DETECTION OF MULTI-DIMENSIONAL STRUCTURES IN GRANULAR MATERIALS

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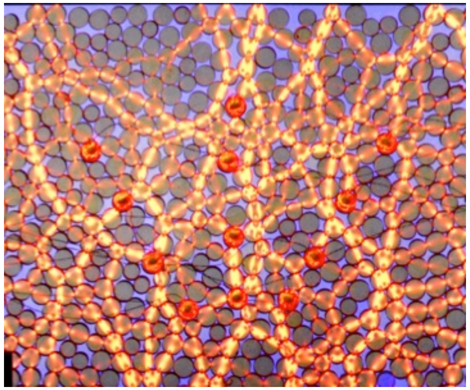

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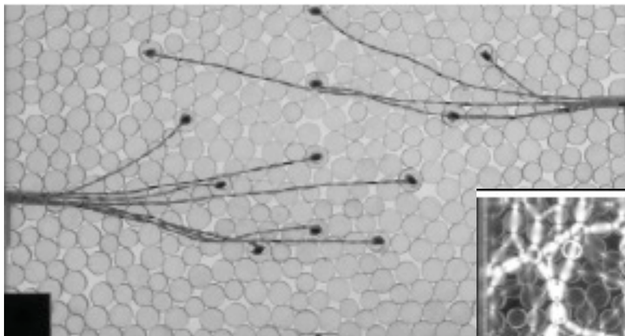
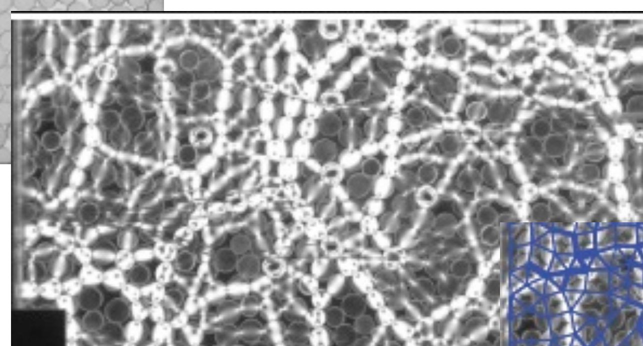
Force Chains in Granular Media

Collection of particles putting pressure on one another through an interaction of gravity and geometry



Photoelastic response shows internal forces



Define:

A_{ij} Contact network

W_{ij} Force network

ω Mean Force

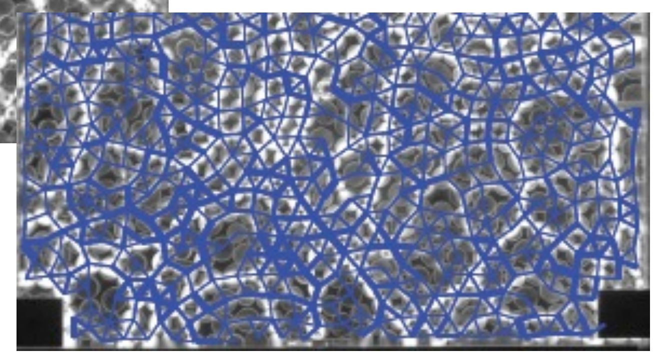
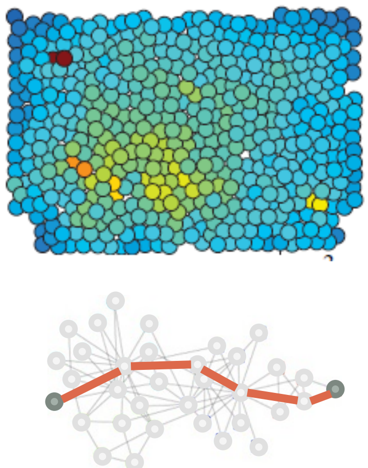


Image processing to obtain network

Probing Multi-Dimensional Structures

System

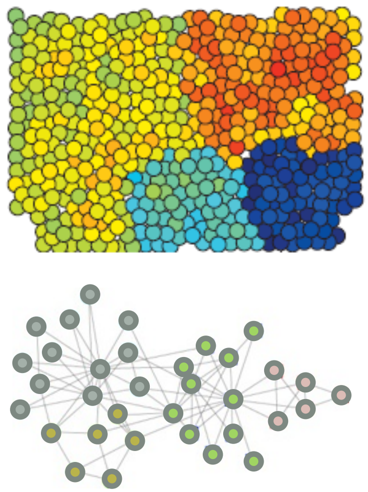


Global Efficiency

$$E(\mathbf{G}) = \frac{1}{N(N-1)} \sum_{i \neq j \in \mathbf{G}} \frac{1}{d_{ij}}$$

