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*Is there a fast parallel nonrandomized (*NC*) algorithm for* PM*?*

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Deterministic parallel algorithms

There are NC algorithms for certain types of graphs:

- $K_{3,3}$-free graphs(Vazirani [Vaz89]),
- graphs having polynomially many p.m.'s (Grigoriev & Karpinski [GK87], also Agrawal, Hoang, & Thierauf [AHT07])
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- planar biparite graphs (Datta, Kulkarni, & Roy [DKR10] and Tewari & Vinodchandran [TV12])

Our Work

Bipartite PM *and* SEARCH-PM *are in* quasi-NC.

That is, PM *and* SEARCH-PM *on bipartite graphs have uniform circuits of depth* $O(\log^2 n)$ *and size* $2^{O(\log^2 n)}$.

We also give an RNC^2 *algorithm for bipartite* PM *using* $O(\log^2 n)$ *random bits.*

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Bipartite perfect matching in RNC

G bipartite with bipartition $L = \{u_1, \ldots, u_{n/2}\}$ and $R = \{v_1, \ldots, v_{n/2}\}$.
Given a weight function $w : E \to \mathbb{Z}^+$, we extend w to sets of edges: for $S \subseteq E$, define $w(S) := \sum_{e \in S} w(e)$.
Define the $n/2 \times n/2$ matrix $A_w = [a_{i,j}]$ as

$$a_{ij} = \begin{cases} 2^{w(e)} & \text{if } e = (u_i, v_j) \in E, \\ 0 & \text{if } (u_i, v_j) \notin E. \end{cases}$$

Then

$$\det(A_w) = \sum_{M \text{ a p.m. of } G} \operatorname{sgn}(M)\, 2^{w(M)}.$$

If G has no p.m., then $\det(A_w) = 0$ for any w.
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Definition

A weight function w is isolating if G has a unique minimum weight p.m. with respect to w.

If w is isolating, then $\det(A_w) \neq 0$, because the minimum weight term in $\det(A_w)$ does not cancel with other terms, which are strictly higher powers of 2.

Lemma (Isolation Lemma [MVV87])

Let $w(e)$ chosen uniformly at random from $\{1, \ldots, 2m\}$ for each edge e independently. Then w is isolating with probability $\geq 1/2$.

If w is isolating, then computing $\det(A_w)$ gives the correct answer. This can be done in NC^2 (Berkowitz [Ber84]).

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We want to derandomize this lemma!

Let $E = \{e_0, \ldots, e_{m-1}\}$, and define $w(e_i) = 2^i$ for all $i < m$.
w is clearly isolating, ...
but we cannot compute $\det(A_w)$ efficiently, because the matrix entries are too big.
Instead, we reduce the weights modulo small numbers j:

Definition

Fix an integer $j > 1$. Define the weight function $w_{\bmod j}$ as

$$w_{\bmod j}(e) := w(e) \bmod j$$

for all $e \in E$.
For some t we choose later, define the set of weight functions

$$W_t := \{w_{\bmod j} \mid 2 \leq j \leq t\} .$$

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Fix an integer $j > 1$. Define the weight function $w_{\bmod j}$ as

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for all $e \in E$.
For some t we choose later, define the set of weight functions

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We want to derandomize this lemma!

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Circulation

Let $C = \langle e_1, \ldots, e_p \rangle$ be a cycle of G with edges given in cyclic order.
(p is even because G is bipartite.)

