An integral lift of contact homology

Jo Nelson

IAS and Columbia University

IAS short talks 2015

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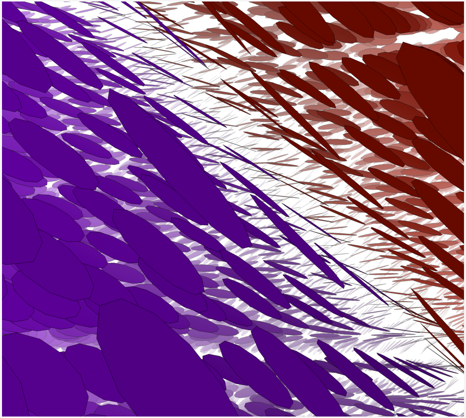

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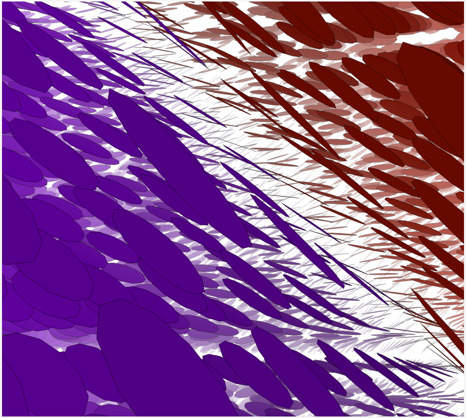

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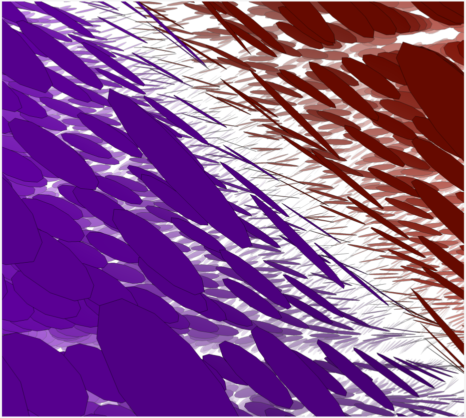

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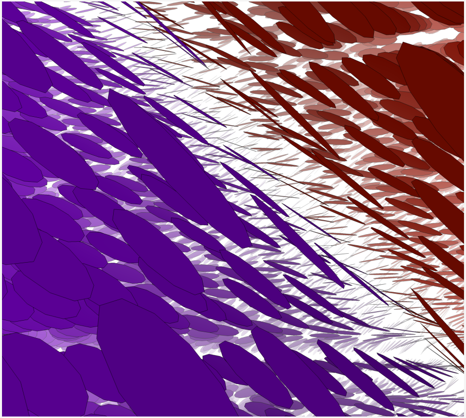

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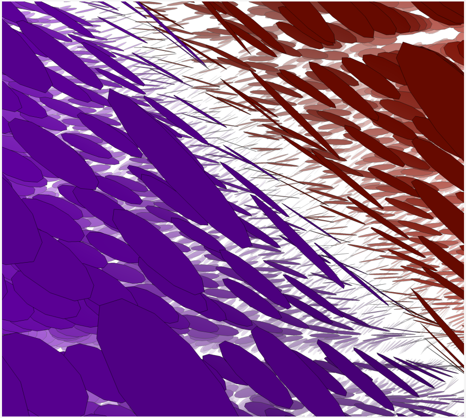

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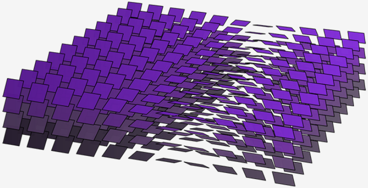

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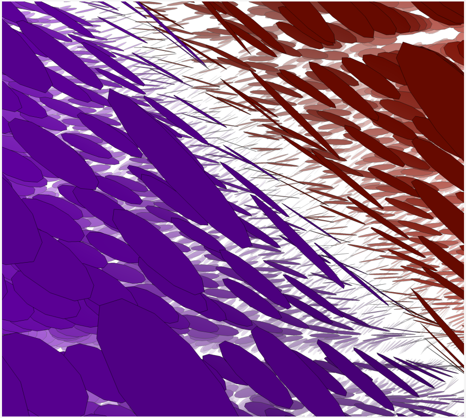

