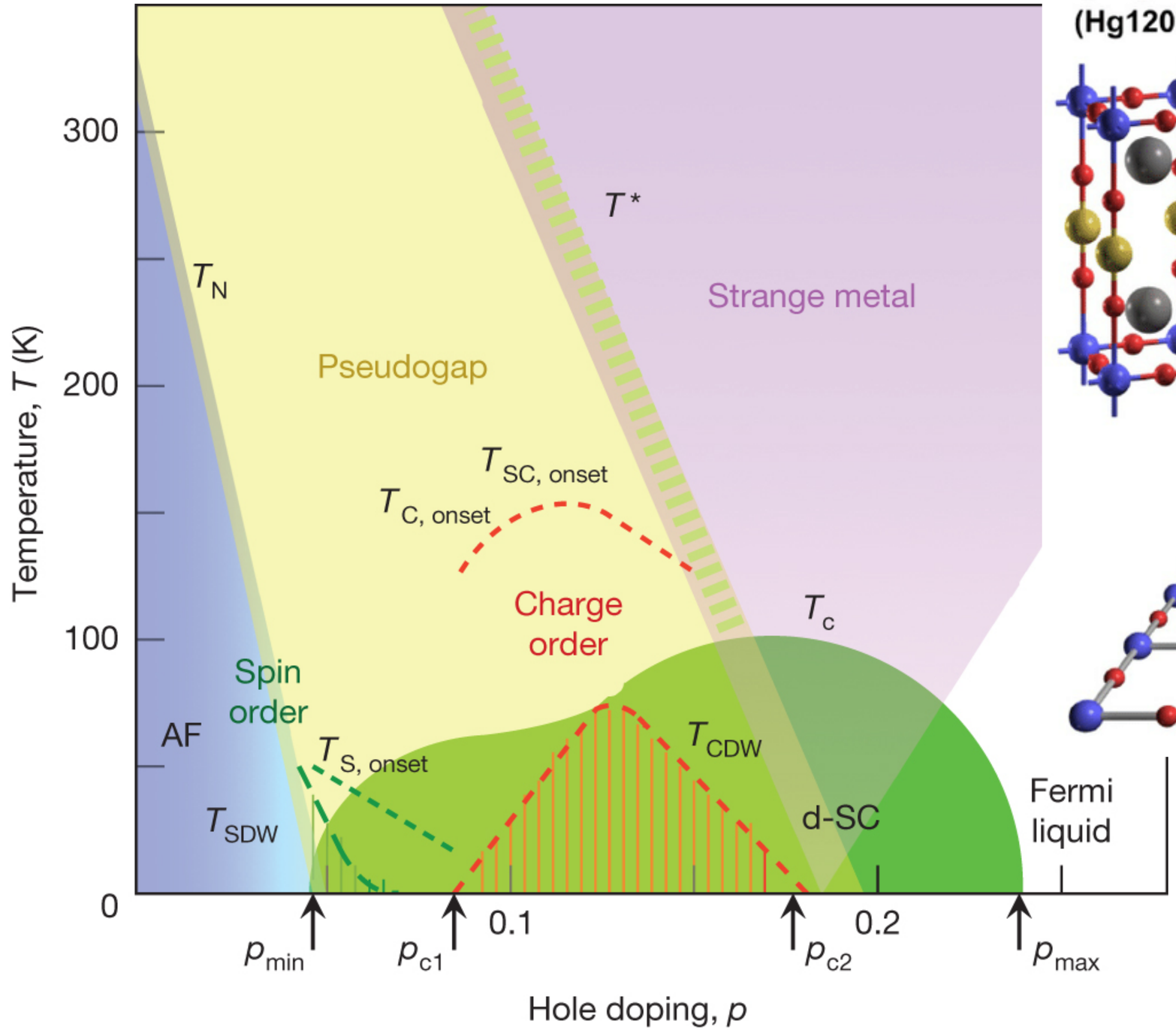
Observation of the Yamaji effect in a cuprate and FL* theory of the pseudogap

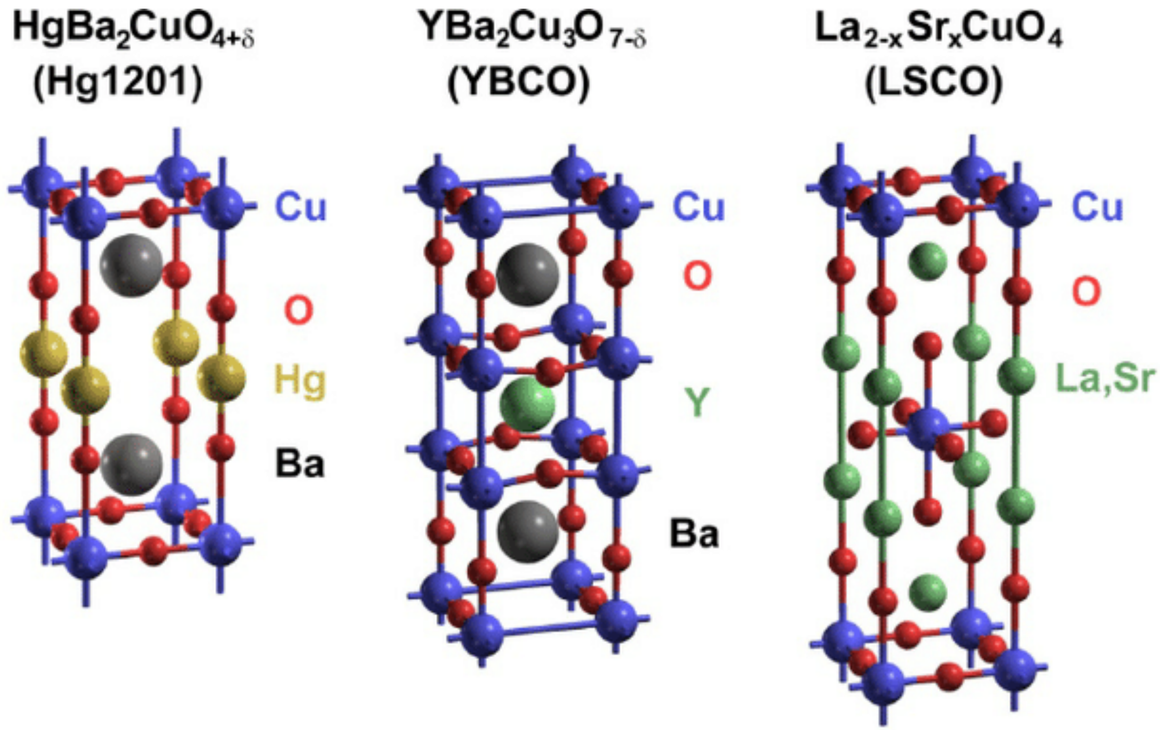
Simons Ultra-Quantum Matter Collaboration Meeting
Institute for Advanced Study, Princeton
October 20, 2025

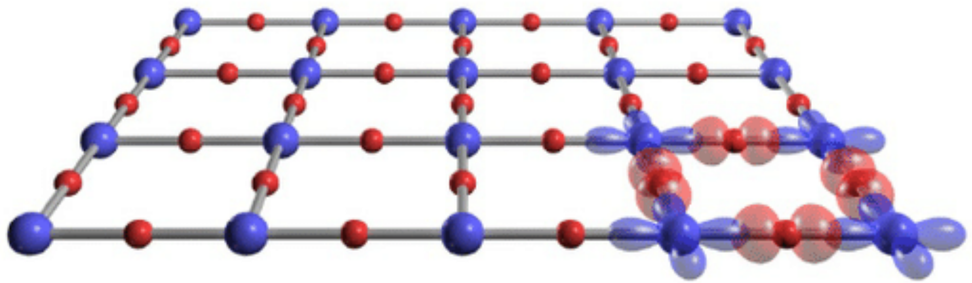
Subir Sachdev

PHYSICS
HARVARD

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)







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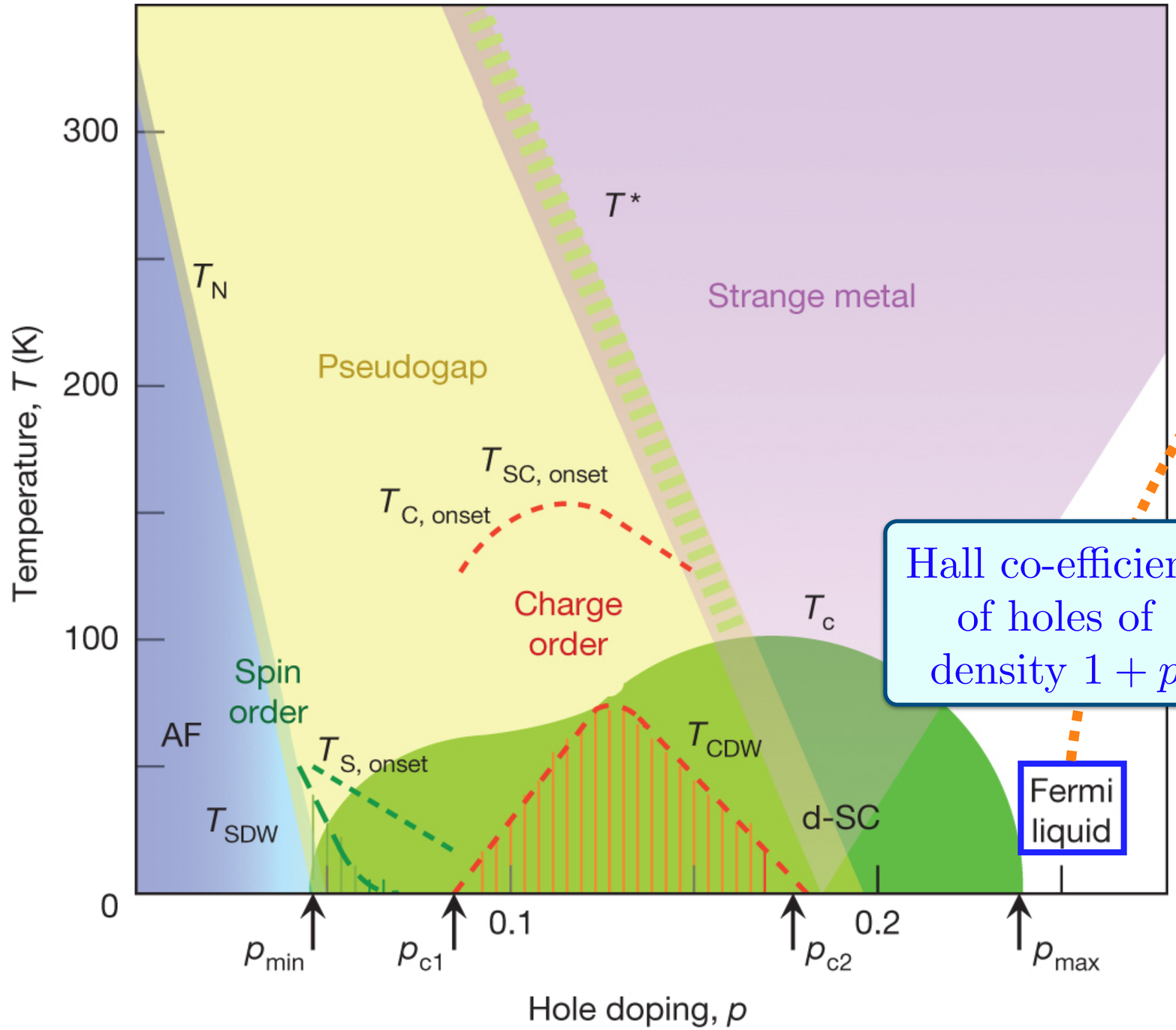
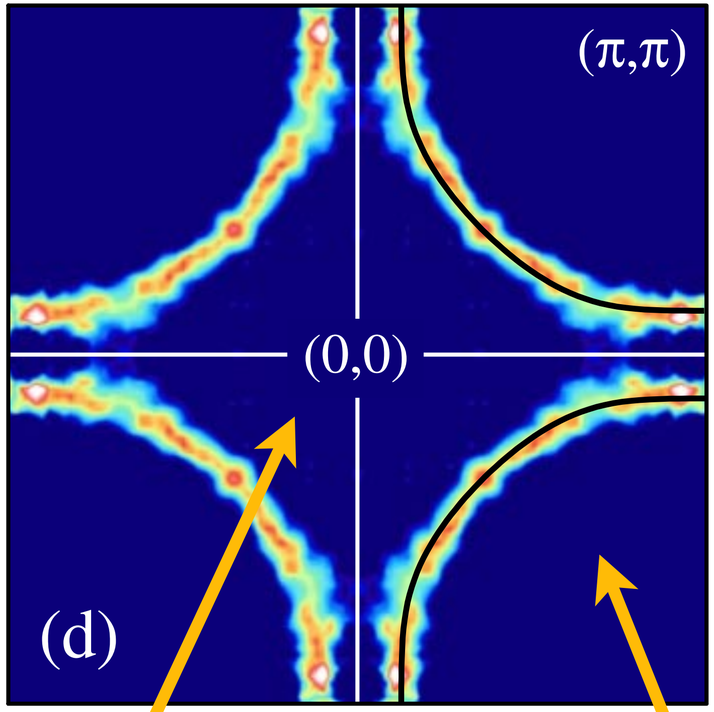


M. Platé *et al.*, PRL **95**, 077001 (2005)



Hall co-efficient of holes of density $1+p$

1-p electrons

1+p holes

Luttinger, 1960: Area enclosed by the Fermi surface is the same as that for free fermions *with the same symmetry.*

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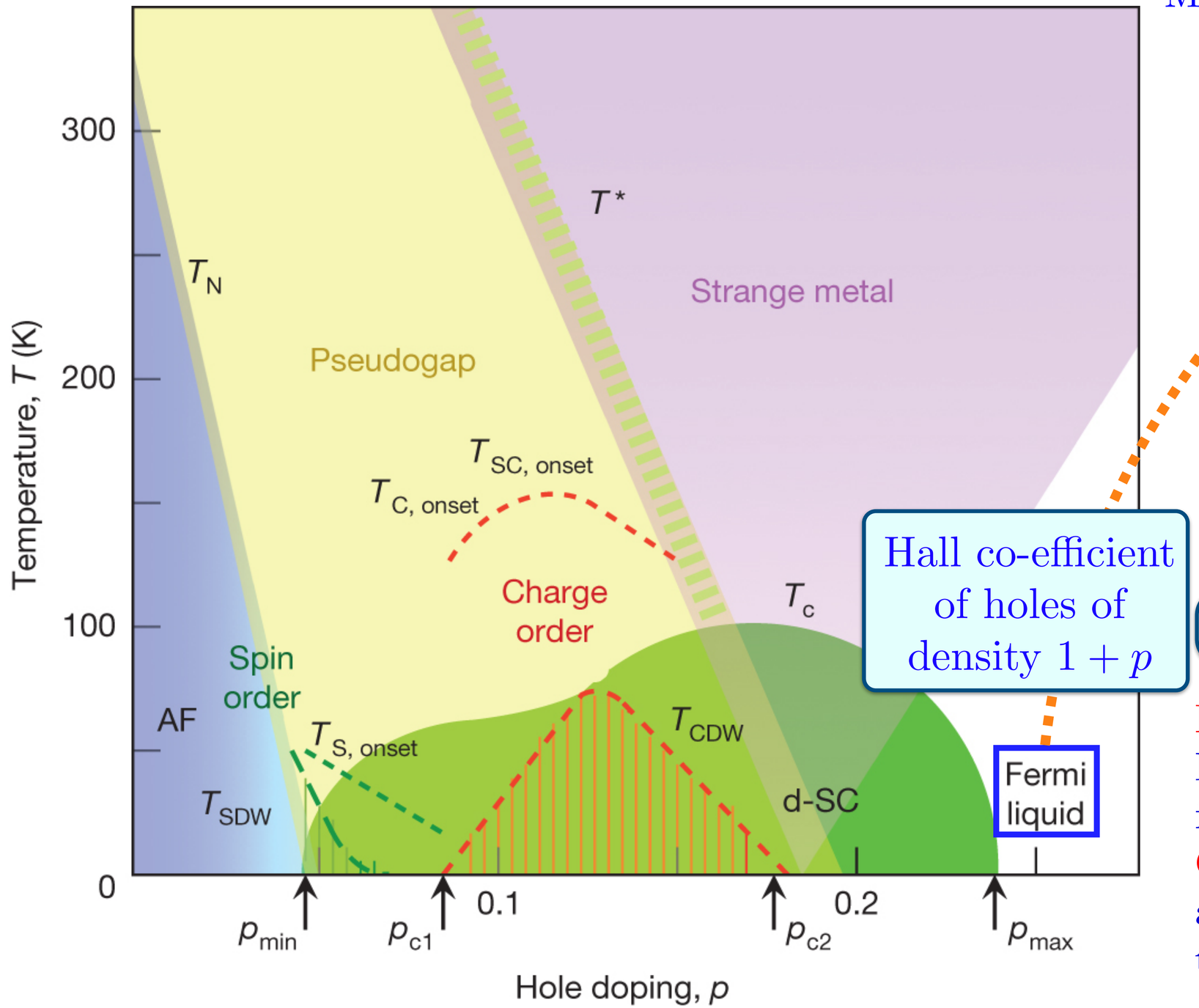