- Efficiency of global signal transmission

2D Domain

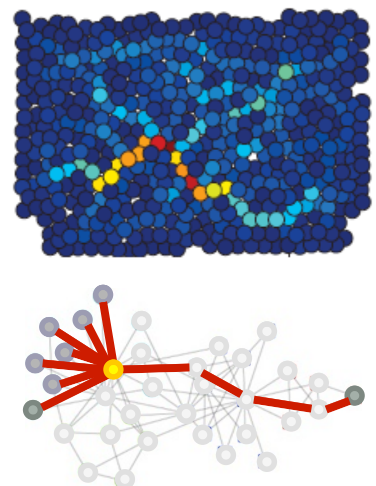


Modularity

$$Q = \sum_{ij} [A_{ij} - P_{ij}] \delta(g_i, g_j)$$

- Local geographic domains

1D Curves

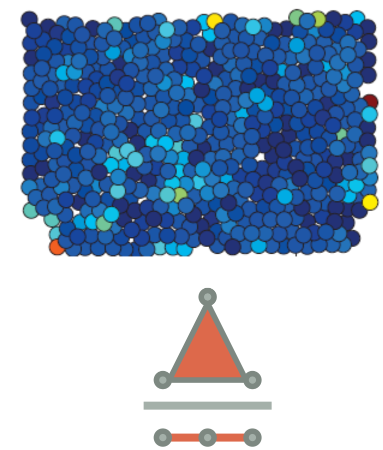


Geodesic Node Betweenness

$$B_i = \sum_{j,m,i \in \mathcal{G}} \frac{\psi_{j,m}(i)}{\psi_{j,m}}$$

- Bottlenecks or centrality

0D Particles

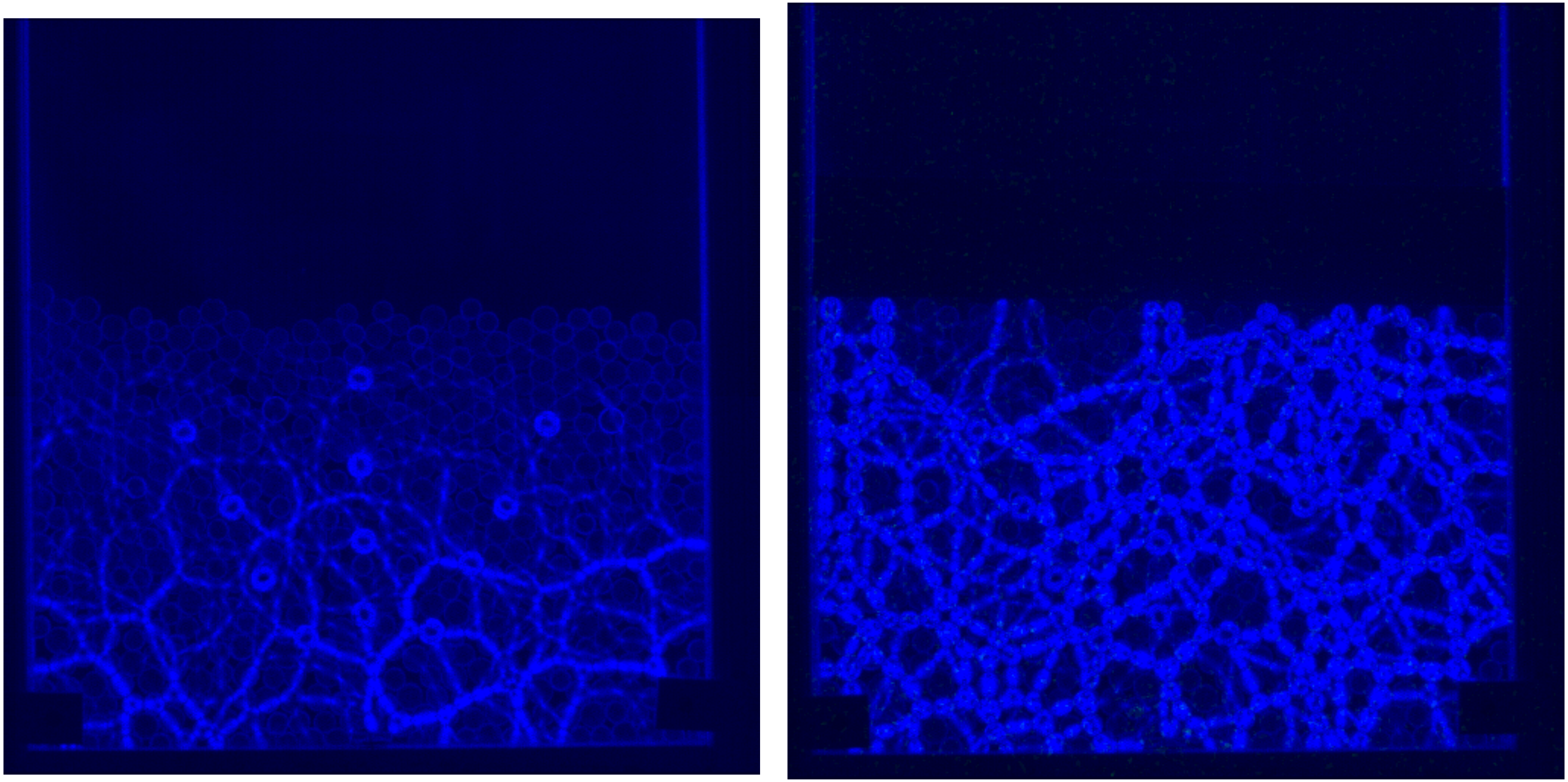


Clustering Coefficient

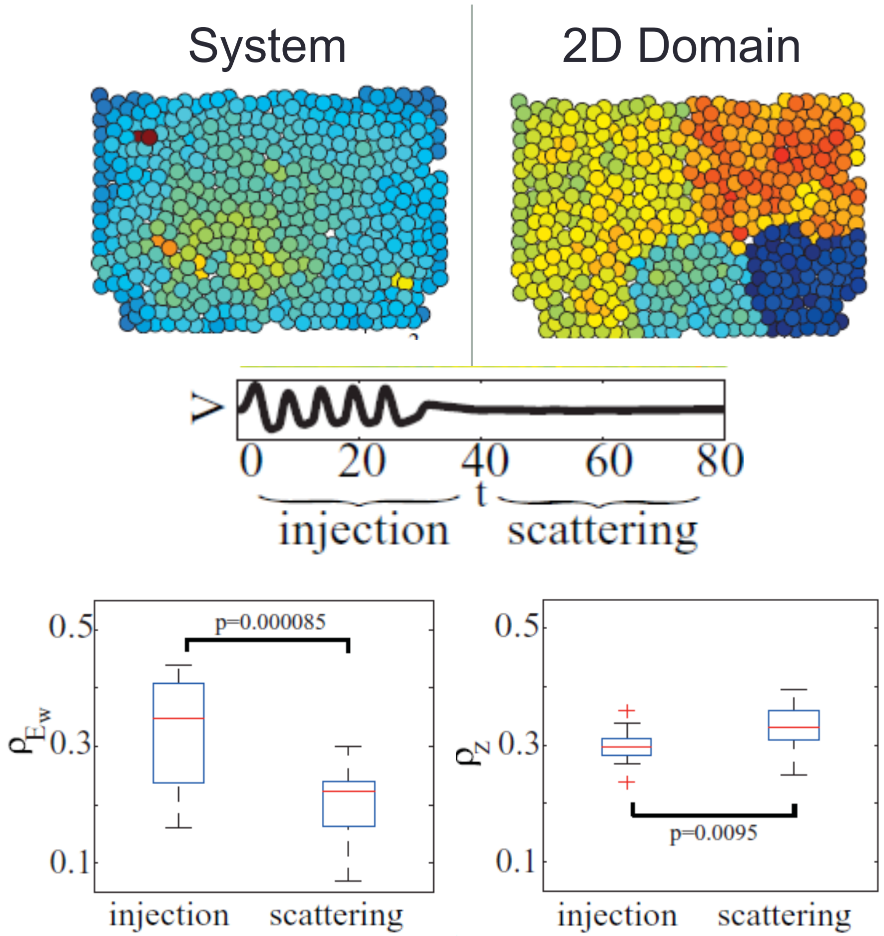
$$C_i = \frac{\sum_{mj} A_{mj} A_{im} A_{ij}}{k_i(k_i - 1)}$$

- Local loop structures

Network Markers of Acoustic Transmission?