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Given a weight function w, the circulation of C with respect to w is

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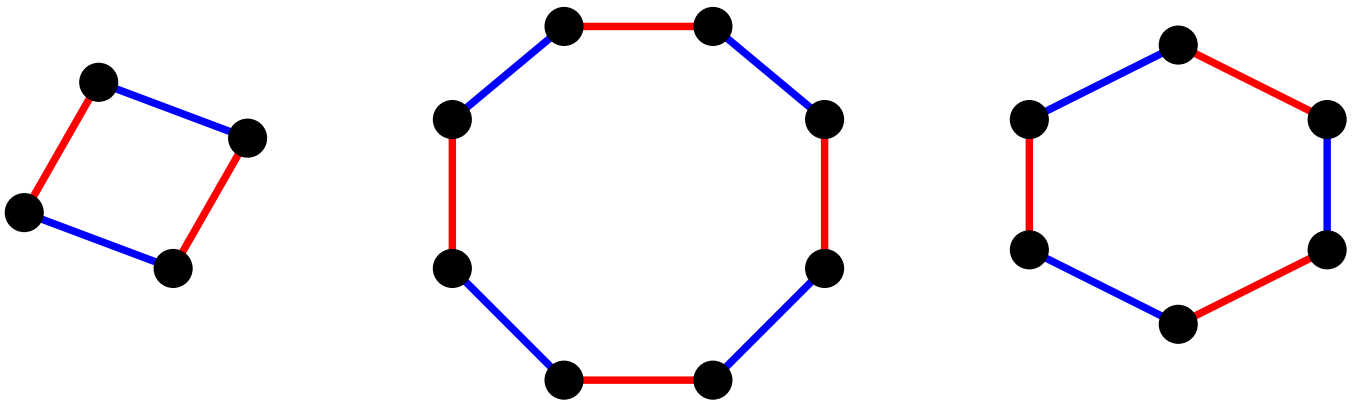

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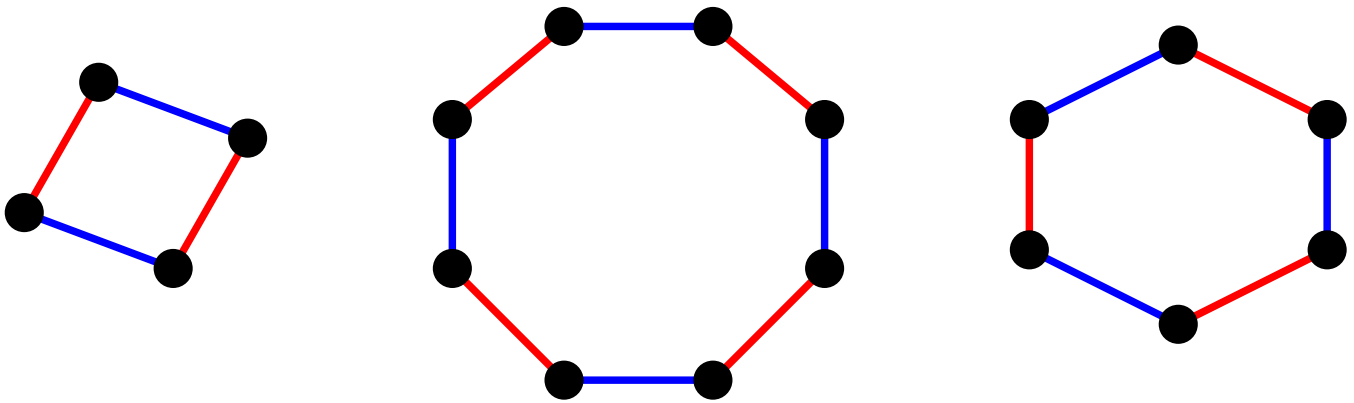

Forcing nonzero circulation

We would like to choose a weight function from W_t that gives nonzero circulation to as many cycles as possible.
We cannot do this for all cycles at once, so we work in stages, starting with short cycles.

Lemma ([CRS95])

Let s be a positive integer, and let $t = n^2 s$. Then for any set of s many cycles $\{C_1, \ldots, C_s\}$ there exists a weight function $w \in W_t$ that gives nonzero circulation to all of $C_1, \ldots, C_s$.

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The derived graph

Suppose G has a p.m., and w is a weight function on G.