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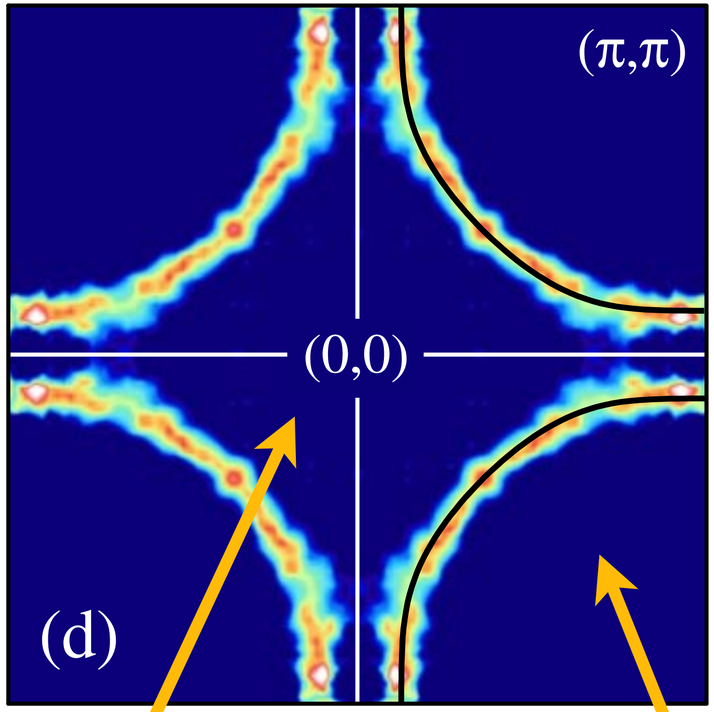

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Oshikawa, 2000: Area constrained by a 't Hooft anomaly of global U(1) and translations

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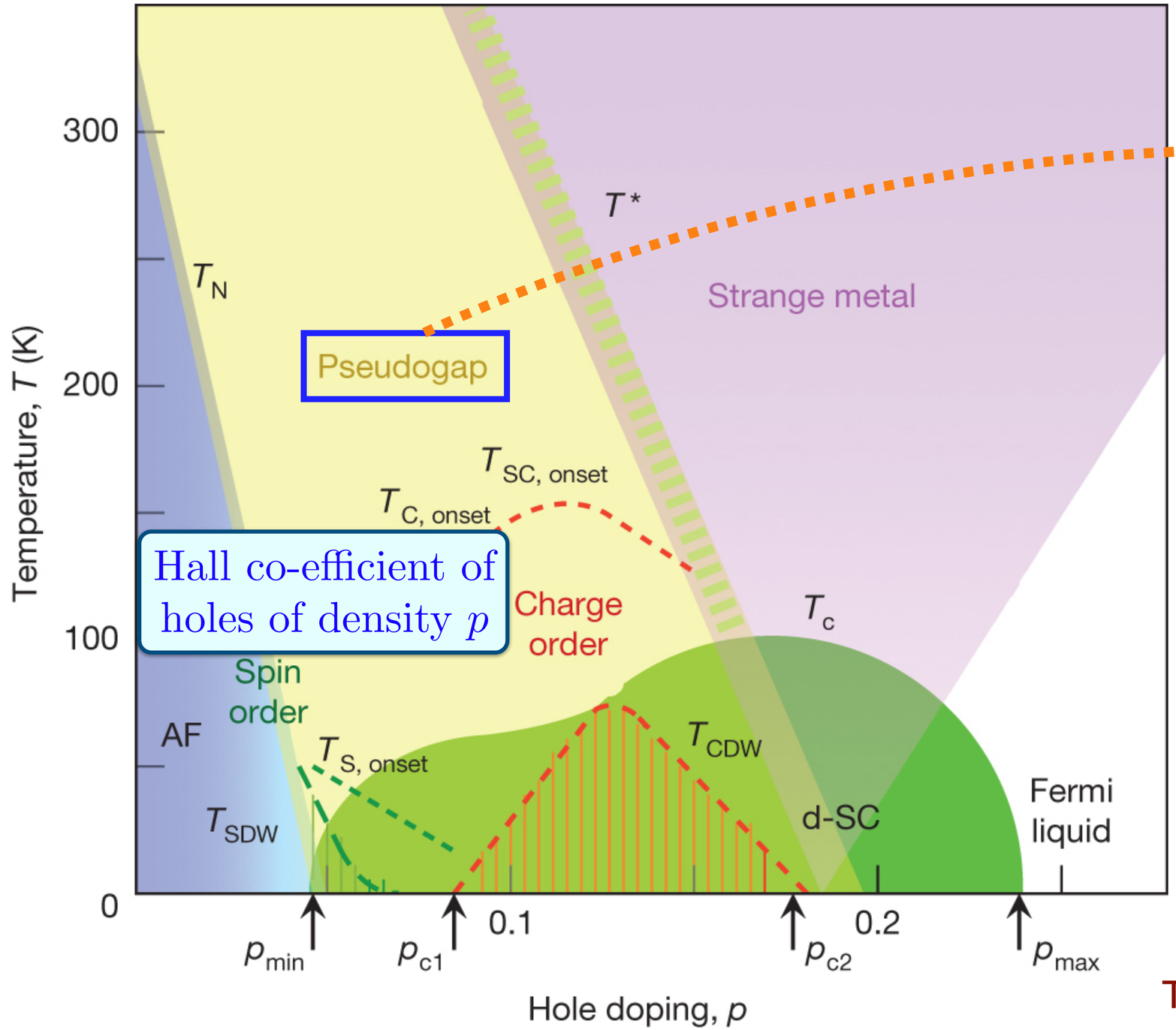
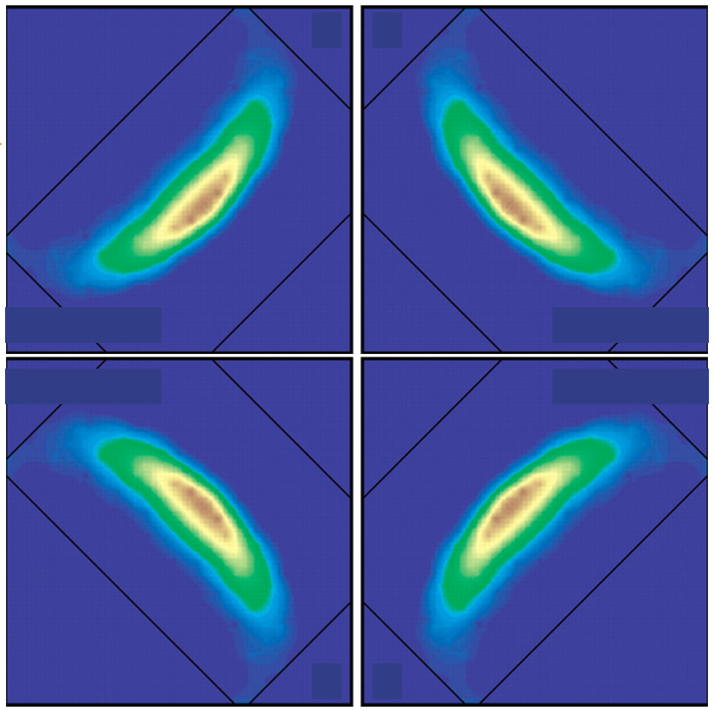
Kyle M. Shen *et al.*, Science **307**, 901 (2005)





`Fermi arcs' ?

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003)

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nature physics
arXiv:2411.10631
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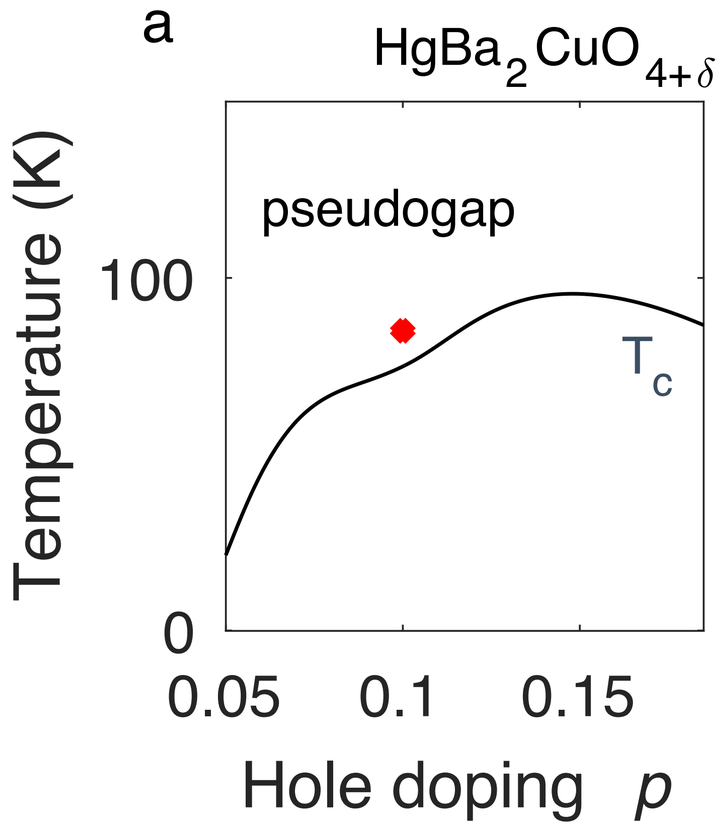

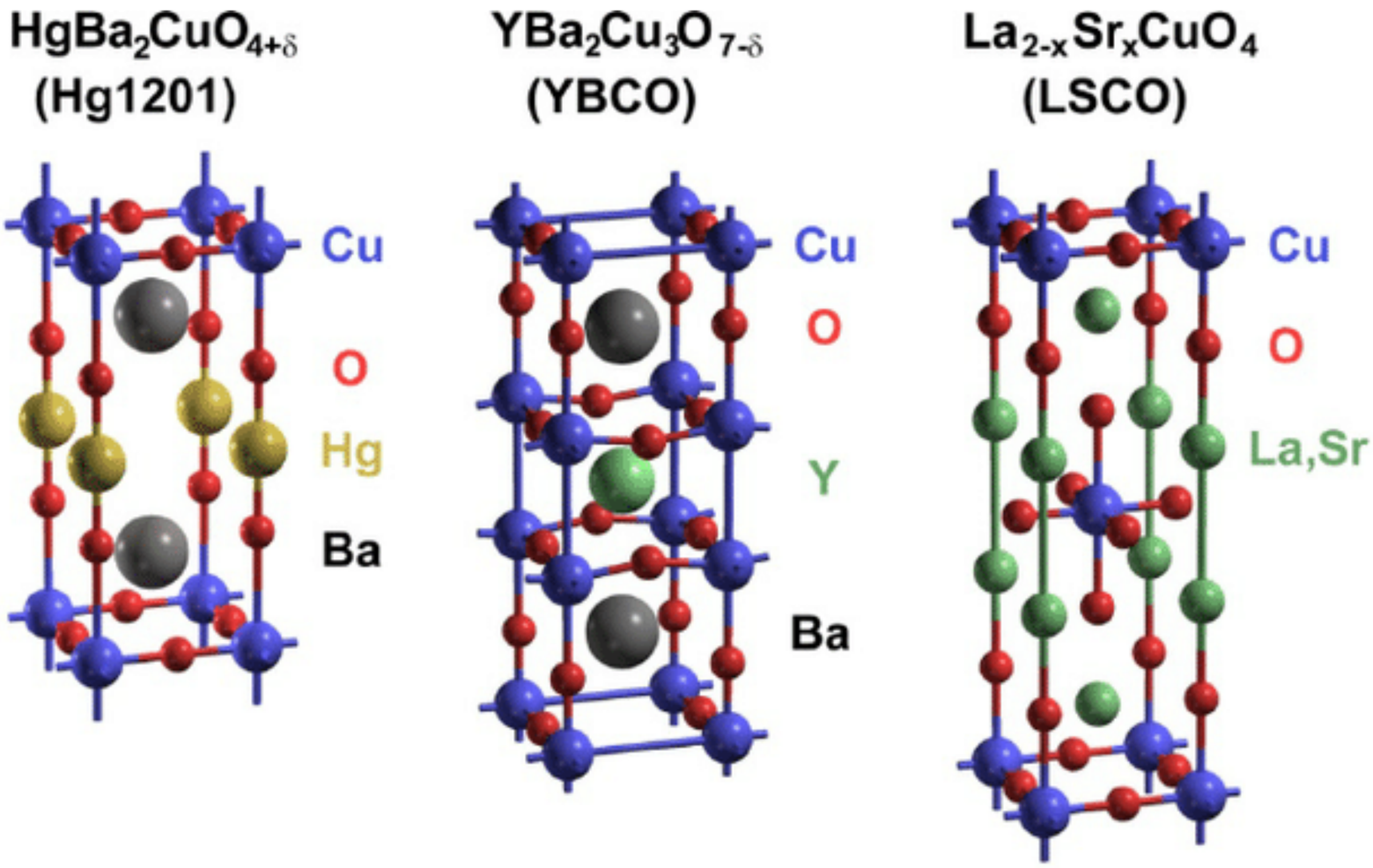

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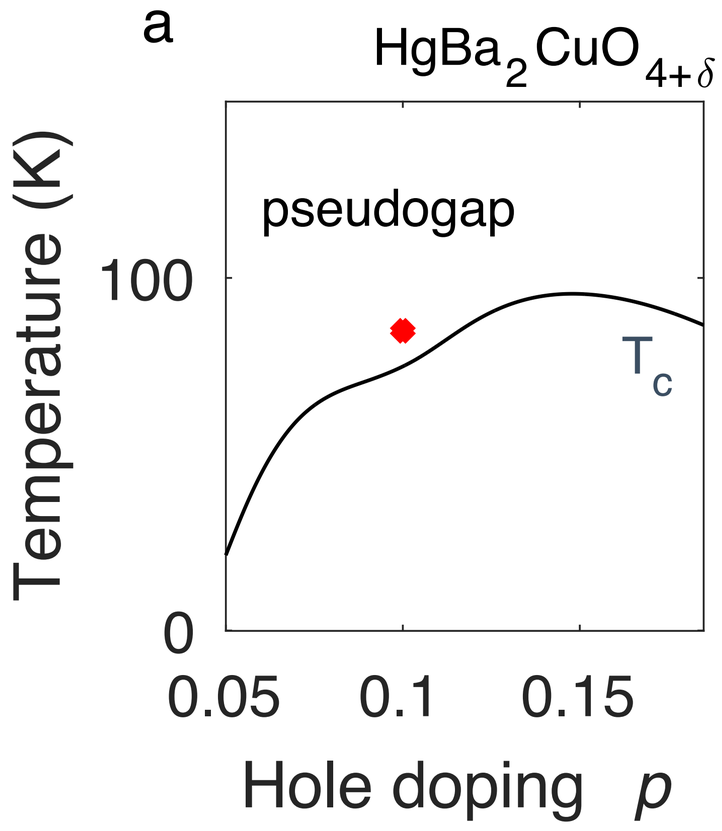

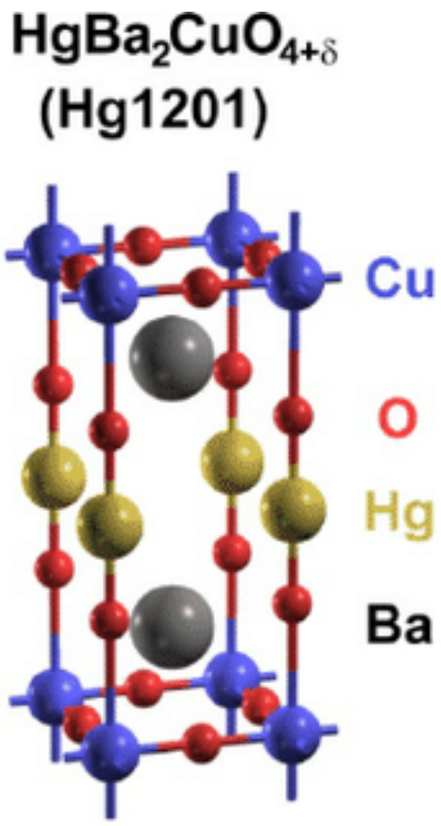

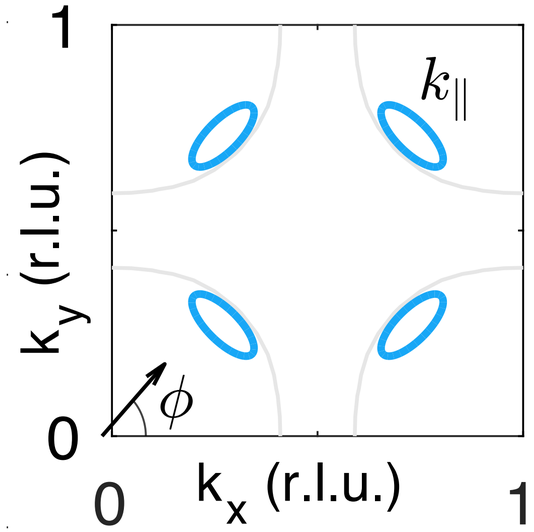

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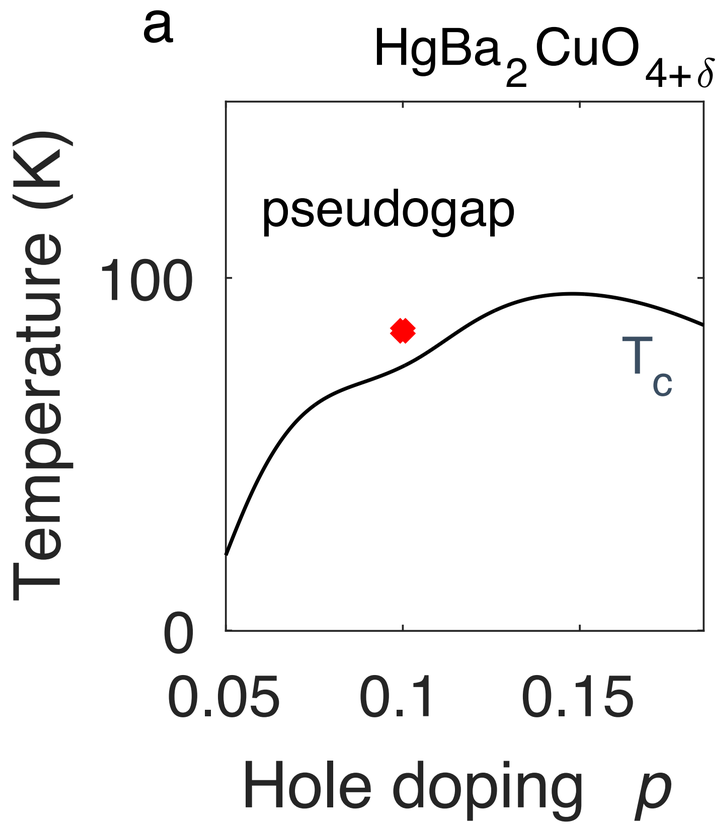