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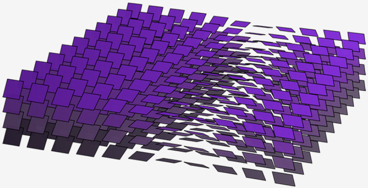

Here: $\alpha = dz - ydx$

Reeb flow

Choose a contact form α.

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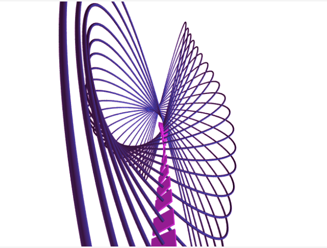
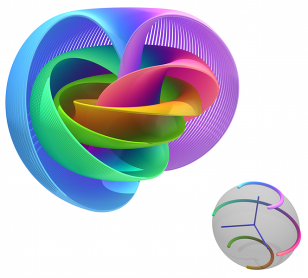


Patrick Massot



http://www.nilesjohnson.net/hopf.html

Global Invariants

The Weinstein Conjecture

Let (M, ξ) be a closed oriented contact manifold. Then for any contact form α for ξ, the Reeb vector field R_α has a closed orbit.

Proven in dimension 3 by Taubes in 2007.

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Conjecture (Hutchings-Taubes)

The only contact 3-manifolds that admit exactly two simple Reeb orbits must be either a sphere or a lens space...Otherwise there are always infinitely many simple periodic Reeb orbits!

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Assume: M closed and α nondegenerate

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- $C_*(\alpha) = \mathbb{Q}\langle\{\text{closed Reeb orbits}\} \setminus \{\text{bad Reeb orbits}\}\rangle$

A new hope...

Gradient flow lines no go; use finite energy pseudoholomorphic cylinders $u \in \mathcal{M}_{\tilde{J}}(\gamma_+; \gamma_-)$, where $\gamma_\pm$ are $T_\pm$-periodic Reeb orbits.

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Conjeorem (Eliashberg-Givental-Hofer '00)

Assume a minimal amount of things. Then $(C_*(\alpha), \partial))$ forms a chain complex and $H(C_*(\alpha), \partial)$ is independent of α and $\tilde{J}$.

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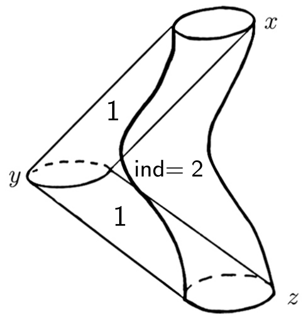

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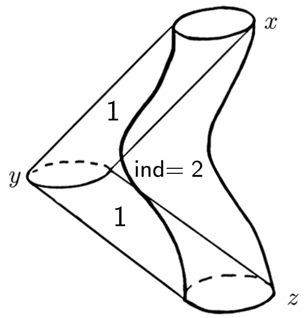

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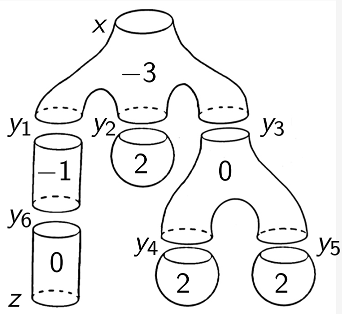

Adding to 2 becomes hard

A graduate student strikes back

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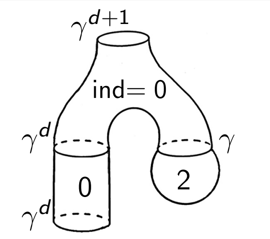

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Still stuck on Invariance....