Definition

The derived graph of G with respect to w is the subgraph $G^{(w)} := (V, E')$, where E' is the union of all w-min weight p.m.'s of G.

Key Lemma

All cycles in $G^{(w)}$ have zero circulation with respect to w.

We proved this lemma using linear algebra.
Later, an alternate combinatorial proof was found by Rao, Shpilka, & Wigderson (reported in Goldwasser & Grossman [GG15]).

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Removing short cycles from $G^{(w)}$

In steps, we remove cycles from derived graphs whose lengths are increasing powers of 2.

The next lemma says there are not too many of these.

Lemma

Let G be a graph with no cycles of length $\leq r$ for some even r. Then G has $\leq n^4$ cycles of length $\leq 2r$.

We can give all these cycles nonzero circulation by some weight function $w \in W_t$, where $t = n^6$.

So these cycles cannot exist in the derived graph with respect to w.

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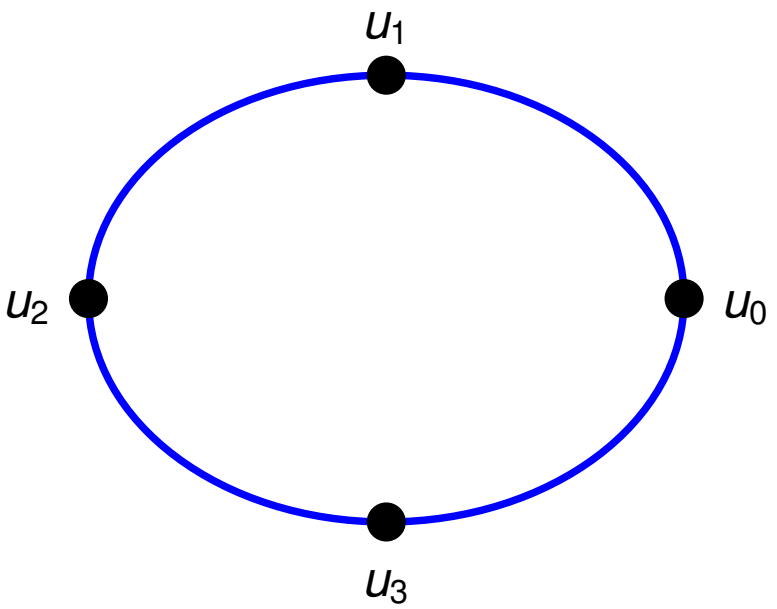

- Given a cycle C of length $\leq 2r$, choose four vertices u_0, u_1, u_2, u_3 on the cycle
- such that the distance between adjacent vertices is $\leq r/2$.
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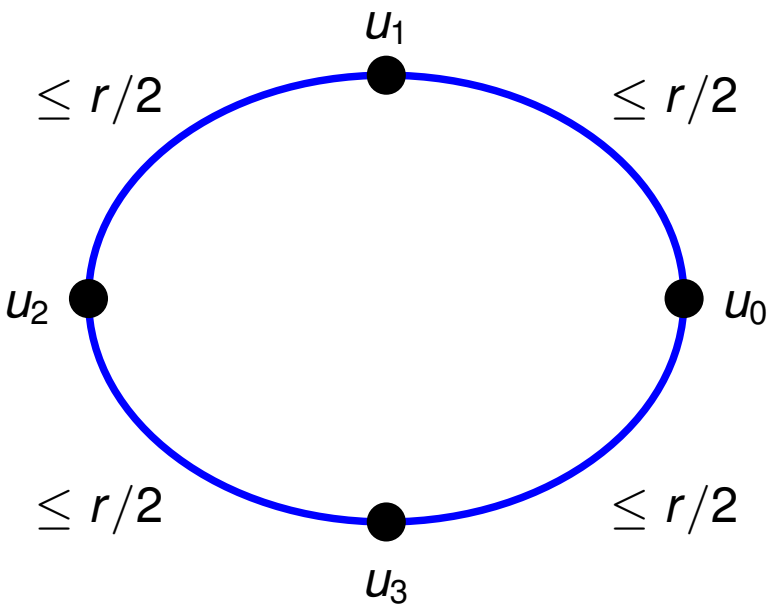