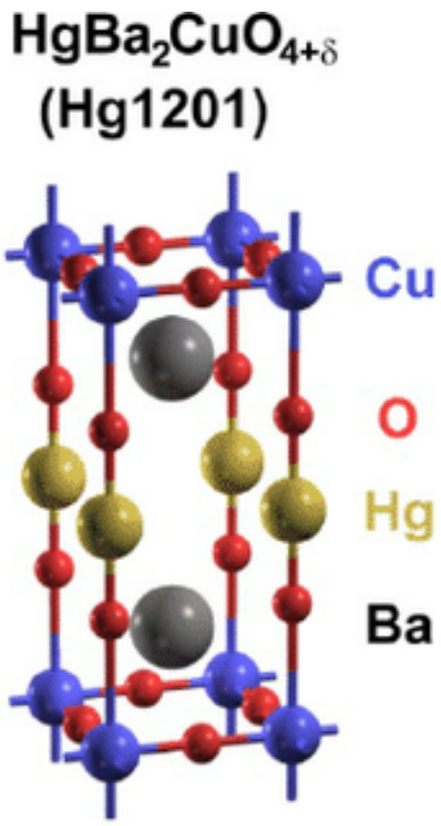

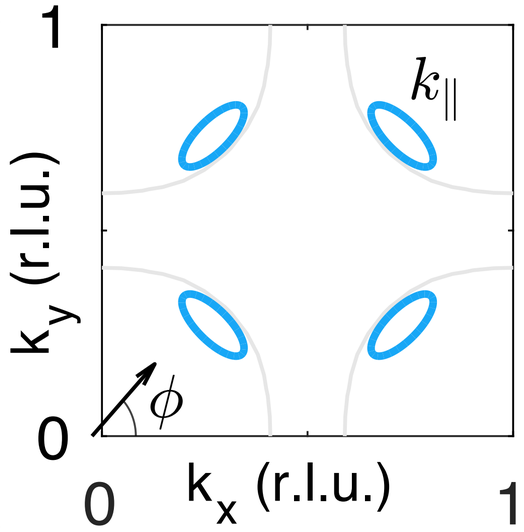

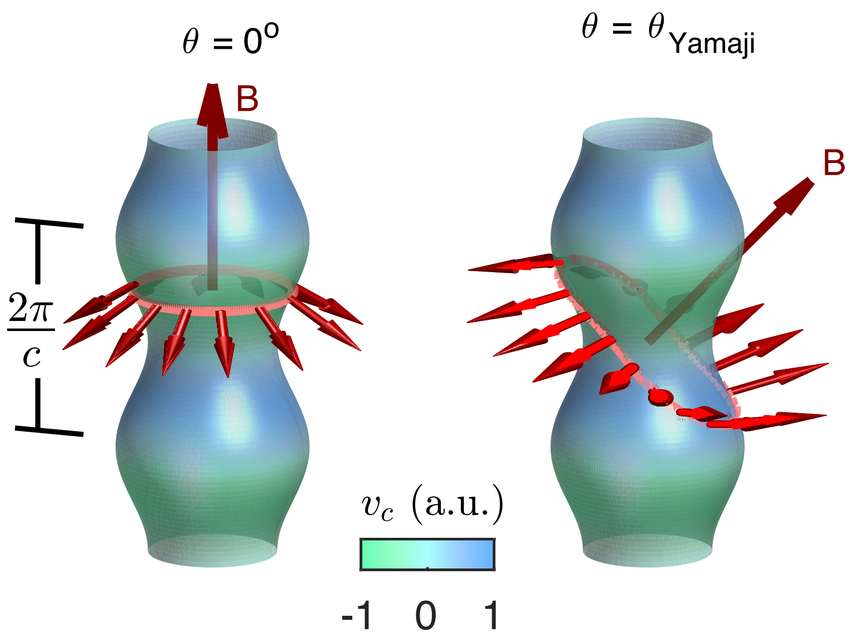

At the Yamaji angle, the orbits in the plane orthogonal to $\boldsymbol{B}$ have an area which is independent of momentum in the c direction, to first order in the hopping along the c direction.

K. Yamaji JPSJ **58**, 1520 (1989)

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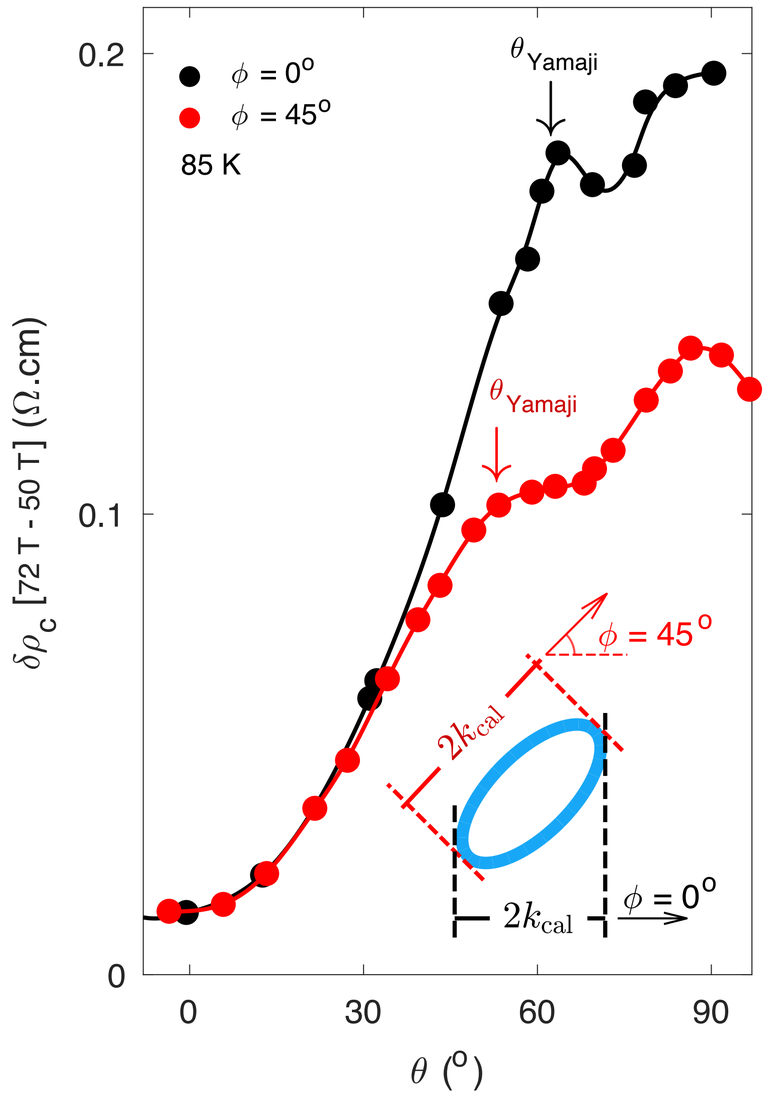
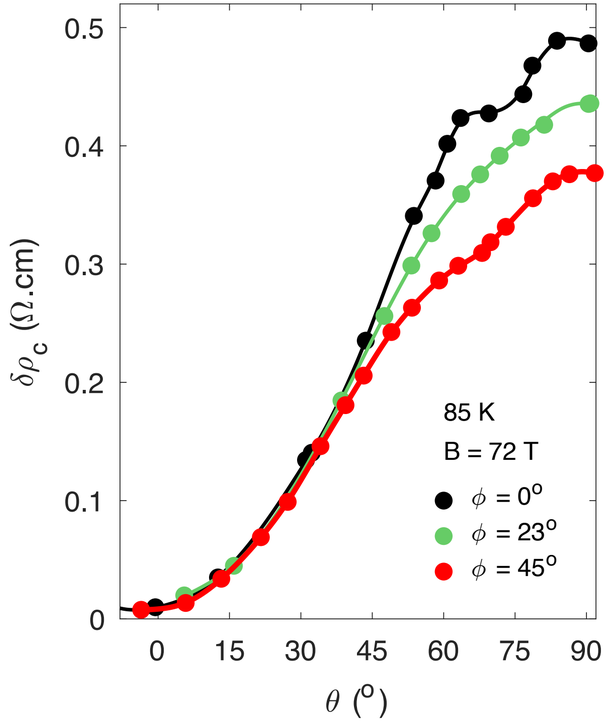
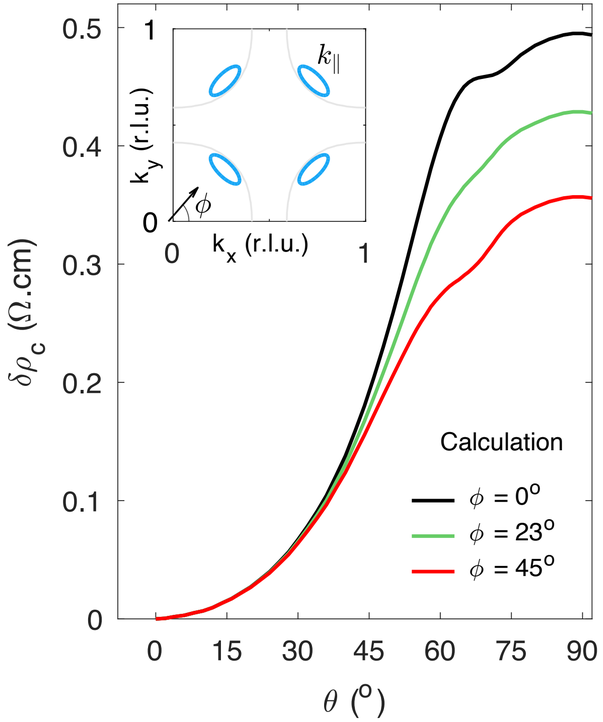




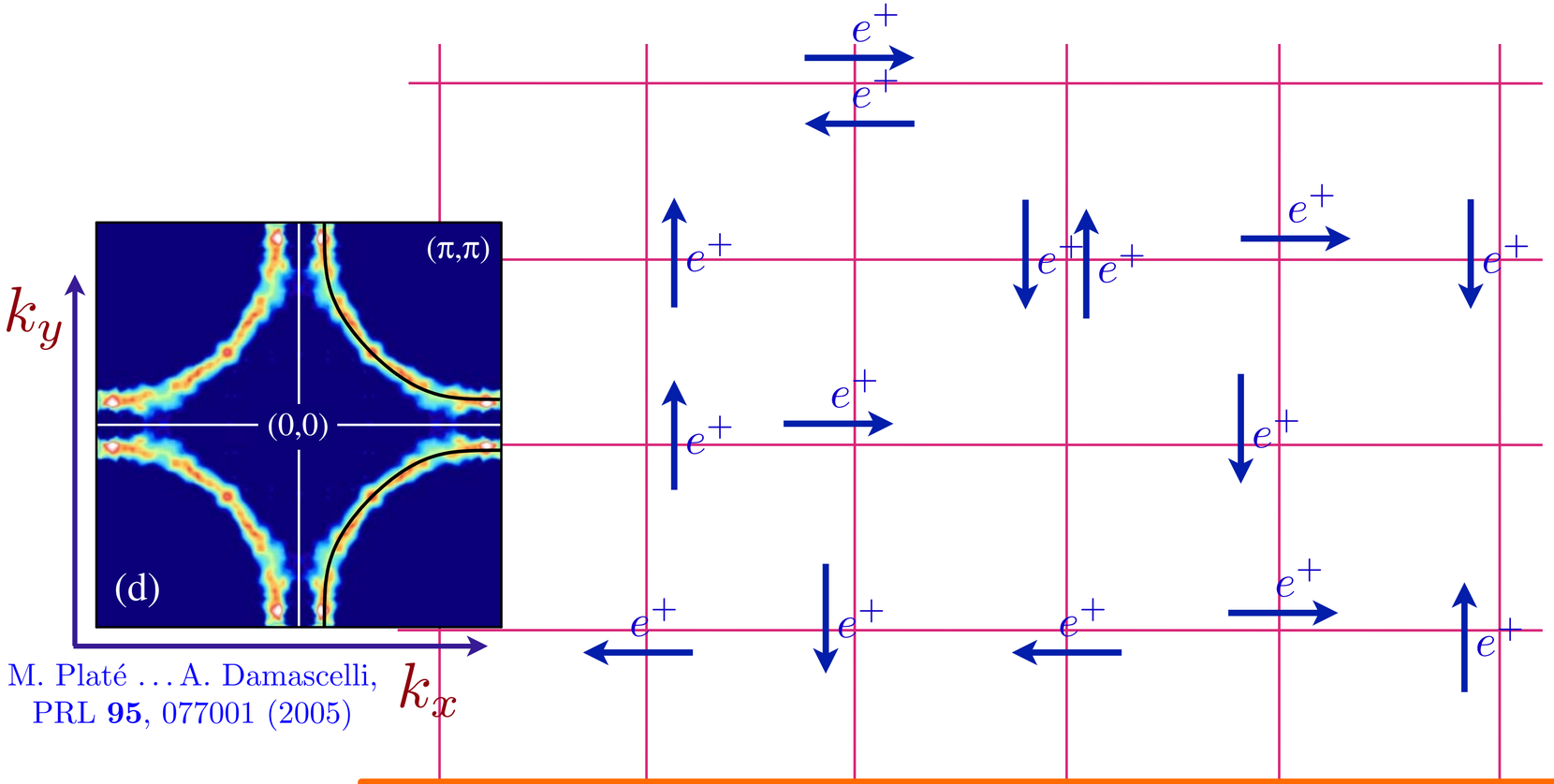


Doping
$p = 0.1$

"The small size of the pockets determined from the Yamaji effect is . . . approximately 1.3% of the Brillouin zone area"

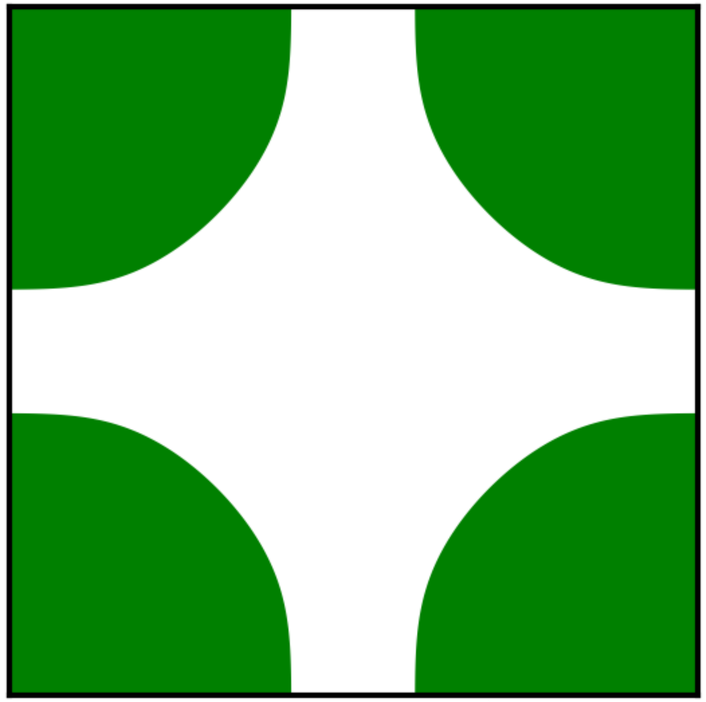
Metals obtained by doping Mott insulators