Spatial Structures Differentially Constrains Sound



Correlation between **nodal efficiency** and acoustic signal intensity

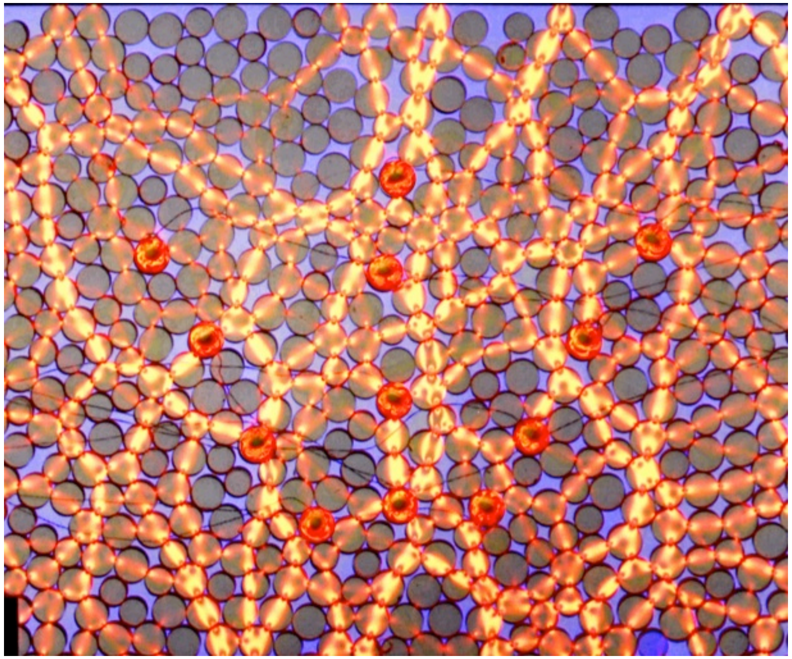
Correlation between **intra-module strength** and acoustic signal intensity

Summary

- Granular materials can be represented as weighted networks.

- Traditional graph-based statistics can be computed to probe multidimensional structures at the:
  - System
  - 2D domains
  - 1D cures
  - 0D particles

- These multi-dimensional structures differentially constrain bulk properties of the material, such as acoustic transmission.



Problem: Nonphysical Assumptions of Traditional Graph Statistics

Traditional Community Detection Techniques uncover 2D domains.

A_{ij}	Contact network
W_{ij}	Force network
ω	Mean Force

$$Q = \sum_{i \neq j \in G} [W_{ij} - P_{ij}]\delta(g_i, g_j)$$

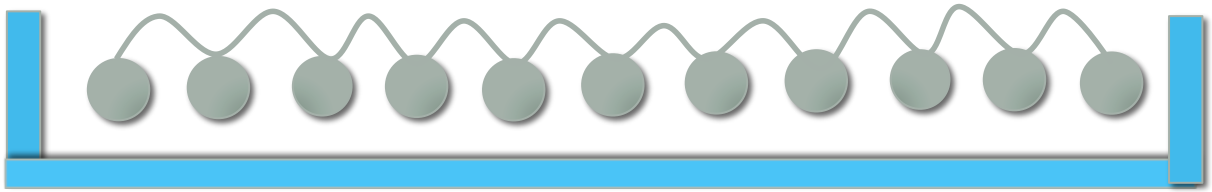
Modularity Quality Function

$$P_{ij} = \frac{k_i k_j}{2m}$$

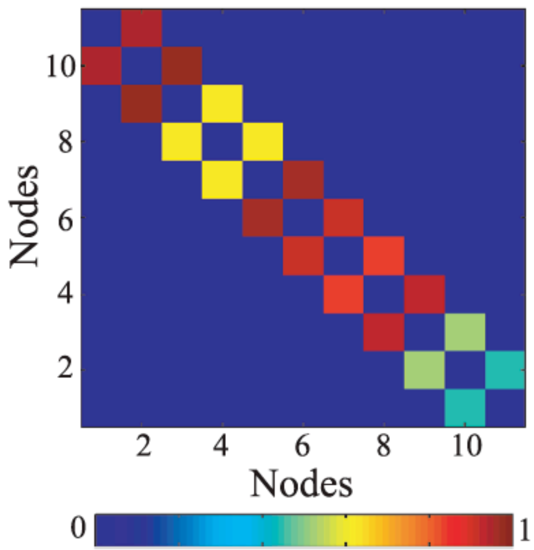
Null Model

Example: Ring Lattice Network

Box

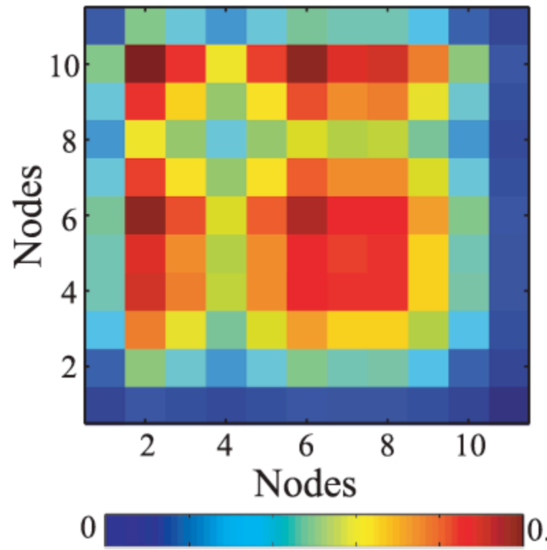


W_{ij} Force Network



Connectivity

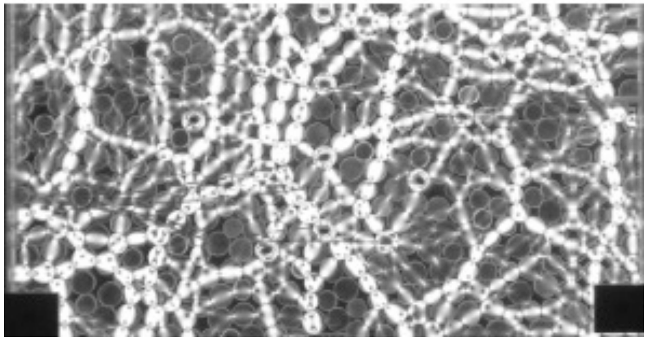
$$P_{ij} = \frac{k_i k_j}{2m}$$ Null Model