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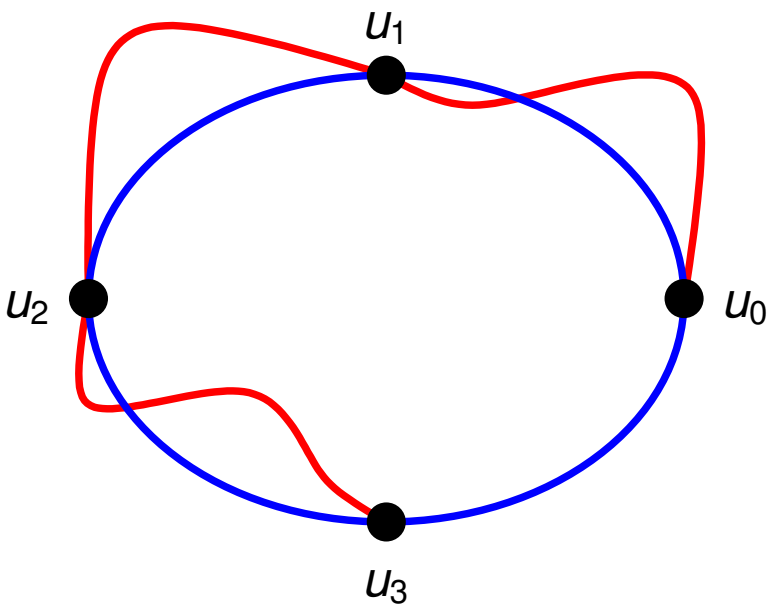

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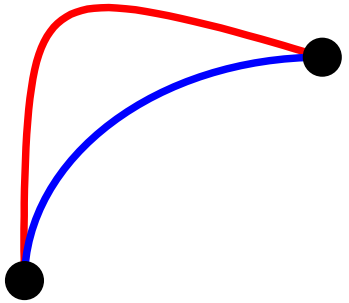

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The sequence of derived graphs

Start with $G_0 := G$, a bipartite graph with a p.m.

- Choose $w_1 \in W_t$ such that all cycles in G_0 of length ≤ 4 have nonzero circulation.
- Let $G_1 := G^{(w_1)}$. G_1 has a p.m. and no cycles of length ≤ 4.
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- Choose $w_1 \in W_t$ such that all cycles in G_0 of length ≤ 4 have nonzero circulation.
- Let $G_1 := G^{(w_1)}$. G_1 has a p.m. and no cycles of length ≤ 4.
- Choose $w_2 \in W_t$ such that all cycles in G_1 of length ≤ 8 have nonzero circulation.
- Let $G_2 := G_1^{(w_2)}$. G_2 has a p.m. and no cycles of length ≤ 8.
- $\ldots$
- Choose $w_i \in W_t$ such that all cycles in G_{i-1} of length $\leq 2^{i+1}$ have nonzero circulation.
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An isolating weight function for G

We must glue the weight functions $w_1, \ldots, w_k$ together into a single weight function.

Let B be a strict bound on any edge weight from $w_1, \ldots, w_k$ (we may take $B := n^6$).

For every $e \in E$, define

$$w(e) = B^{k-1} w_1(e) + B^{k-2} w_2(e) + \cdots + B^0 w_k(e) .$$

Lemma

If G has a p.m., then w is isolating for G.

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Proof

- Notice that the edge sets of the G_i form a descending chain, ending in a p.m. M of G (the edge set of G_k).
- Let $M' \neq M$ be some other p.m. of G.
- There must be some stage $i < k$ where M and M' are both in G_i but M' is not in G_{i+1}.
- Since M and M' are in $G_1, \ldots, G_i$, they both have the same minimum weight with respect to $w_1, \ldots, w_i$.
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We do not know which $w_1, \ldots, w_k$ work, so we try them all in parallel.