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- And S^1-equivariantize yielding an integral lift of contact homology, $CH_*^{\mathbb{Z}}$...plus $CH_*^{\mathbb{Z}} \otimes \mathbb{Q} \cong CH_*$!

Theorem (Hutchings-N; 2015)

INVARIANCE! Obtained for dynamically convex (M^3, α) *wherein a contractible* γ *has* $\mu_{CZ}(\gamma) = 3$ *only if* γ *is simple.*

The period doubling bifurcation for a simple Reeb orbit

Modify flow by the return map $\phi_r :=$ rotate by $180^\circ \circ \varphi_\epsilon^{X_r}$.

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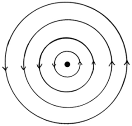


Figure 1: Flow of X_0; before the bifurcation.

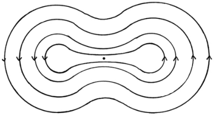
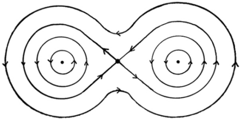


Figure 2: Flow of X_r; for $r = 1/2$.



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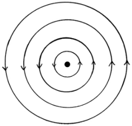

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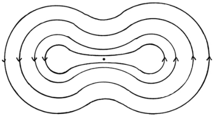

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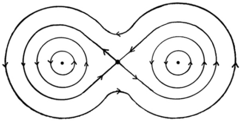

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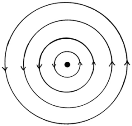

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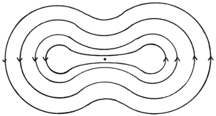

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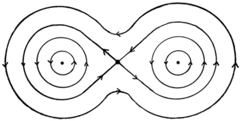

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- At $r = 1$ bifurcation into **negative hyperbolic** orbit h and **elliptic** orbit e_2 of double the period.

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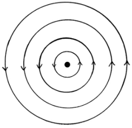

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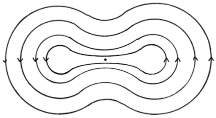


Figure 2: Flow of X_r; for $r = 1/2$.

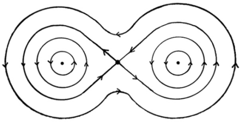


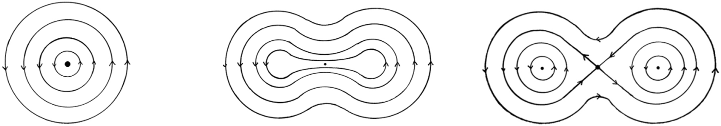
Figure 3: Flow of X_1; after the bifurcation.

- At $r = 0$ have an **elliptic** orbit E,
- At $r = 1$ bifurcation into **negative hyperbolic** orbit h and **elliptic** orbit e_2 of double the period.

Local $CH_*^{\mathbb{Q}} = H_*(\mathbb{Q}\langle \text{good orbits winding } k \text{ times around } N_\gamma\rangle, \partial)$.
For $k = 2$: 1 generator before (E^2) bifurcation and 1 generator after (e_2).

The period doubling bifurcation for a simple Reeb orbit

Modify flow by the return map $\phi_r :=$ rotate by $180^\circ \circ \varphi_\epsilon^{X_r}$.



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Local $CH_*^{\mathbb{Z}}$ sees more!
Local $CH_*^{\mathbb{Z}} = H_*(\mathbb{Z}\langle$good *and* bad winding k times around $N_\gamma\rangle \otimes \mathbb{Z}[[u]])$.

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For the contact form λ_0, the only Reeb orbit in N which winds twice around N is the double cover E^2 of E

$$CH_*^{\mathbb{Z}}(\lambda_0, N) = \begin{cases} \mathbb{Z} & \text{if } * = 0 & \text{(generated by } E), \\ \mathbb{Z}/2 & \text{if } * = 2k+1 & \text{(generated by } u^k E), \\ 0 & \text{otherwise.} \end{cases}$$

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The 2-torsion before the bifurcation sees the bad Reeb orbit that can be created in the bifurcation!

The end!

Thanks!