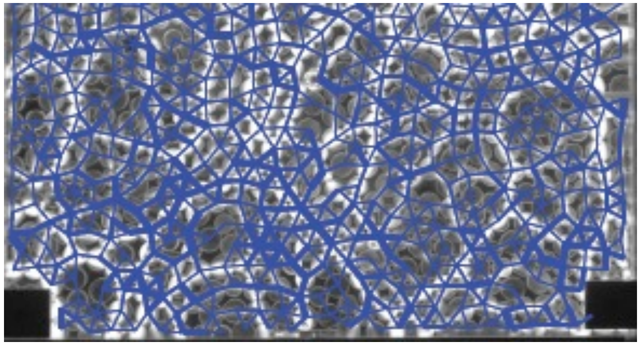
Connectivity

Example: 2-Dimensional Packing

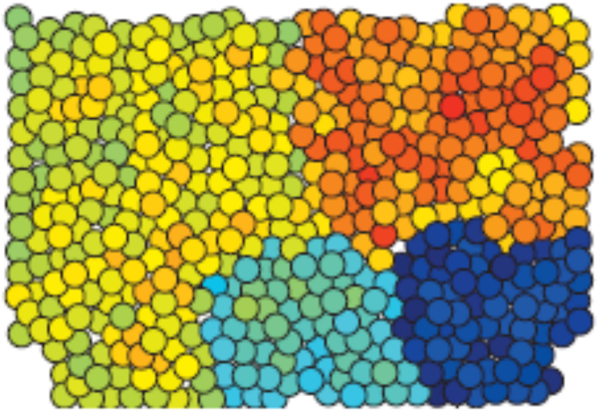
Force Data



Force Weighted Network



2D Domains

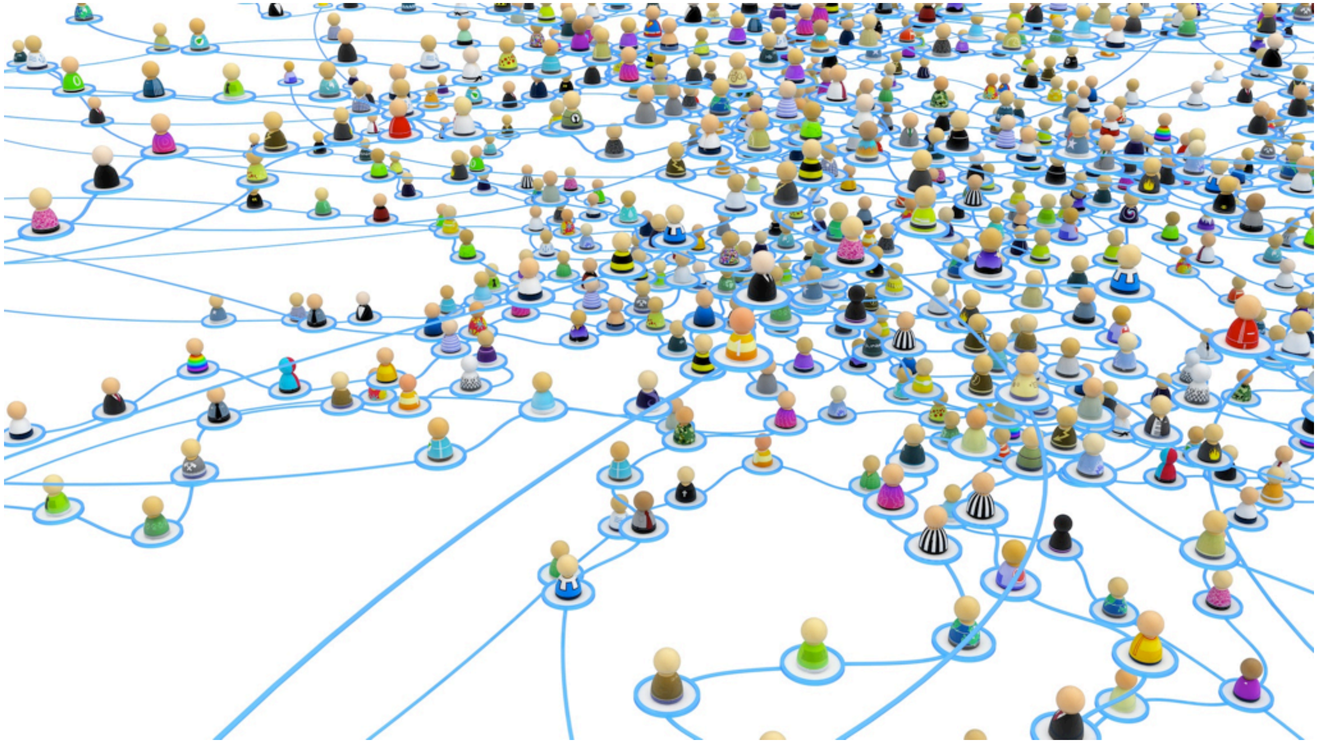


BUT: We have assumed that any particle can contact any other particle.

:: New Physics Alert::

Illustration of a Bigger Problem

The field of network science is built on traditionally non-physical nodes and edges.

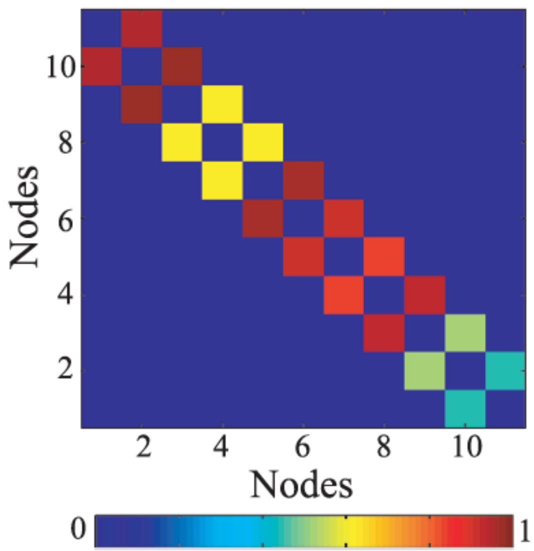


- Radically revamp our toolbox to address physical questions?