Ordinary metal



M. Platé ... A. Damascelli, PRL **95**, 077001 (2005)

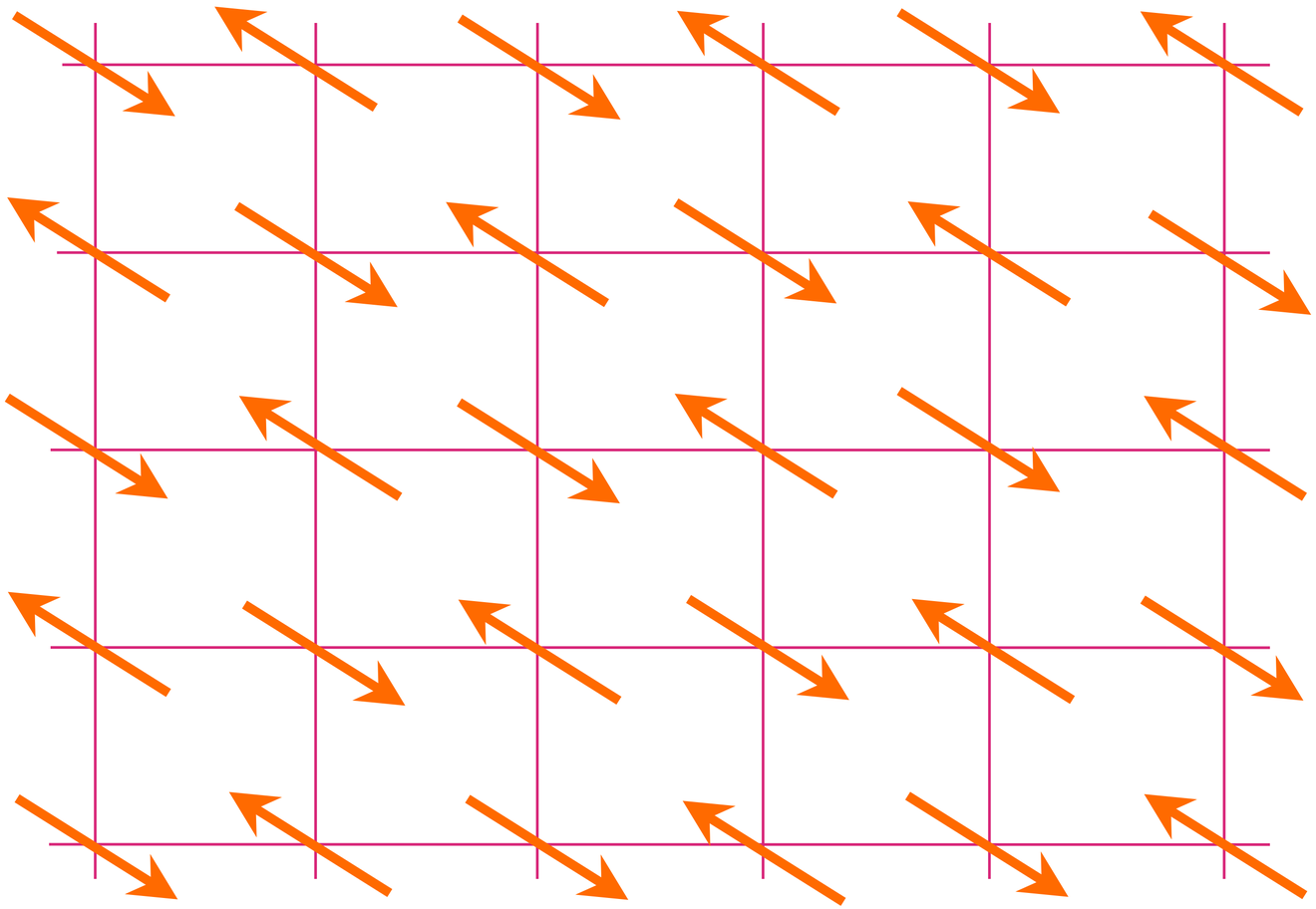
Luttinger area.
No broken symmetry



$$\text{Area } (1+p)/2$$

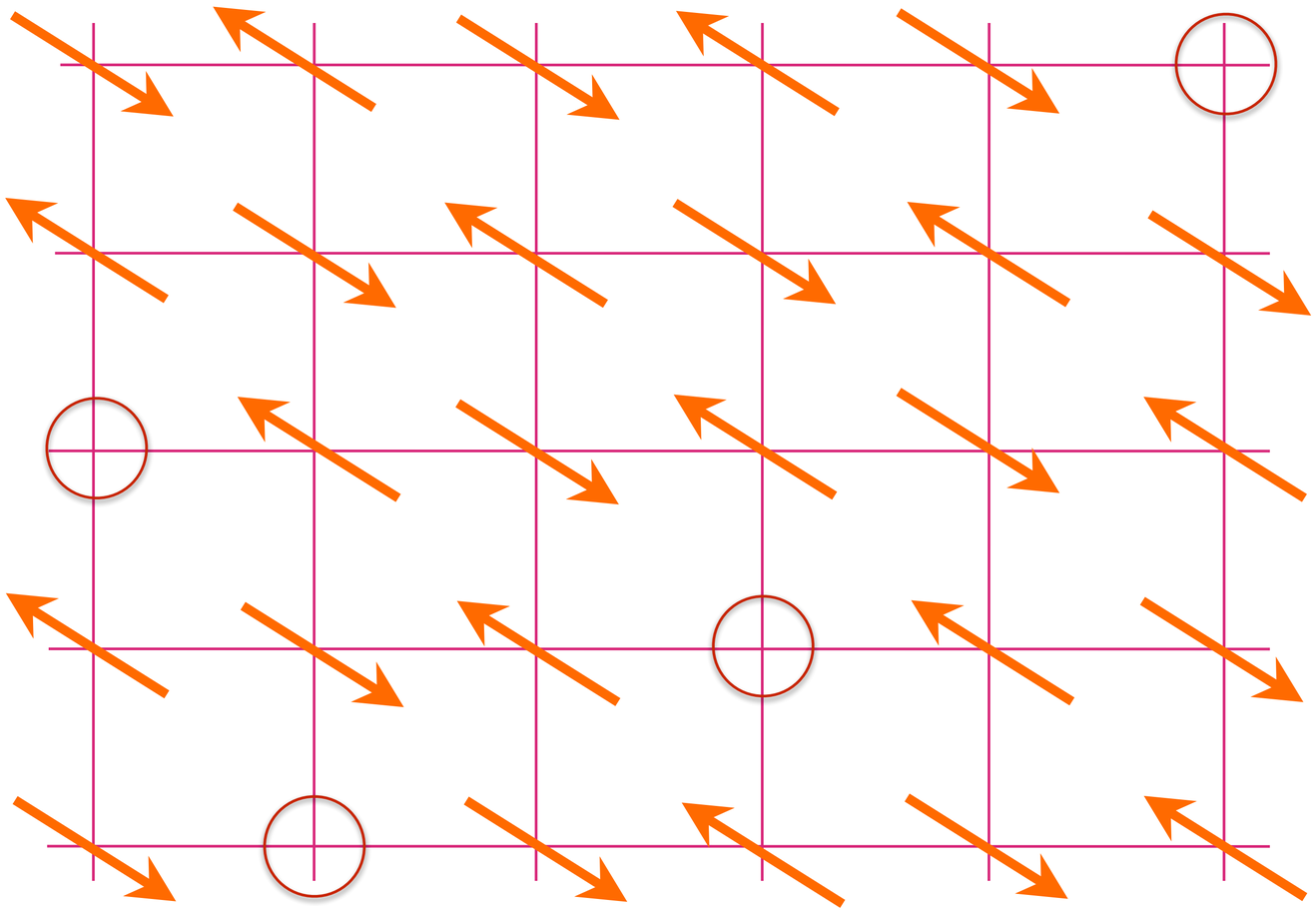
At large p, we obtain a gas of nearly free fermionic holes of density $1+p$ (relative to the filled band with 2 electrons per site)

Insulating antiferromagnet



Doping an insulating antiferromagnet with holes of density p

AF metal



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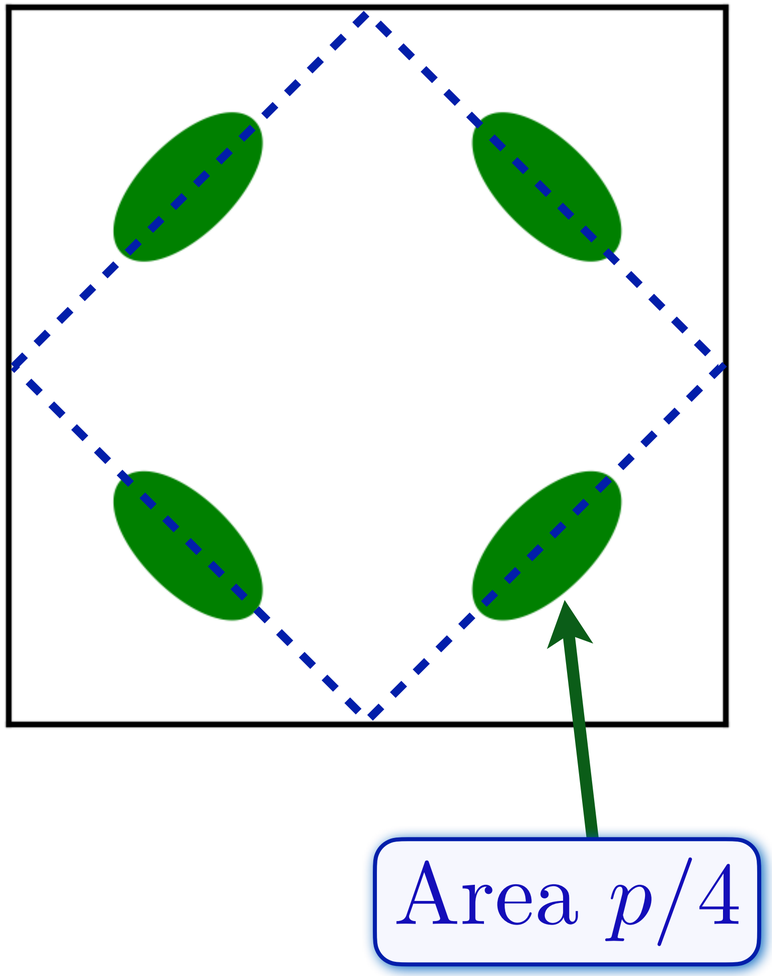

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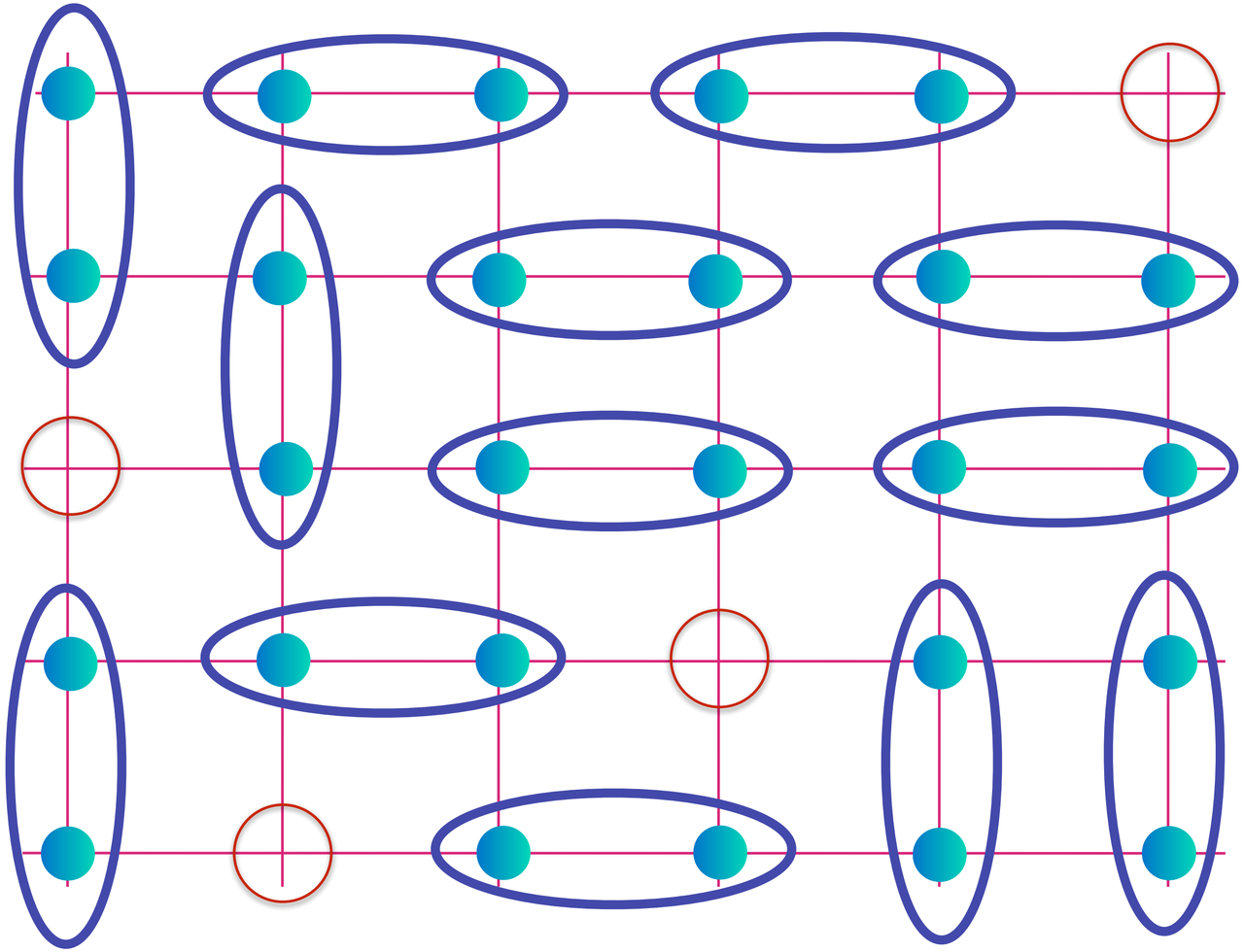
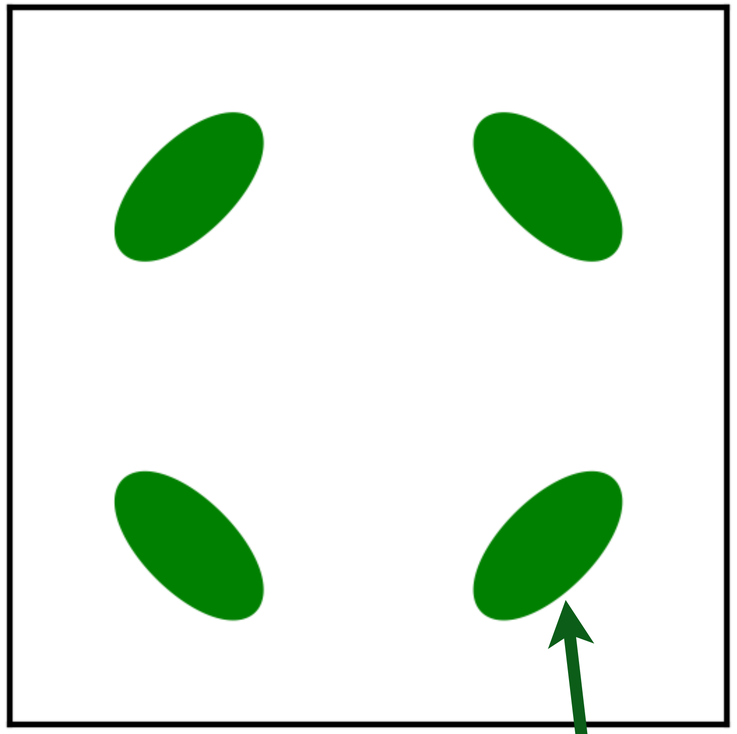
Holon metal

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Spin liquid with density p of spinless, charge $+e$ 'holons'. No coherent inter-layer transport, so no Yamaji effect.





Area $p/4$

$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003);
R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007);
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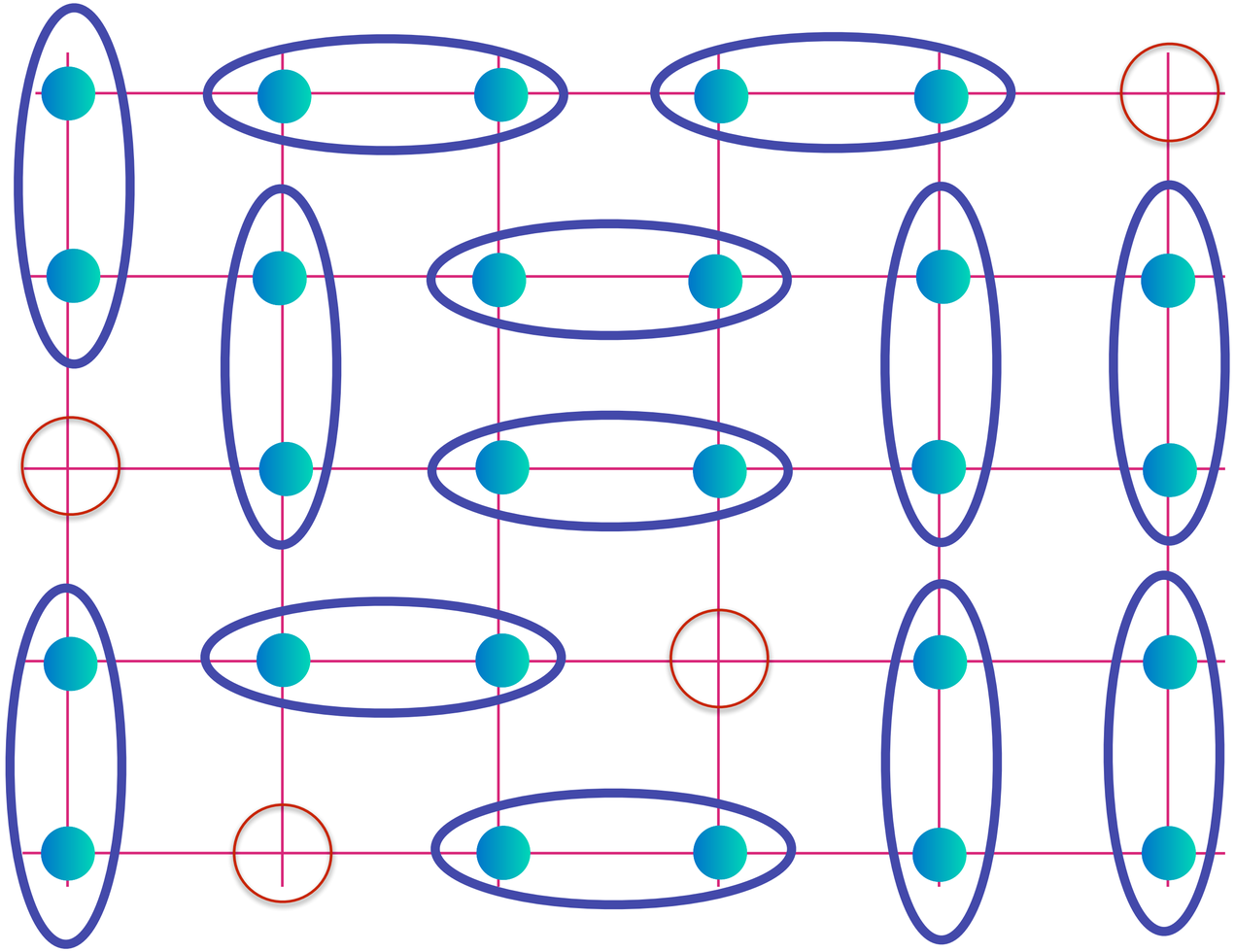