For all $w_1, \ldots, w_k \in W_{n^6}$ in parallel:

- Compute w as above. (One of these choices of w must be isolating.)
- Compute $\det(A_w)$ as in the RNC algorithm of [MVV87].
- If we ever find a nonzero determinant, answer "yes."
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Each w_i takes $6 \log n$ bits to store, so w takes $O(\log^2 n)$ bits. Processing them all in parallel can be done with circuits of size $2^{O(\log^2 n)}$ and depth $O(\log^2 n)$.

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Proving the Key Lemma: The perfect matching polytope

Let $\mathbb{R}^E$ be the m-dimensional real vector space with standard basis labeled by the edges of G.
Then any set $S \subseteq E$ of edges naturally corresponds to its characteristic vector $(s_e)_{e \in E}$, where, for each edge $e \in E$,

$$s_e = \begin{cases} 1 & \text{if } e \in S, \\ 0 & \text{if } e \notin S. \end{cases}$$

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Lemma ([LP86])

If G is bipartite, then a vector $\vec{x} = (x_e)_e$ is in $\mathrm{PM}(G)$ if and only if

$$\begin{aligned} x_e &\geq 0 \,, \\ \sum_{e' \in \delta(v)} x_{e'} &= 1 \,, \end{aligned}$$

for all $e \in E$ and $v \in V$, where $\delta(v)$ is the set of edges incident to v.

The $\Rightarrow$ direction is clear for any graph (not necessarily bipartite). The converse does not hold for general graphs.

We can extend any weight function w to $\mathbb{R}^m$ by linearity:

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Key Lemma (cont.)

Let $\vec{x}_1, \ldots, \vec{x}_t \in \mathrm{PM}(G)$ be vectors corresponding to all the p.m.'s of G with the same minimum weight q.

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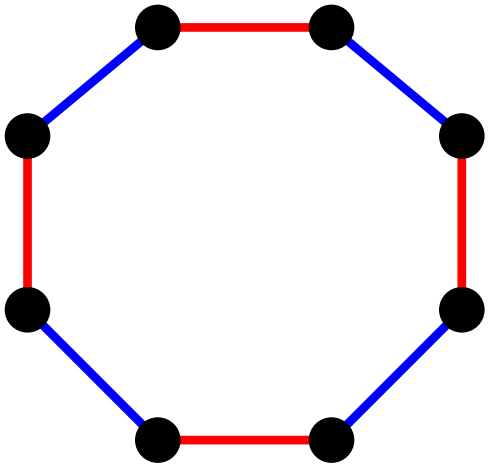

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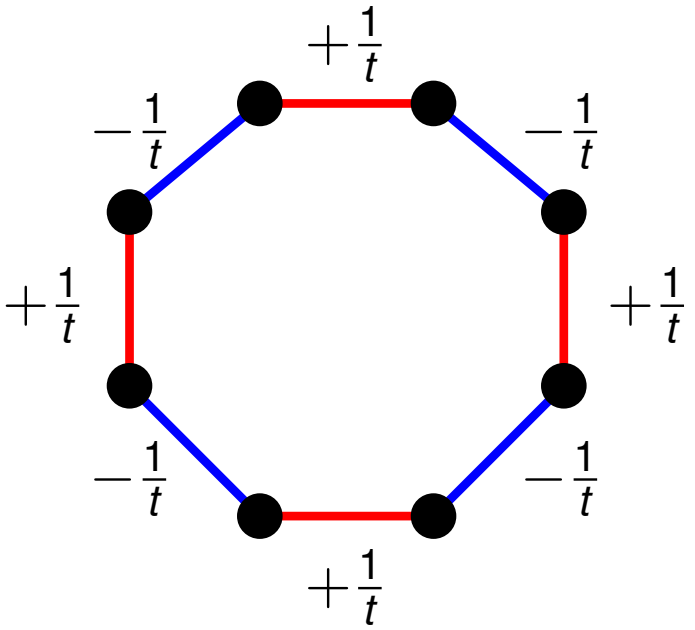