How do we fix this?

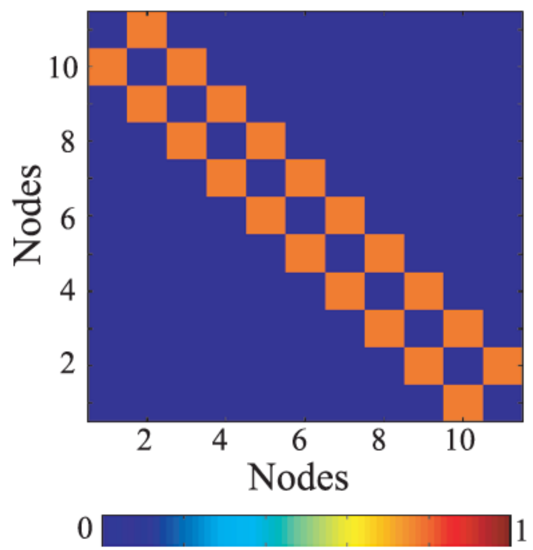
Let's use a custom spatially constrained null model that fixes topology (contacts) while scrambling geometry (forces).

W_{ij} Force Network



Connectivity

$P_{ij} = \omega A_{ij}$ Null Model



Connectivity

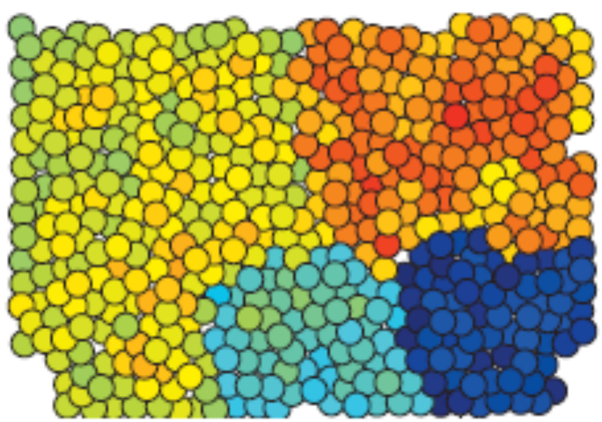
Result

Using the Newman-Girvan null model, we uncover 2D domains;
Using the geographic null model, we uncover structures reminiscent of force chains.

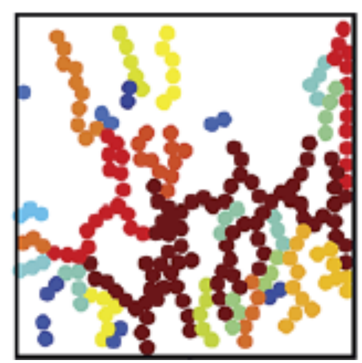
$$P_{ij} = \frac{k_i k_j}{2m}$$

$$P_{ij} = \omega A_{ij}$$

2D Domains



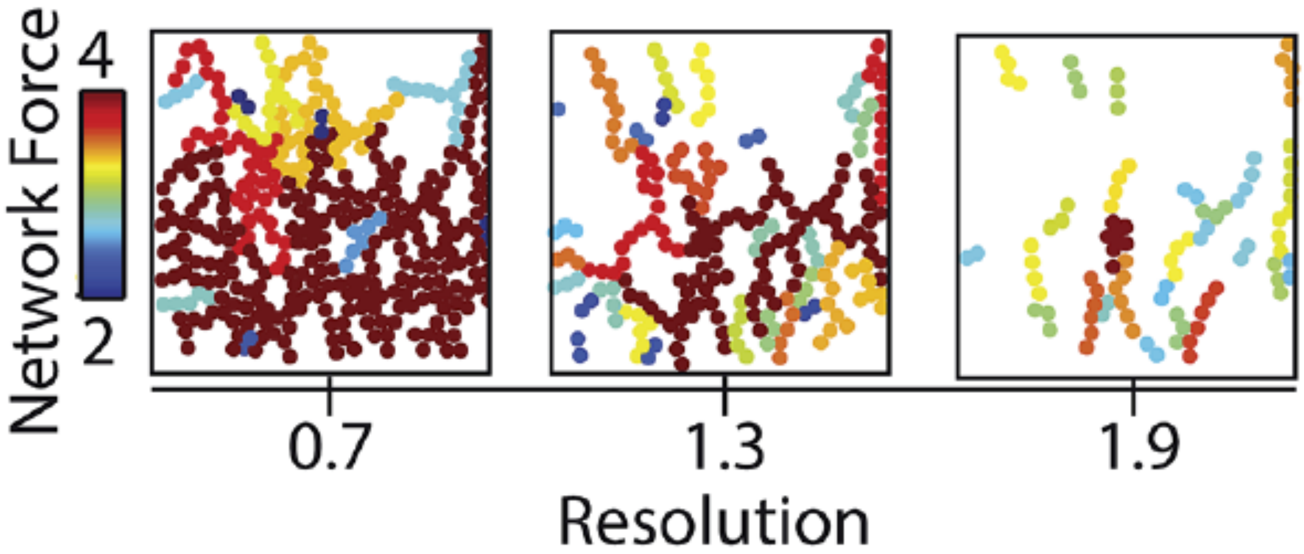
Chain-Like Structures



The Effect of the Resolution Parameter

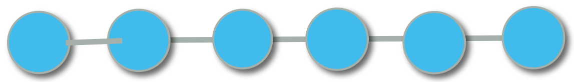
γ = structural resolution parameter

$$Q = \sum_{i \neq j \in G} [W_{ij} - P_{ij}] \delta(g_i, g_j)$$



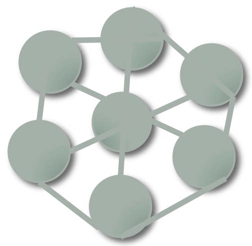
What features do force chains have?