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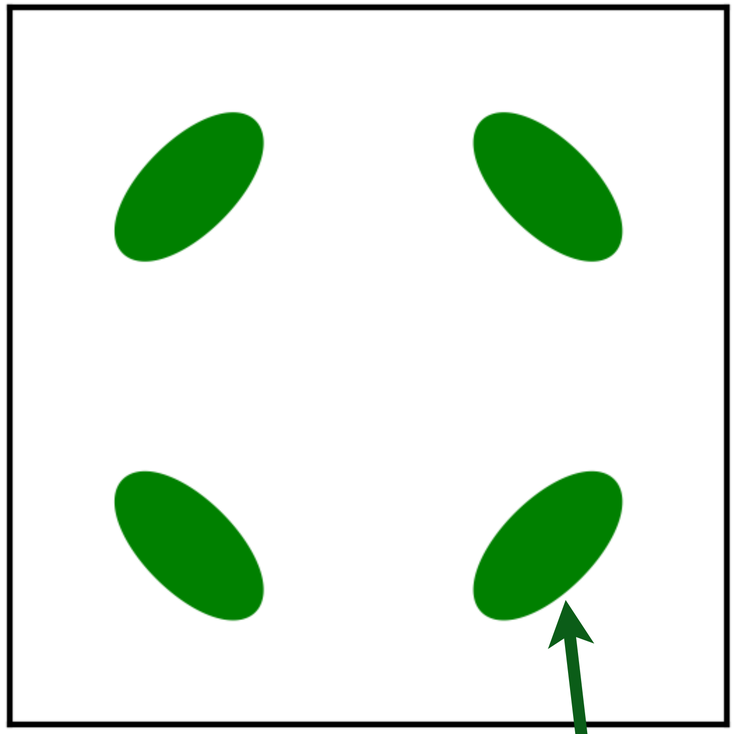

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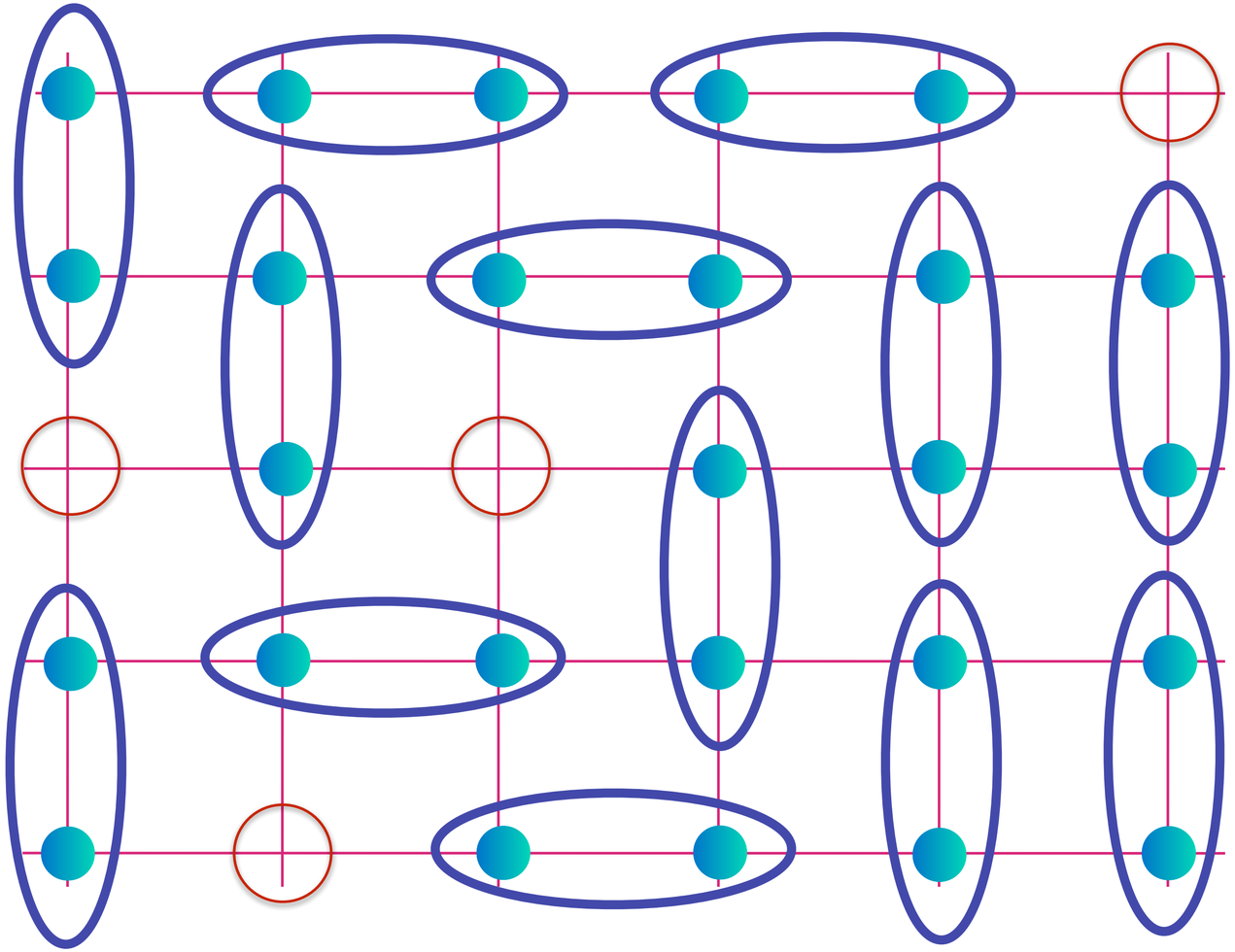

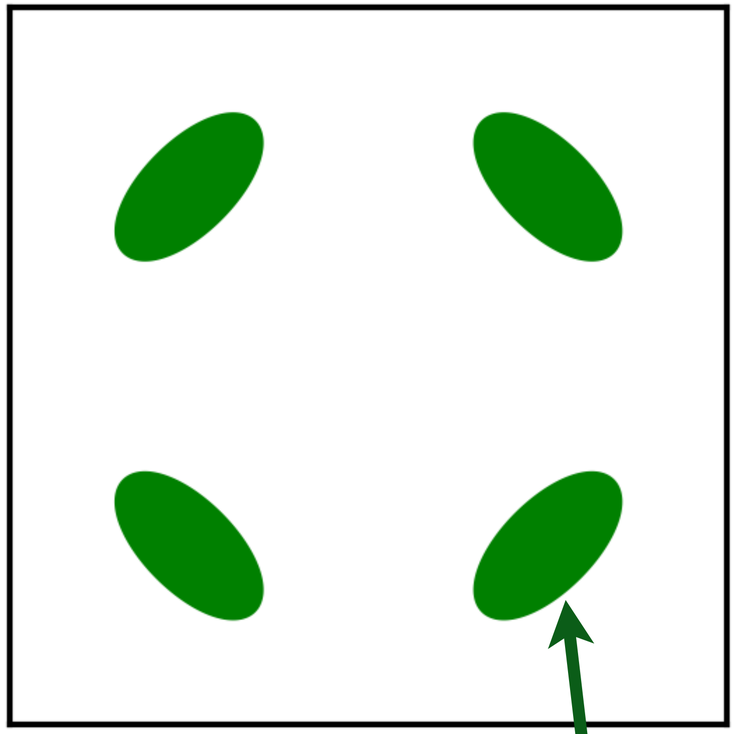

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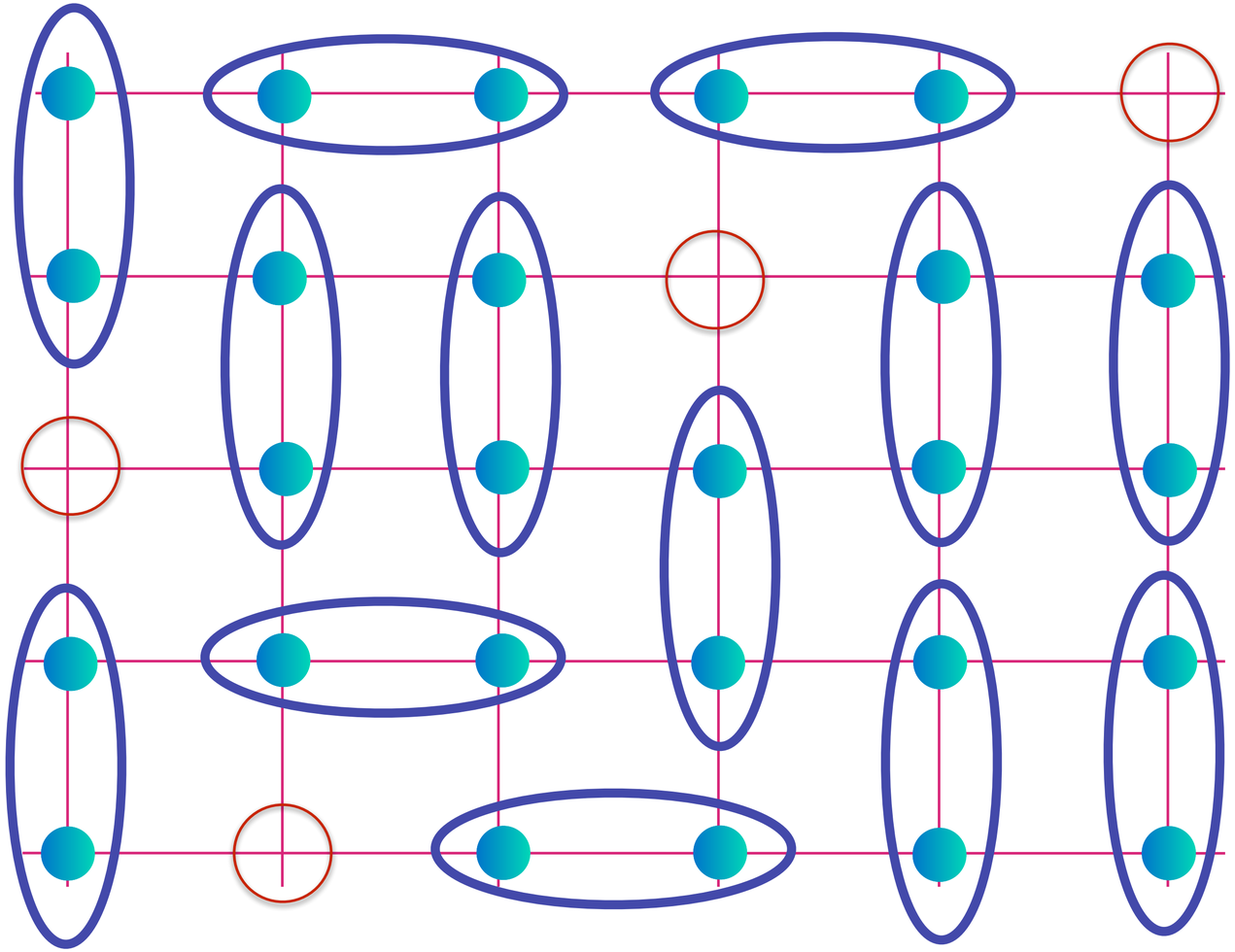

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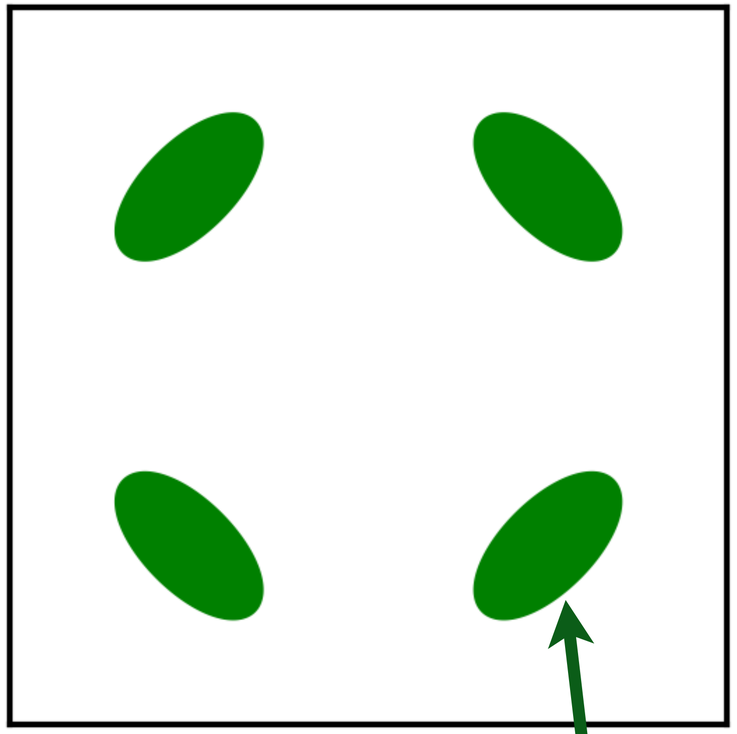

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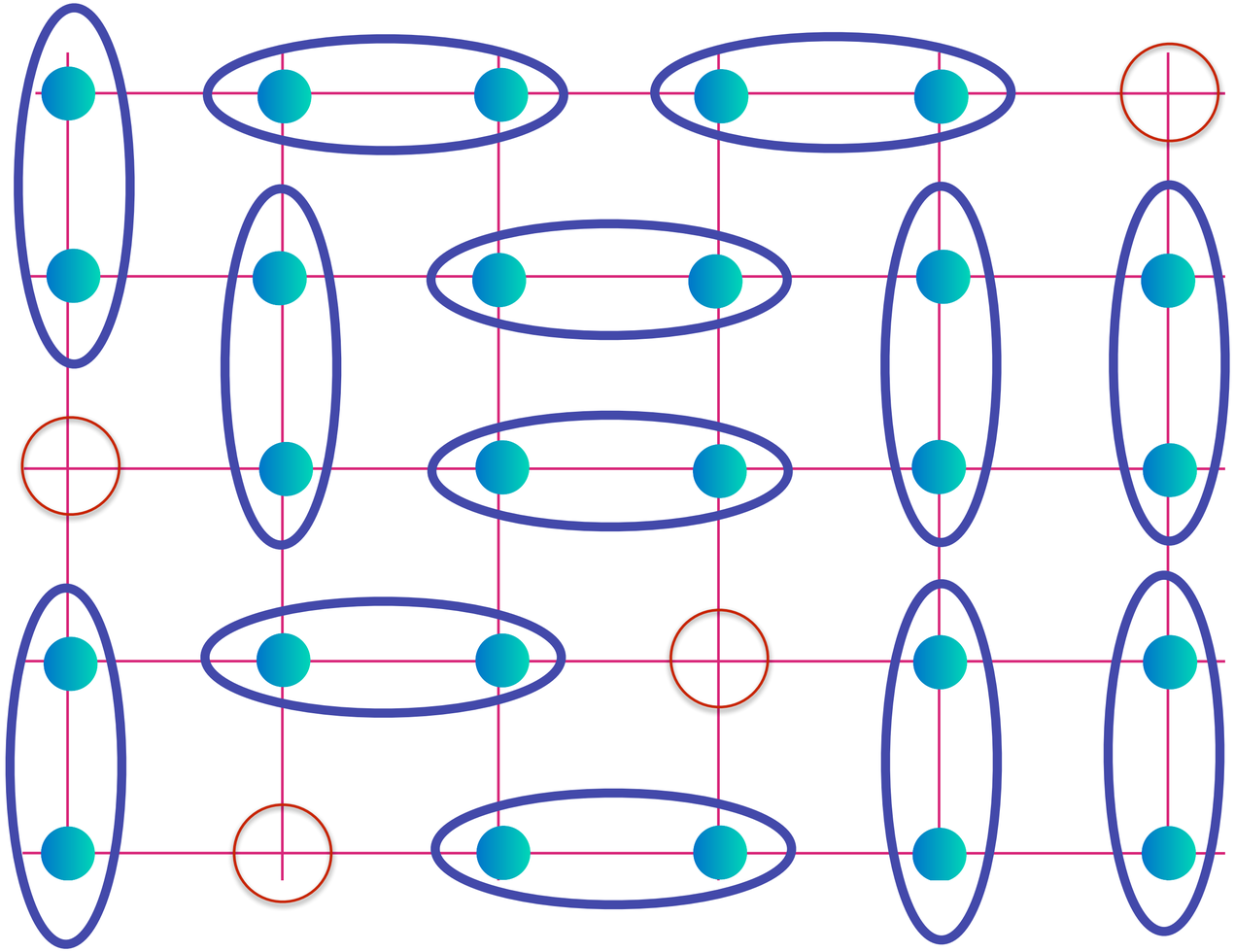