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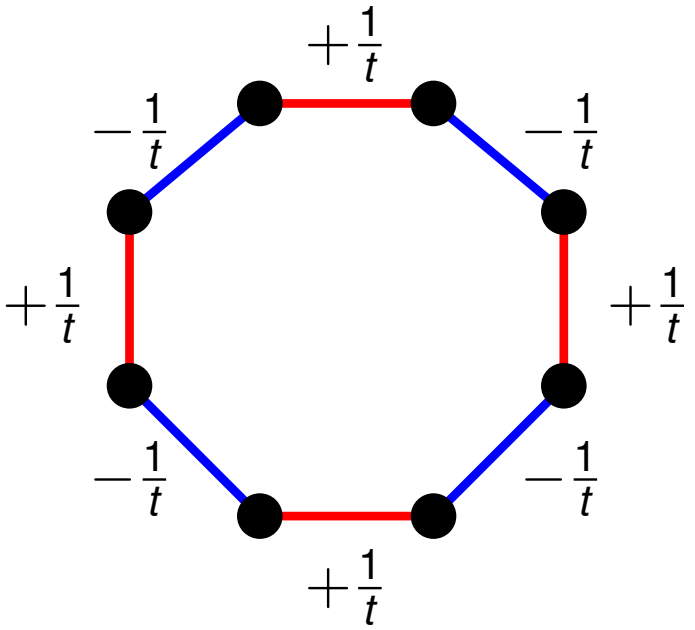

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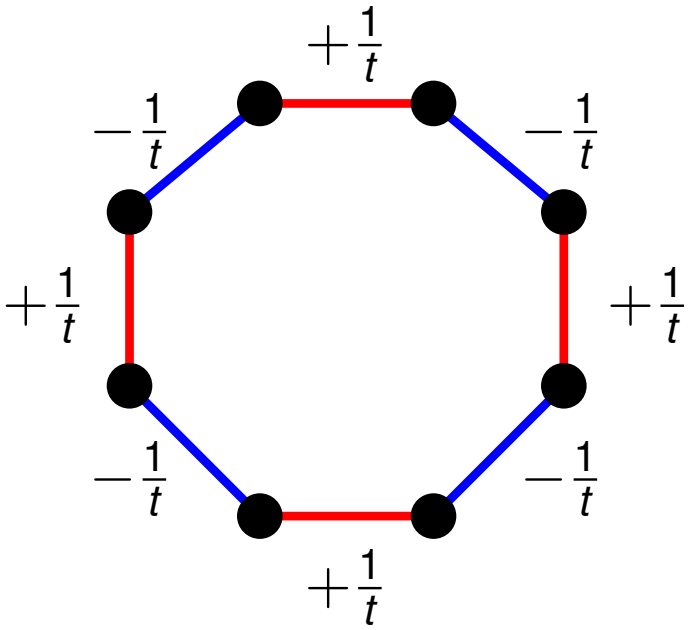

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The RNC algorithm

Recall $w_{\bmod j}(e_i) = 2^i \bmod j$ for each edge $e_i \in E$ and $2 \leq j \leq t$.
Instead of trying all of these weight functions, we let j be a random prime.
Any set of s many cycles has nonzero circulation with high probability
Doing this k times gives random $w_1, \ldots, w_k$.

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Other results

The following are all in quasi-NC:

- bipartite weighted PM with quasi-polynomially bounded integer weights
- maximum bipartite matching
- cycle cover with polynomially bounded integer weights
- subtree isomorphism
- max flow with polynomially bounded integer capacities
- constructing a depth-first search tree

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