Not Interesting:



Line

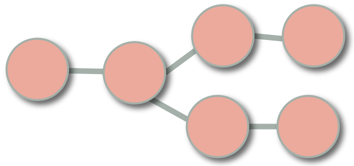
Hop Distance = Physical Distance



Blob

Hop Distance = Physical Distance

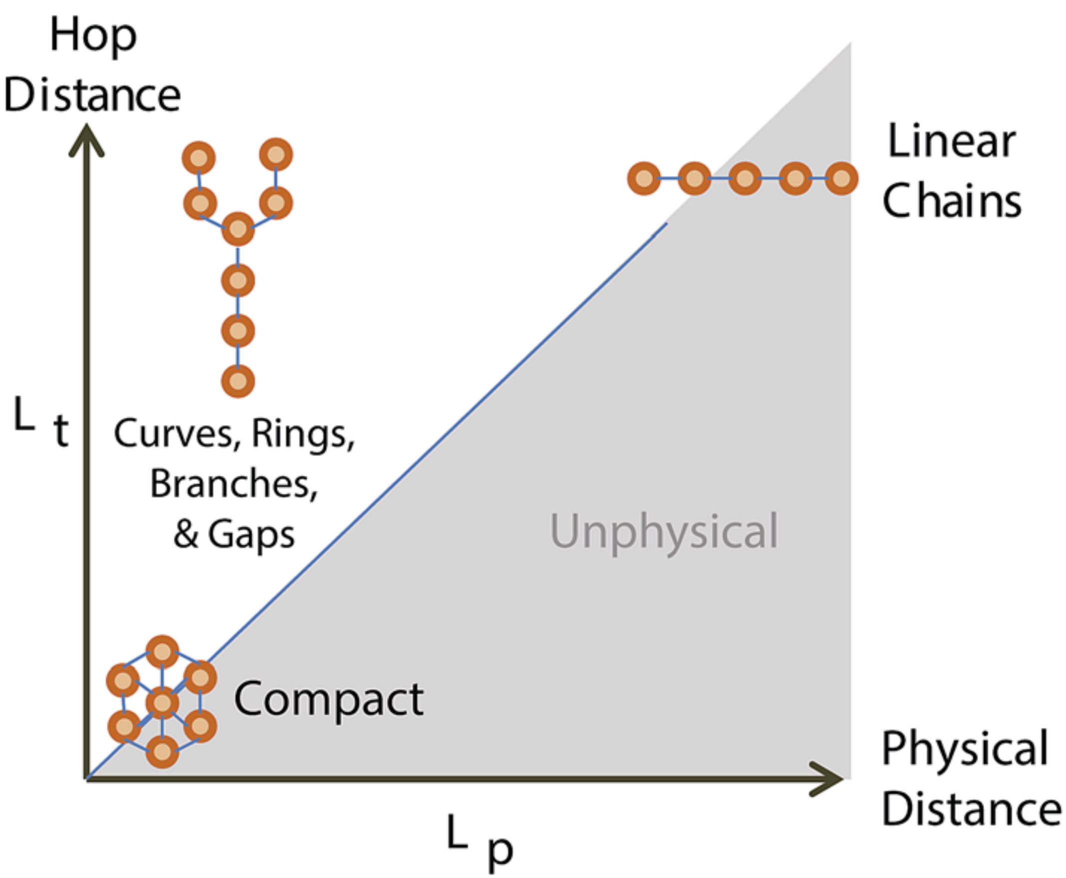
Interesting:



Branch

Hop Distance > Physical Distance

A Statistic for Force Chain Shape: the Gap Factor

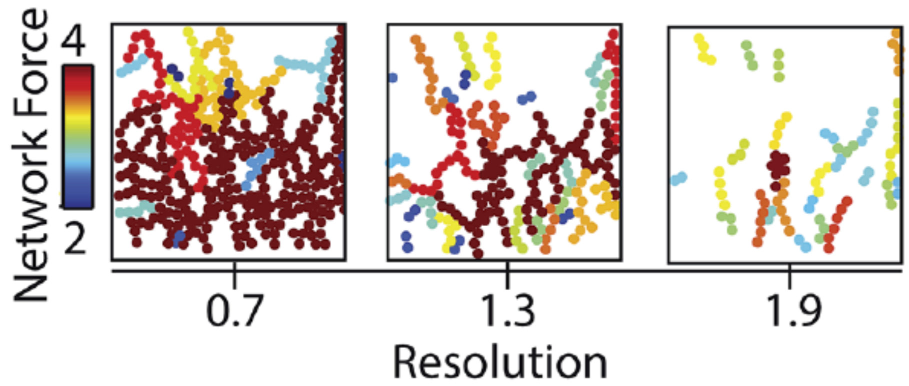


We define the weighted gap factor as:

$$g_c = 1 - \frac{r_c s_c}{s_{\max}}$$

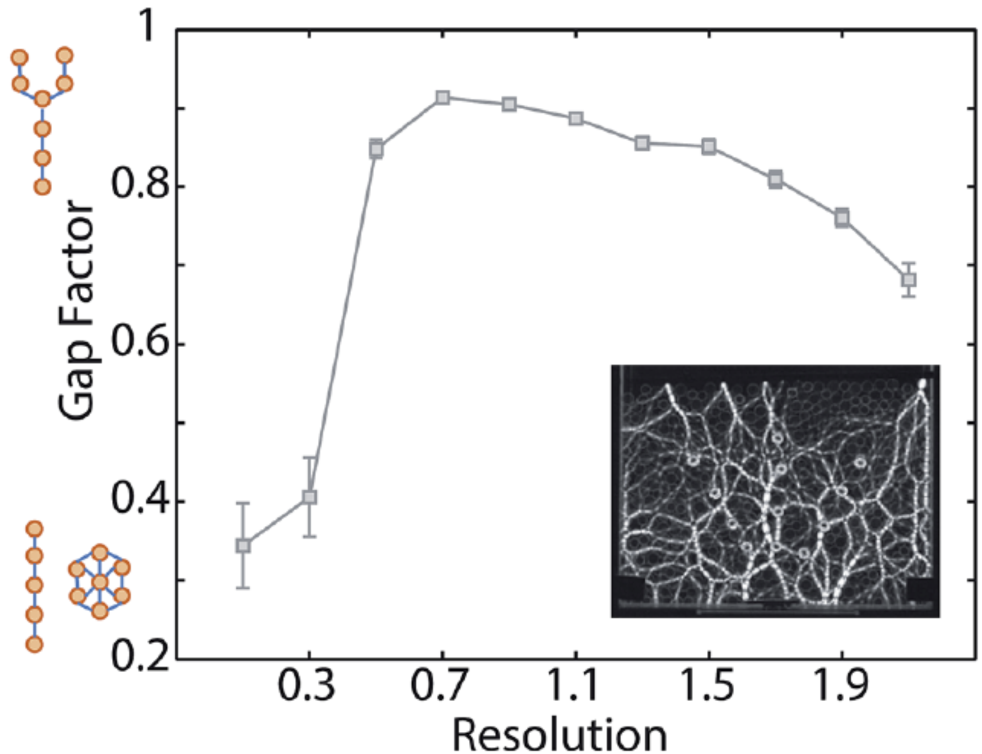
Where r_c is the Pearson correlation coefficient between L_t and L_p within community c, s_c is the size of community c, and s_{max} is the size of the largest community in the packing.