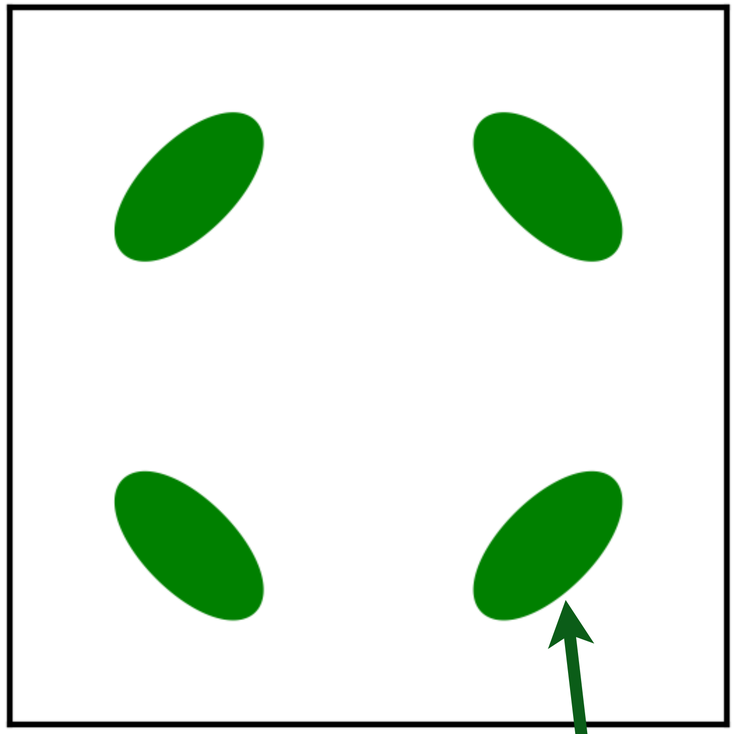

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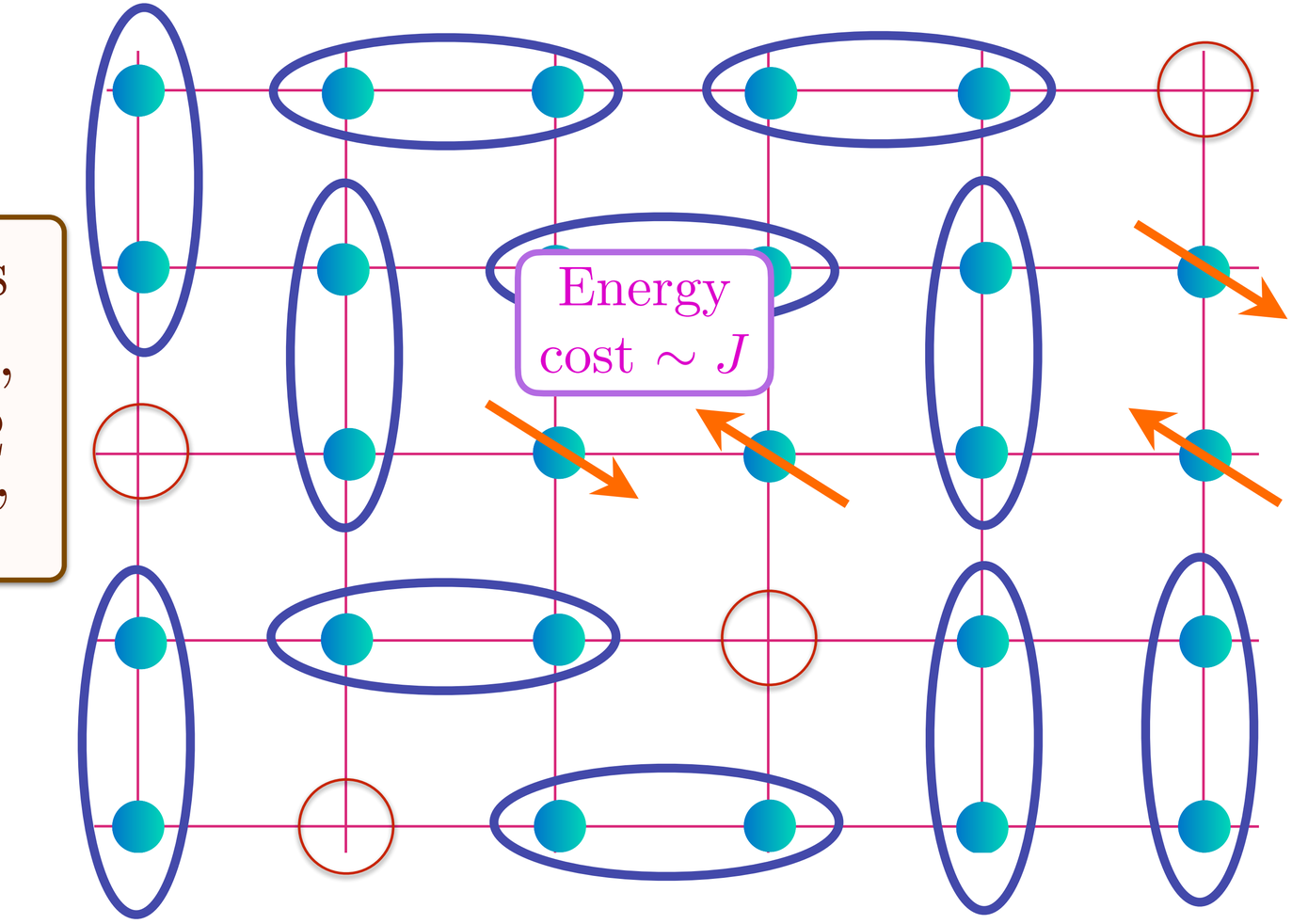

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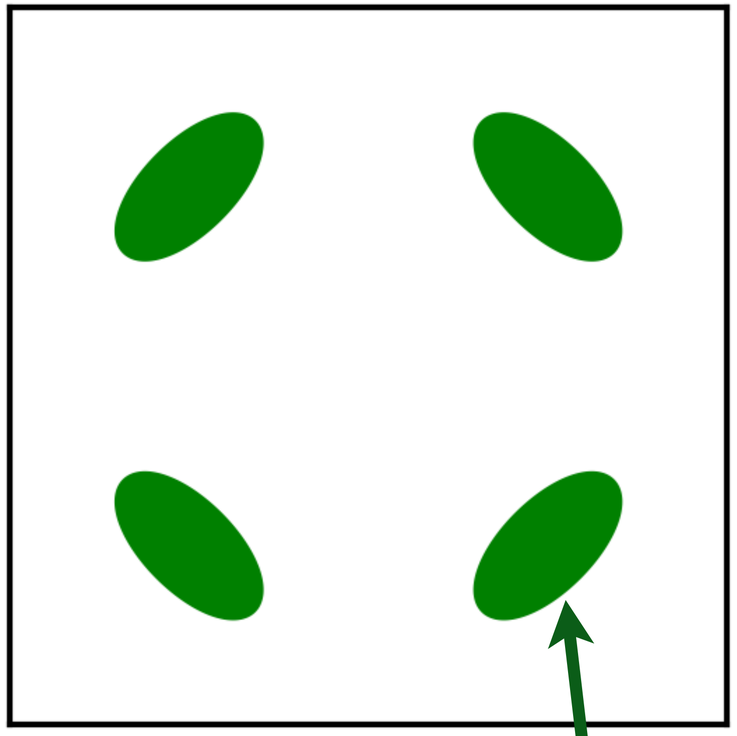

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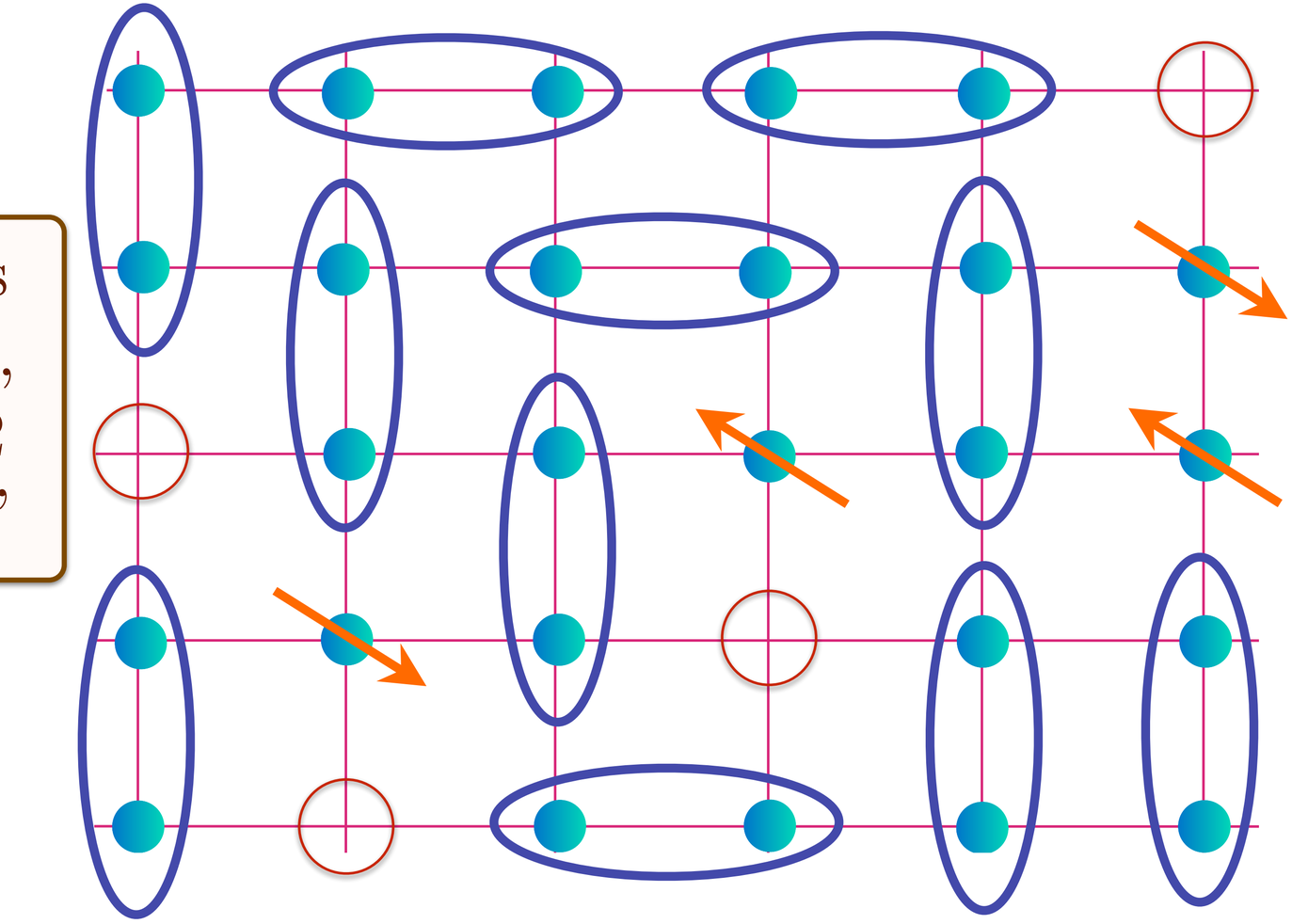

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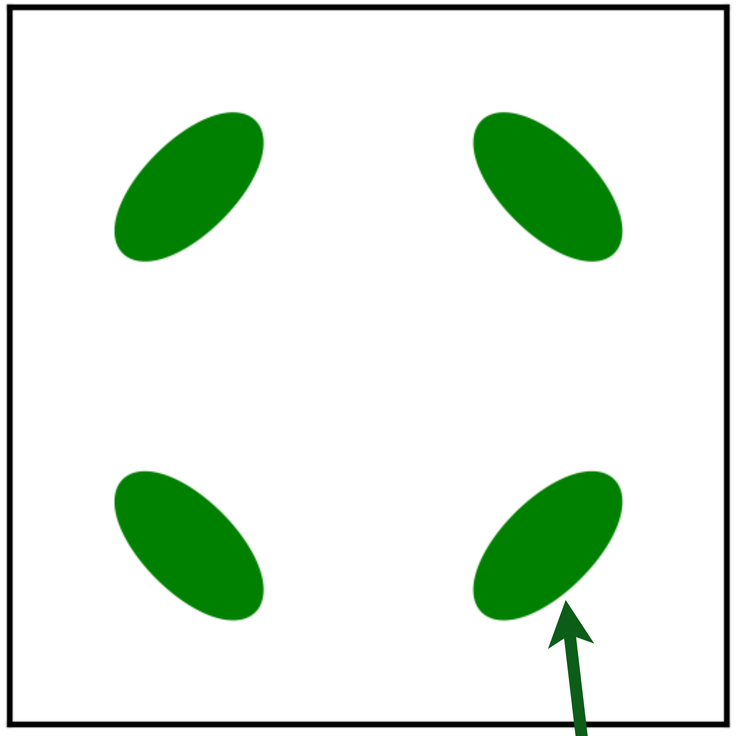

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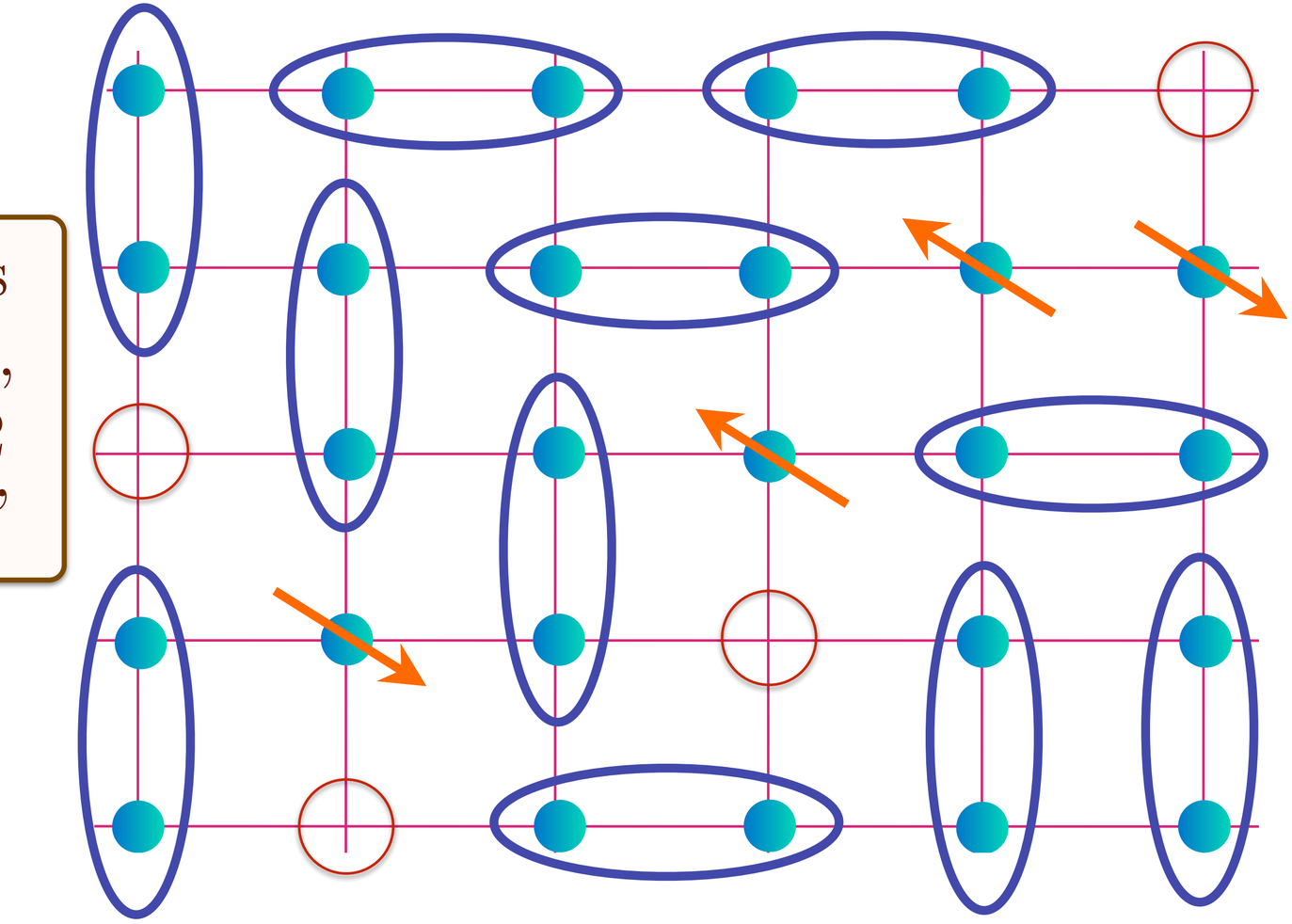

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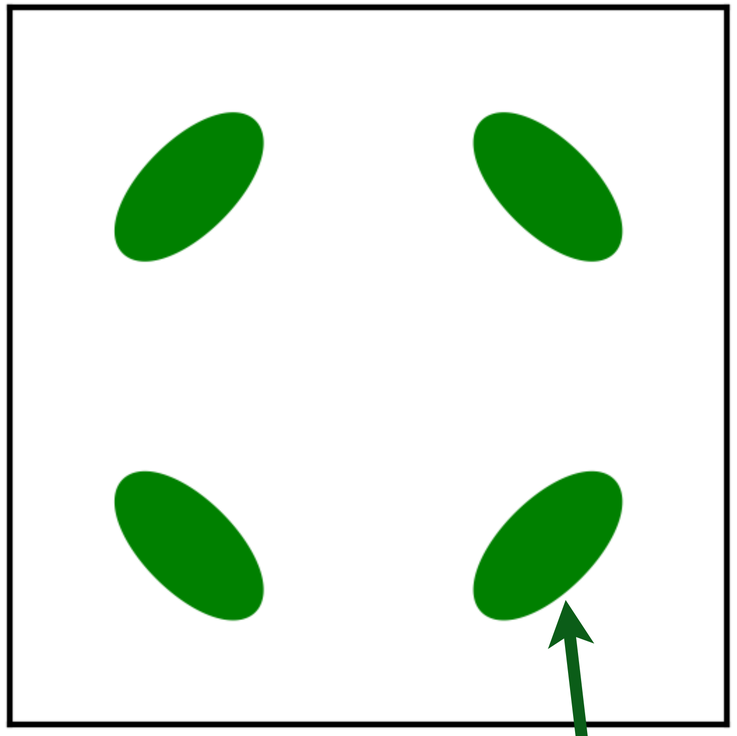

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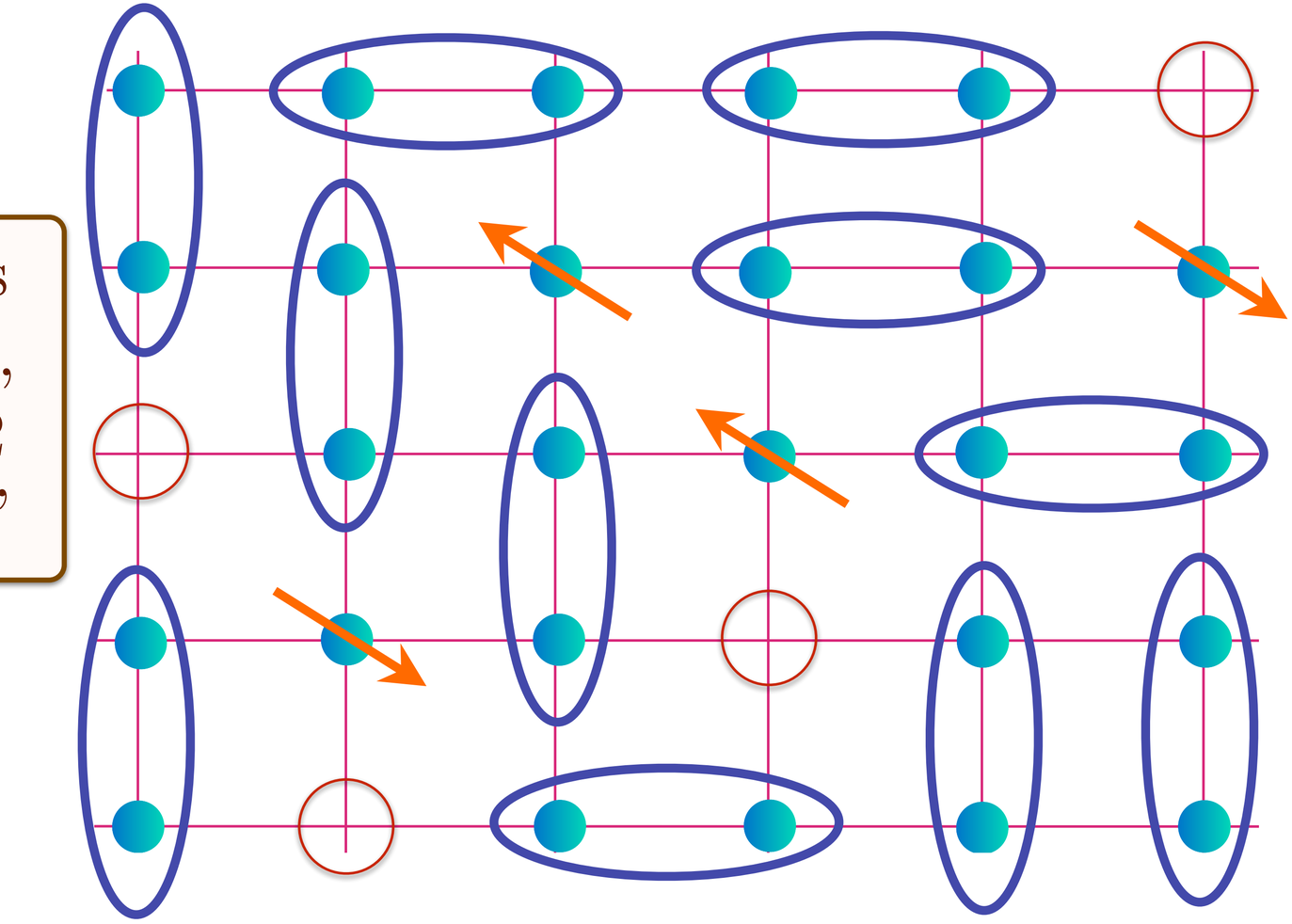

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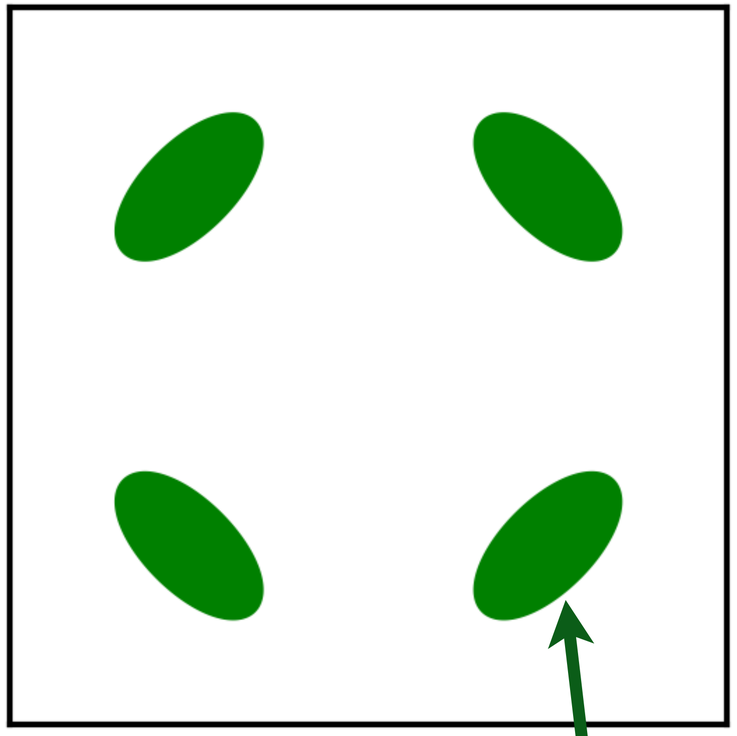

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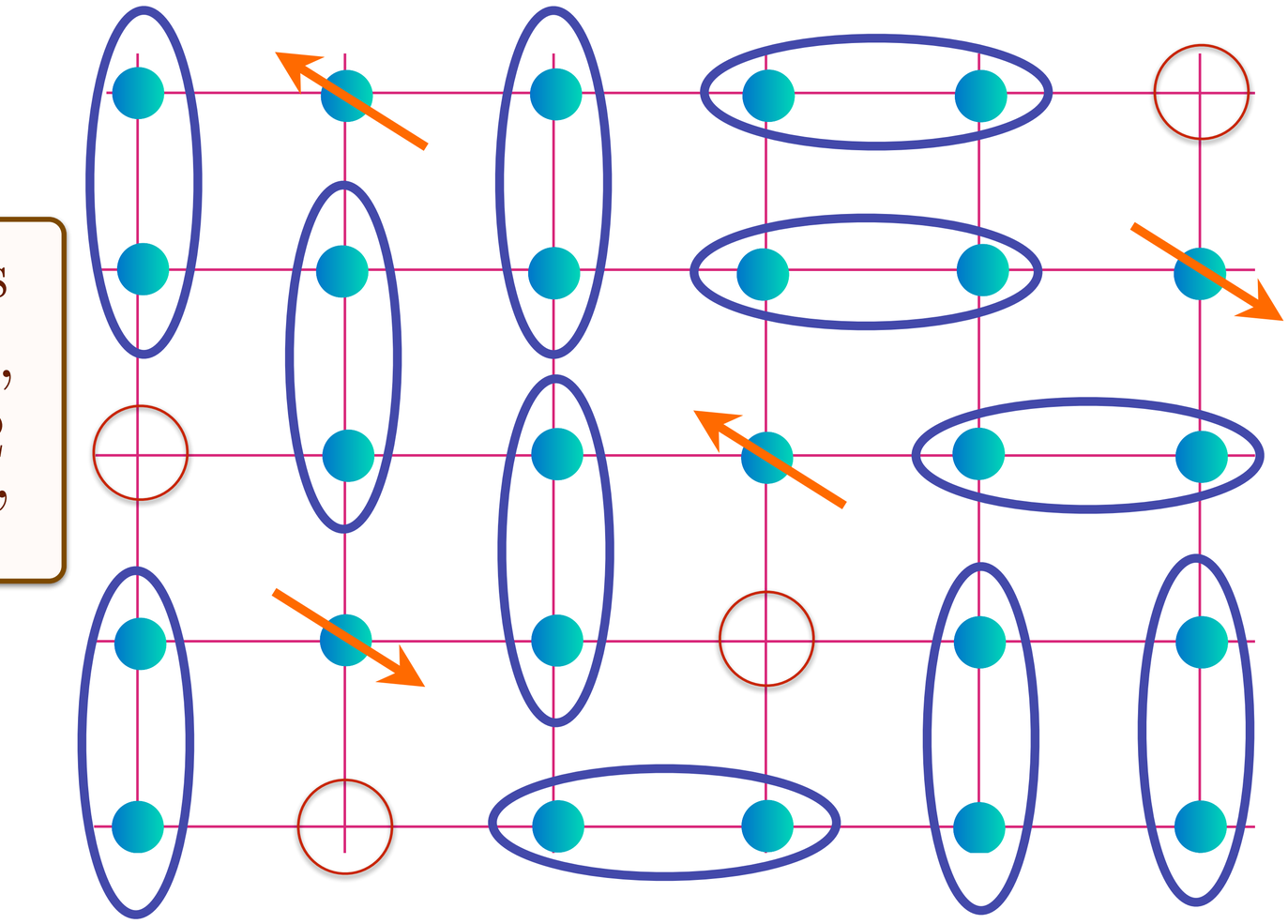

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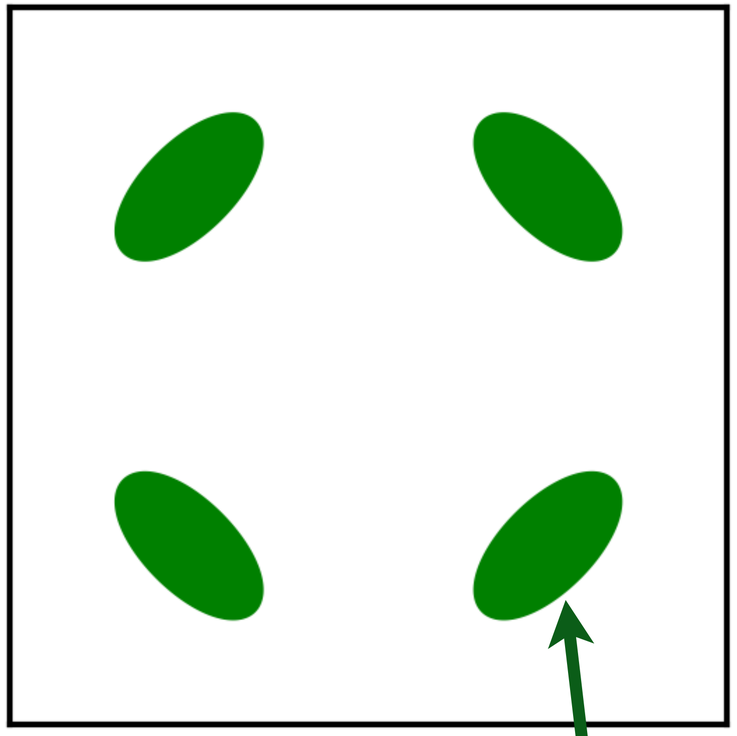

Area $p/4$

Doping an insulating antiferromagnet with holes of density p

Holon metal

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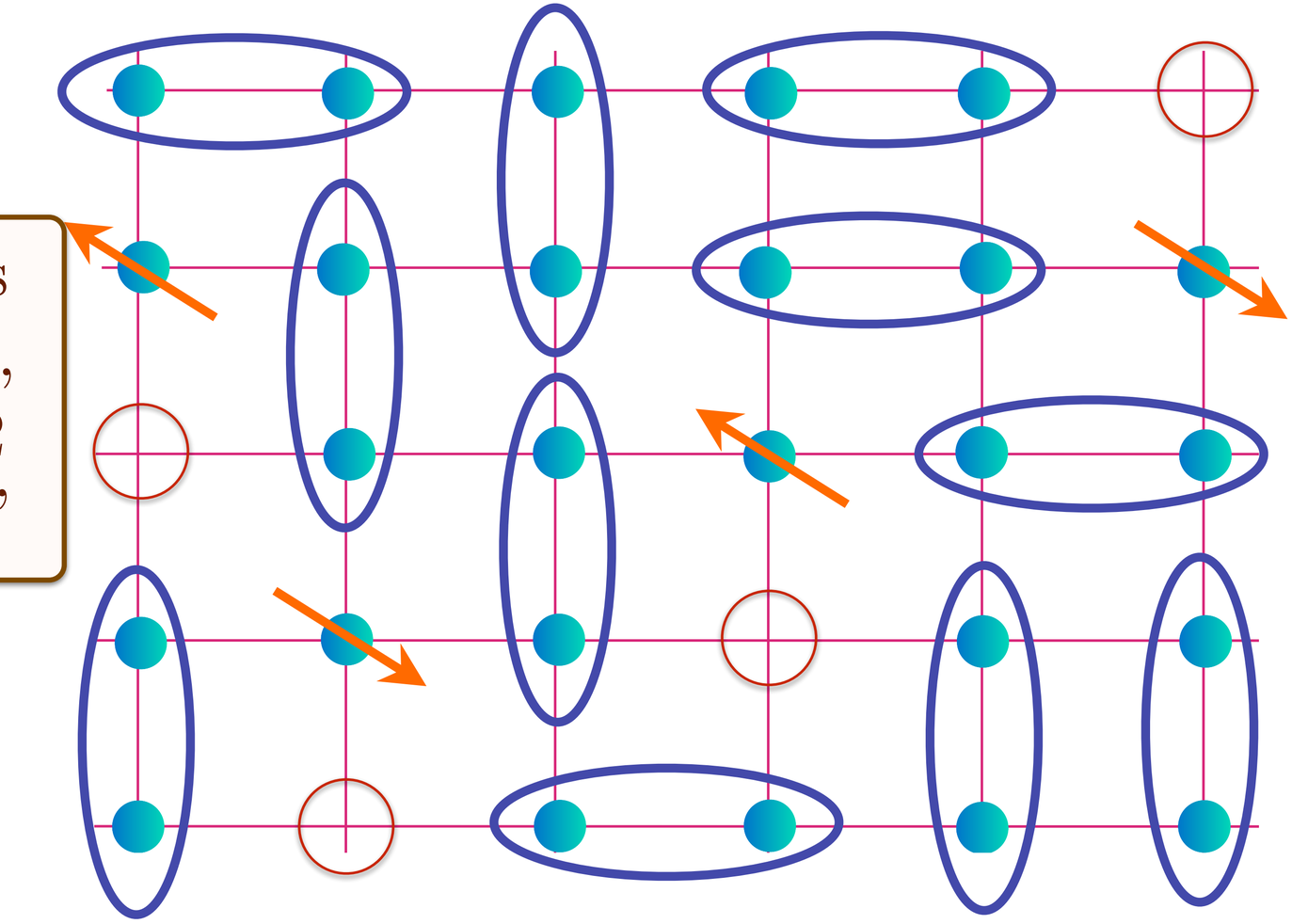

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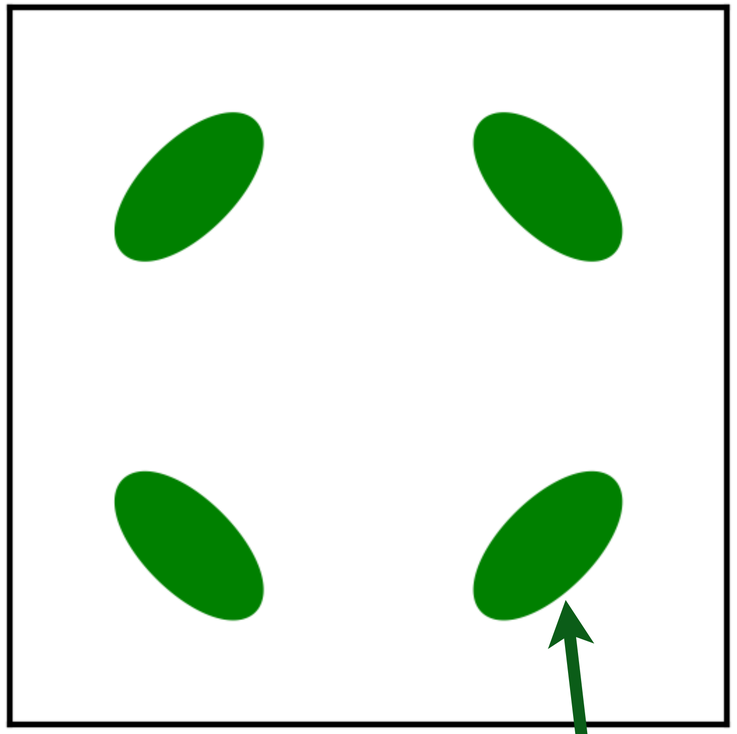

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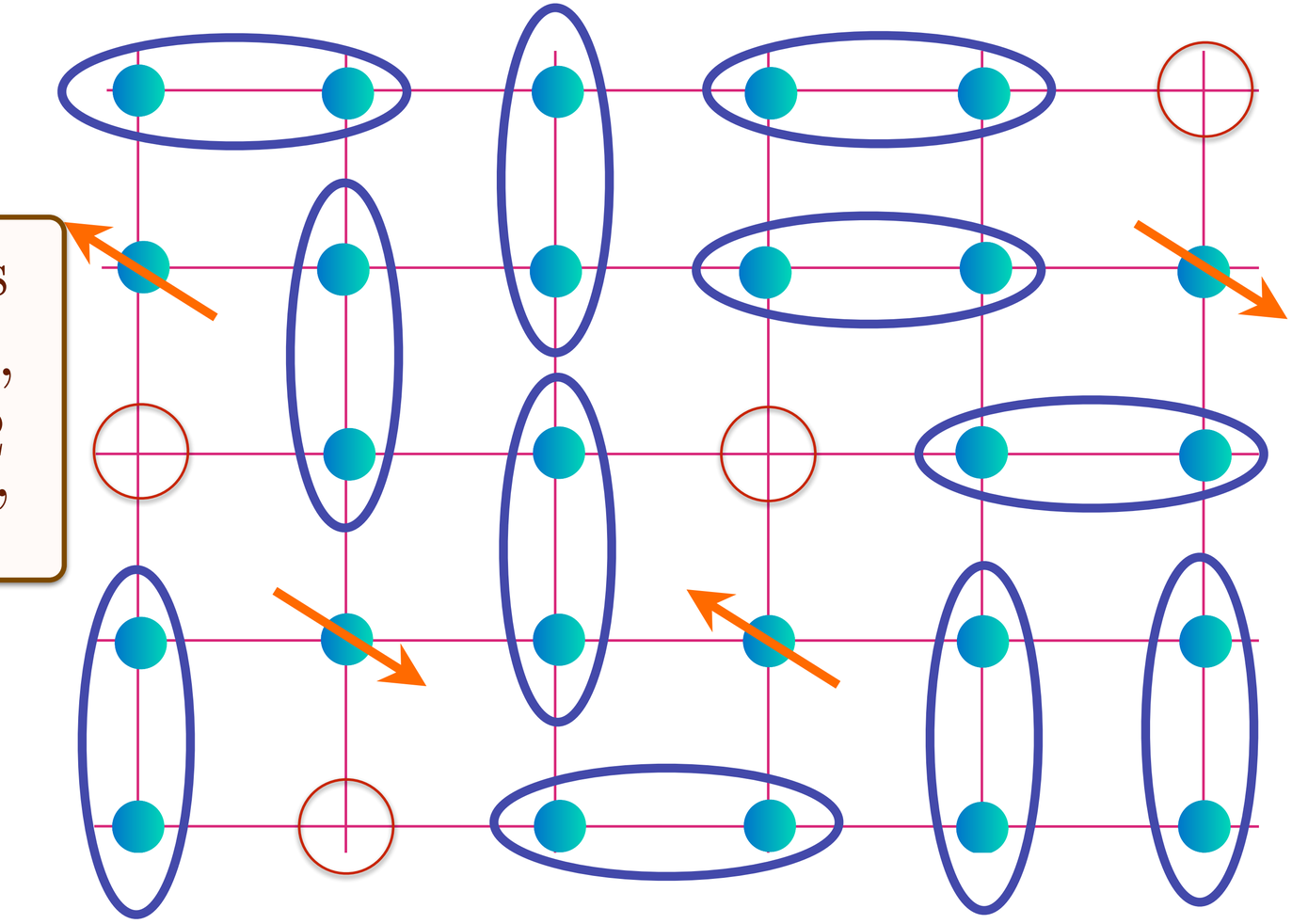