Data-Driven Extraction of Force Chains



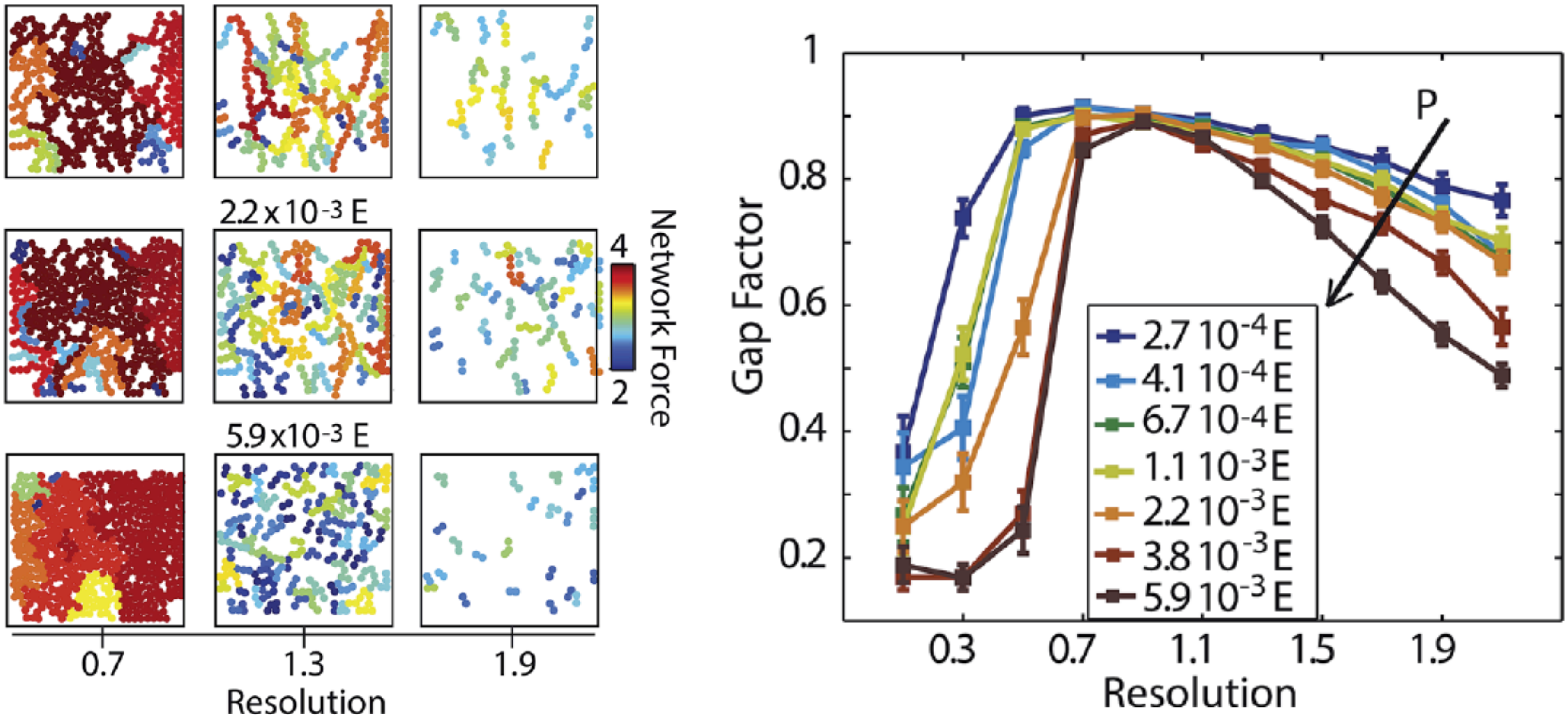
The gap factor is maximized near the resolution at which the most branch-like communities appear.

We therefore use the point of max gap factor to determine γ.

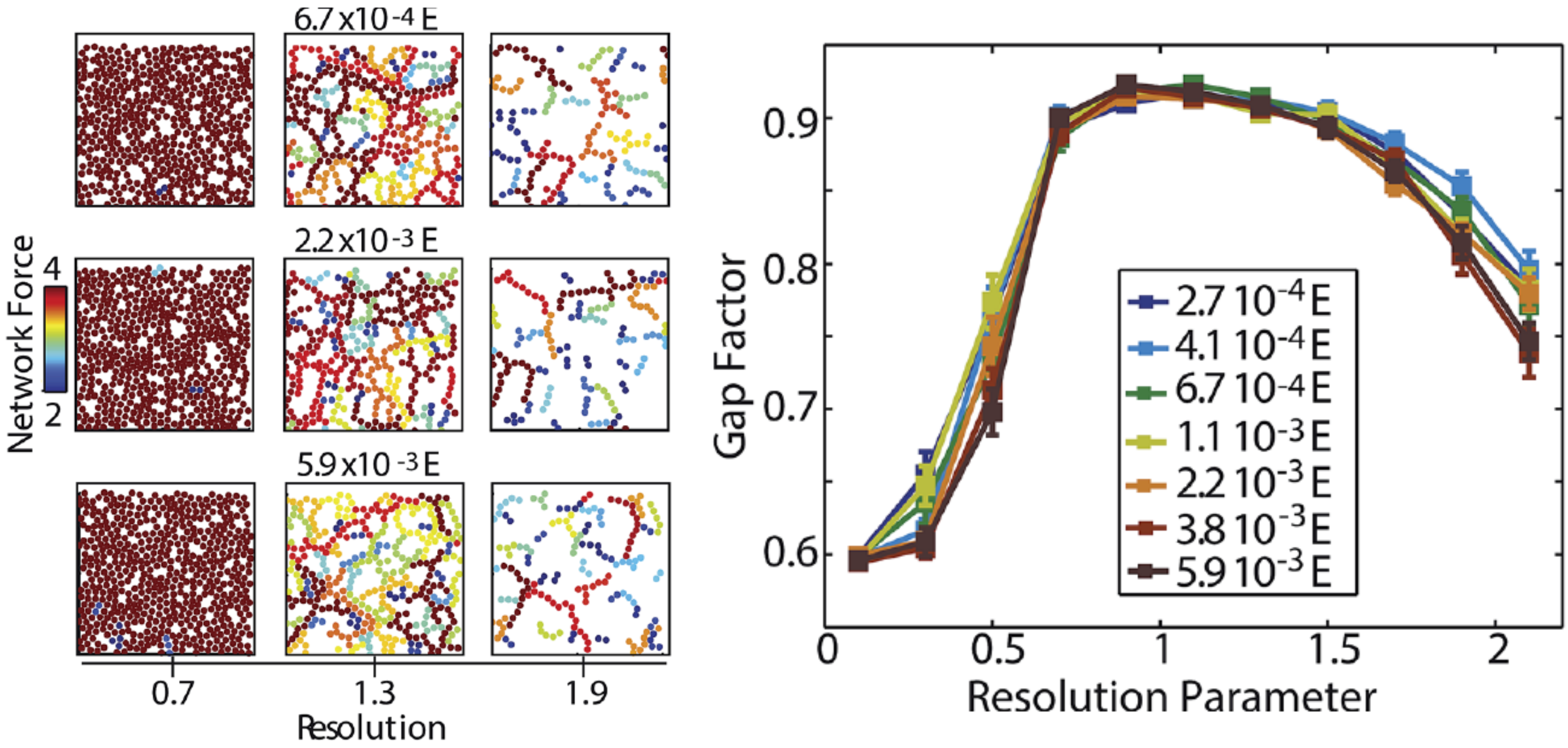


Gap Factor Tracks Shape Changes with Pressure

As pressure increases, force chains become more blob like (at low γ) or more line-like (at high γ), decreasing the gap factor.



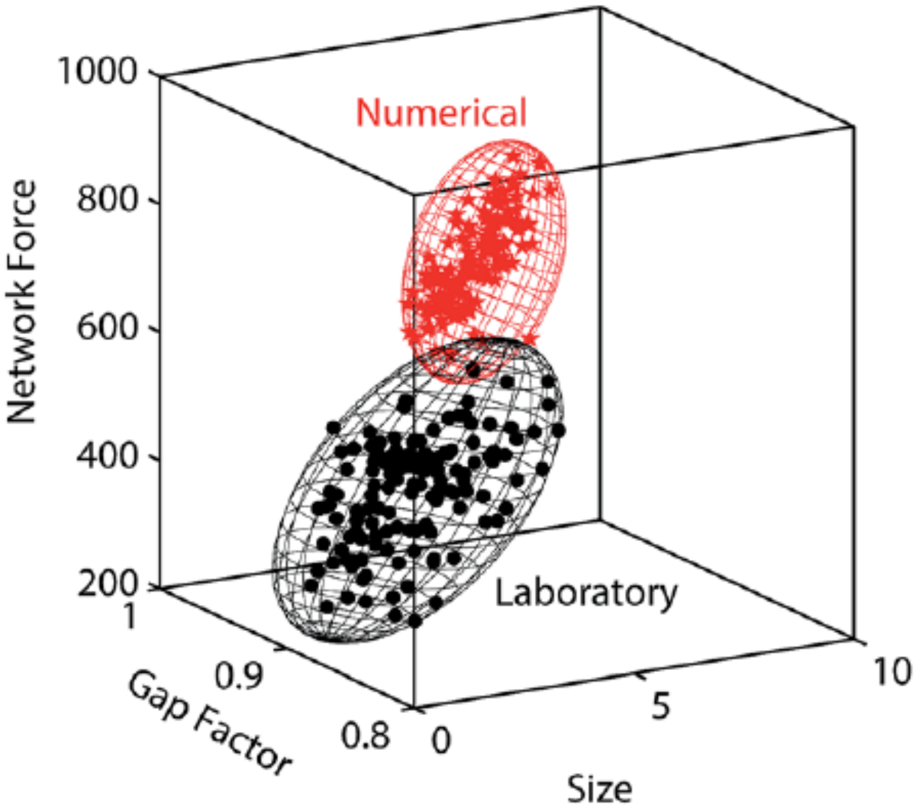
Effects of Pressure Weaker in Simulated Frictionless Packings

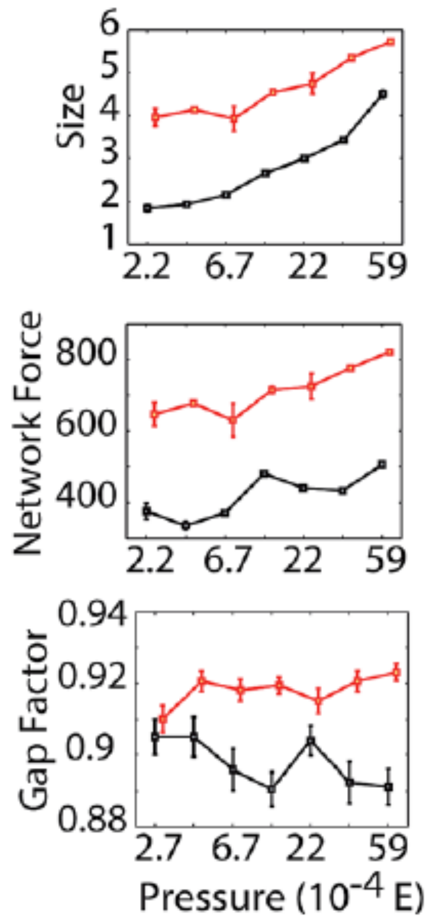


- Suggests that these tools are useful in distinguishing different types of packings.

Differentiating Packing Types

The size, force, and gap factor of packings distinguishes laboratory experiments from numerical simulations.





Practical Utility

Framework for understanding which features of force chains are universal versus which are governed by particle-particle or particle-environment interactions.

Tool to predict differences in macroscopic behavior based on subtle changes in force chain diagnostics.

Methodology that provides new information:

- which particles are strongly connected within communities
- Spatial distribution of force chains

➢ Are boundaries of force chains mechanically unstable?
Are large strong force chains mechanically stable?