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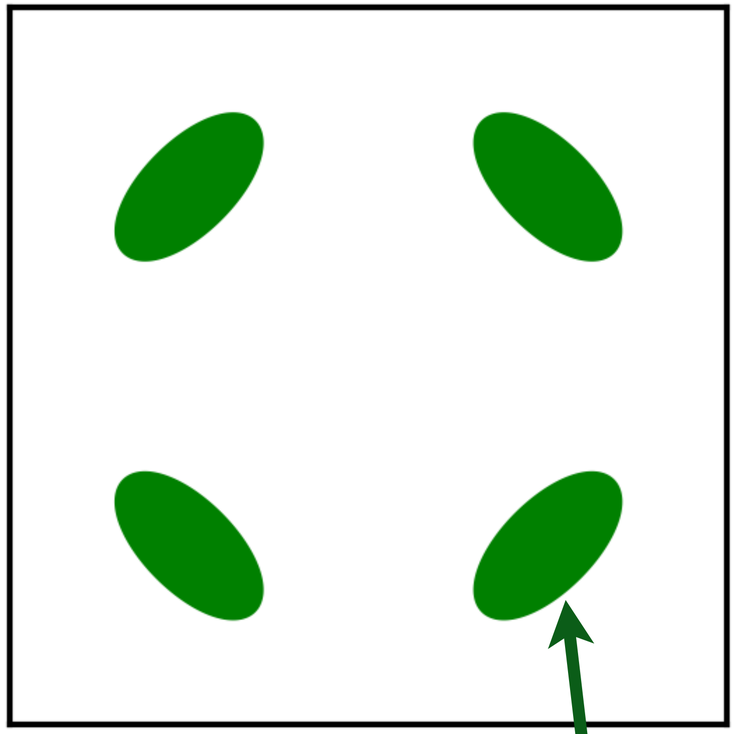

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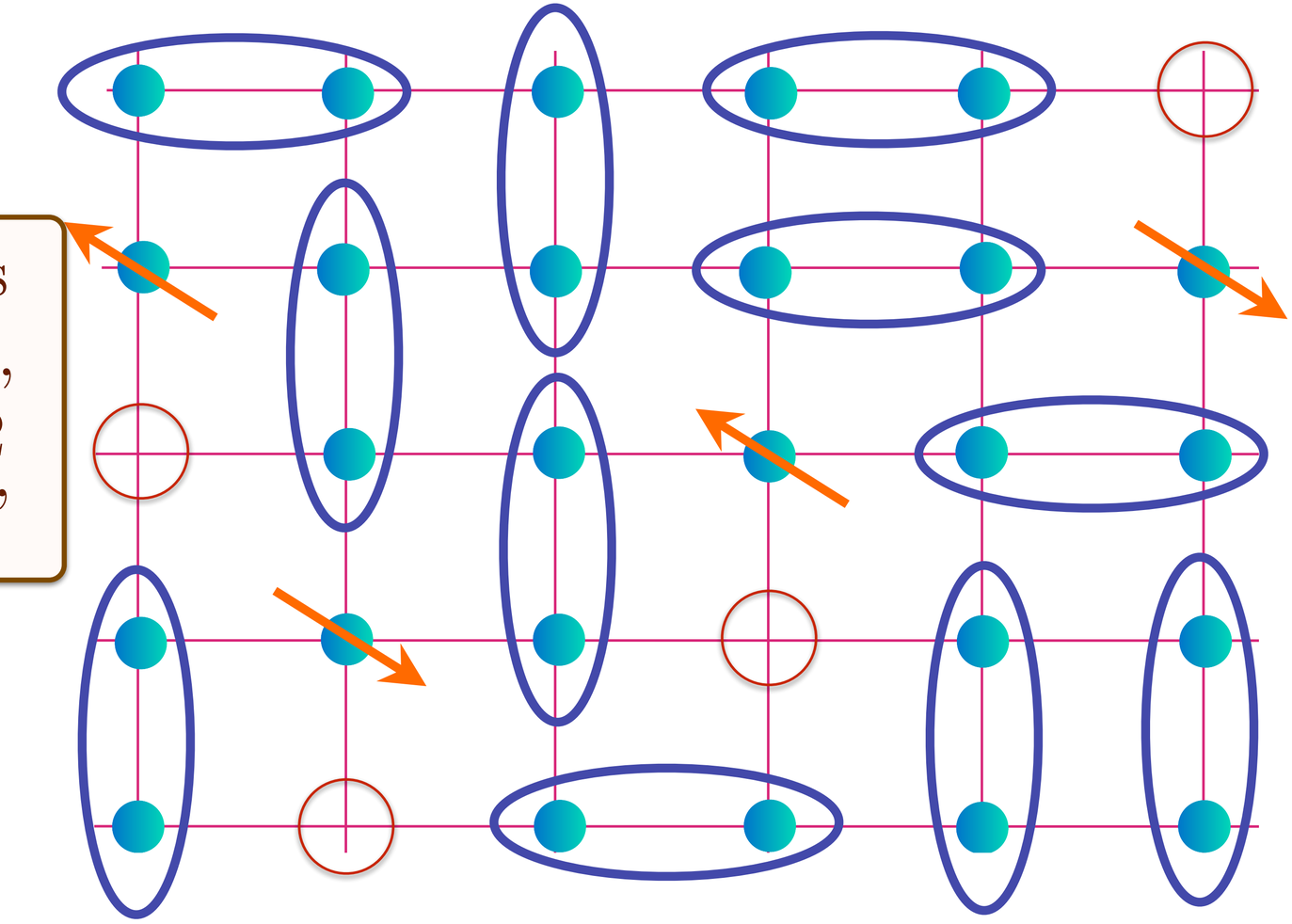

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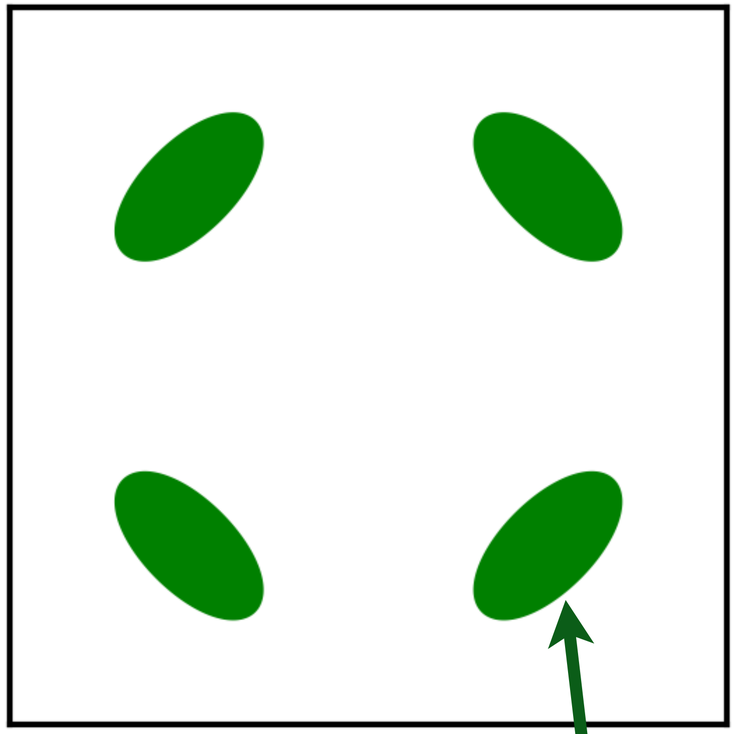

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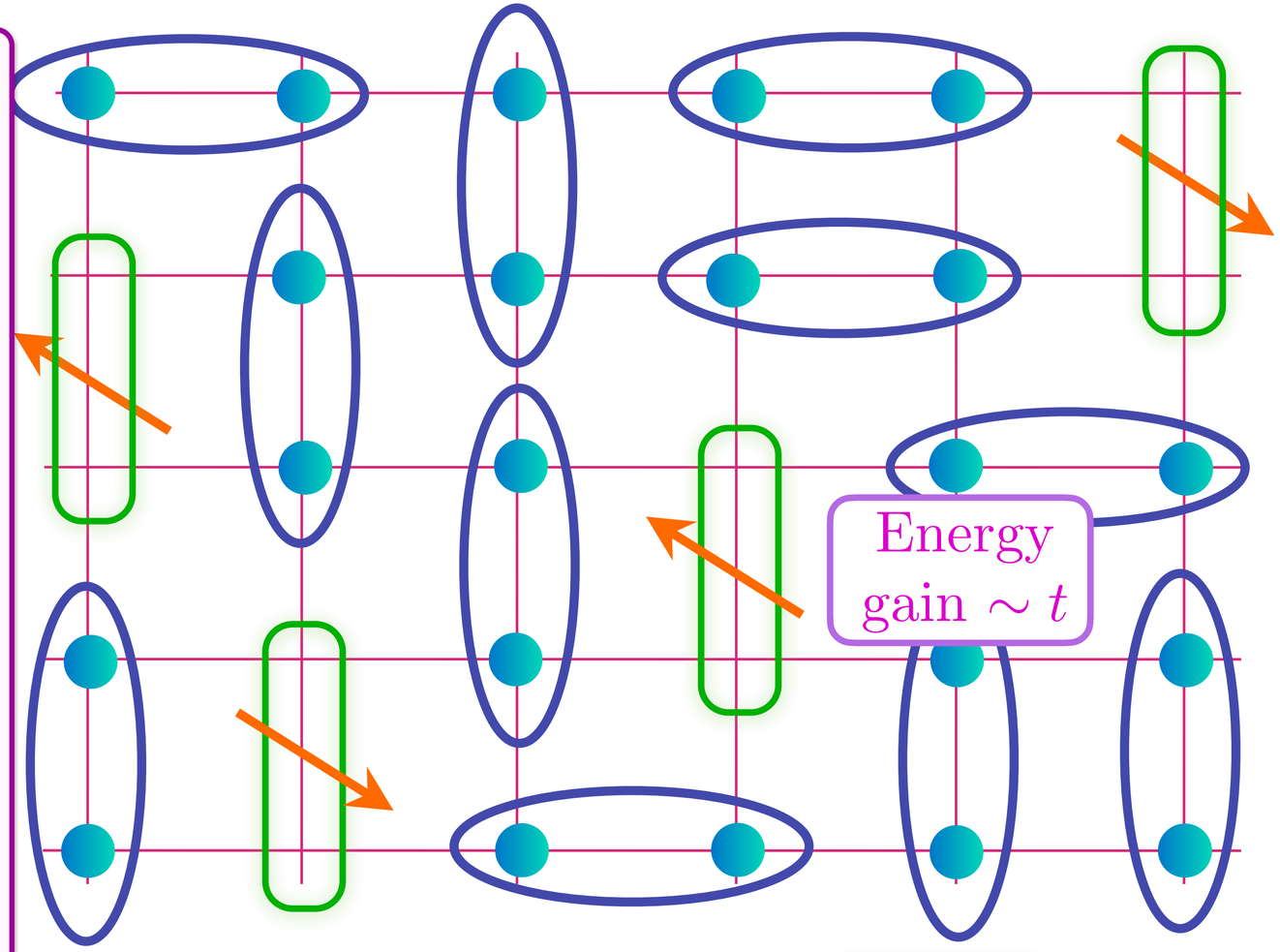

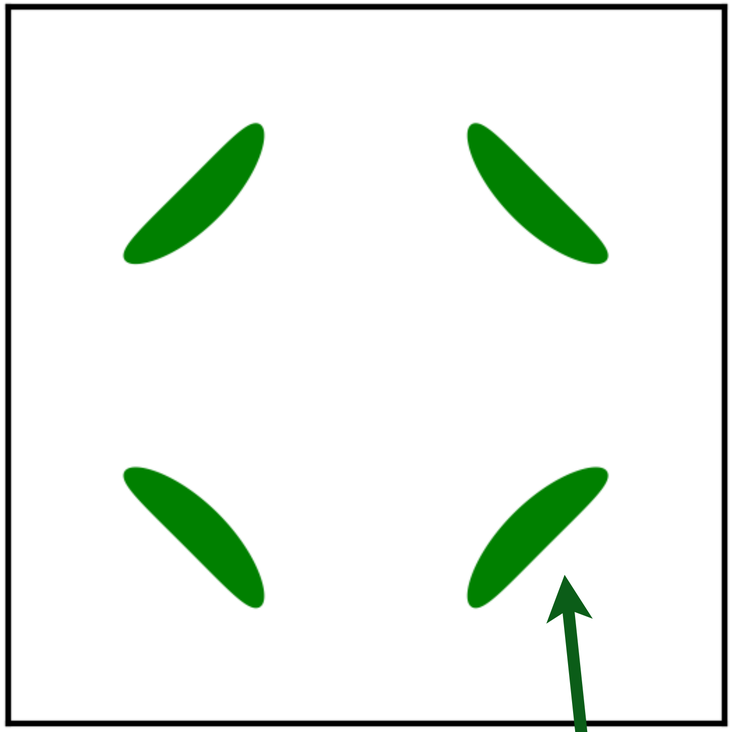

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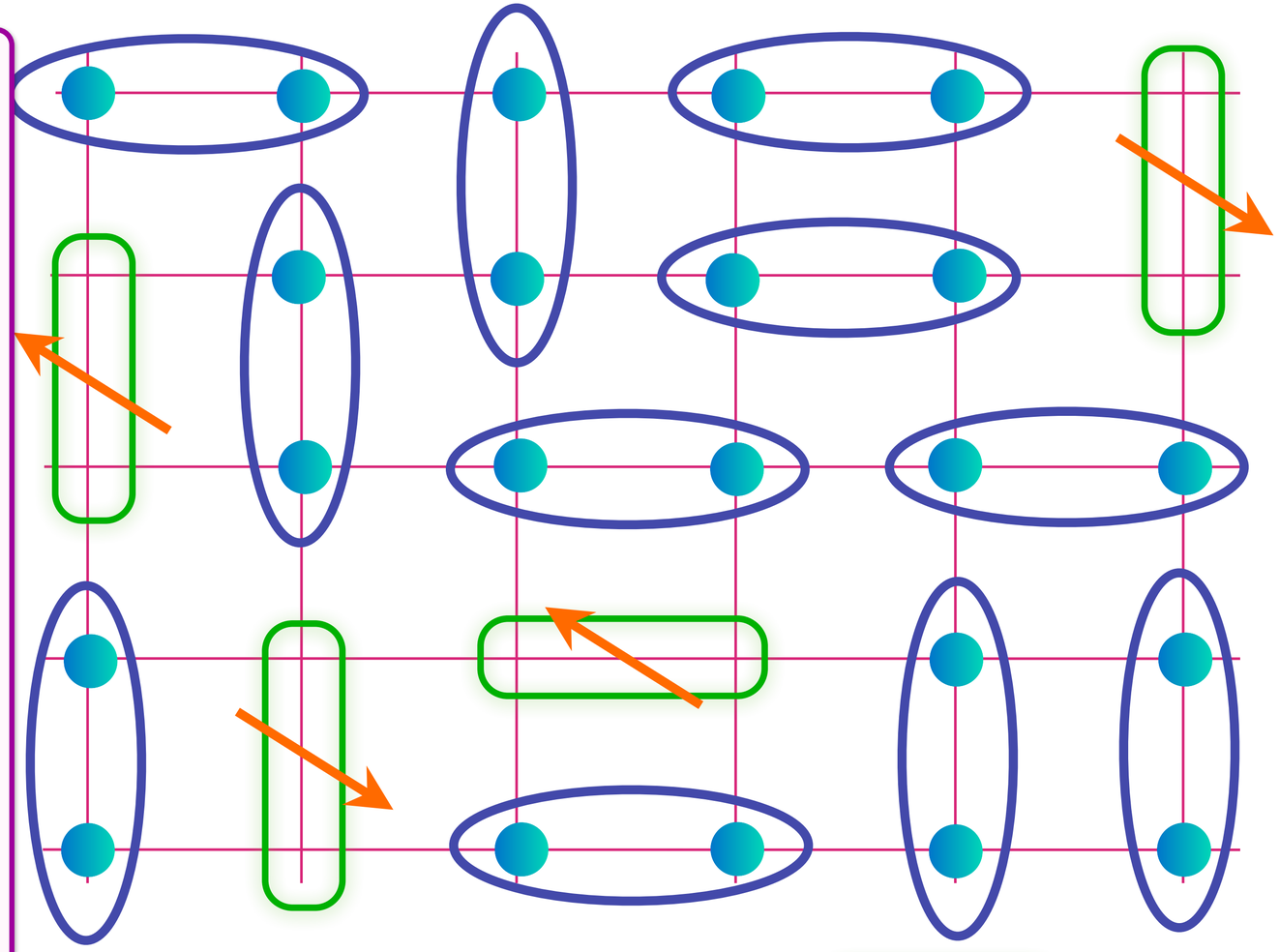

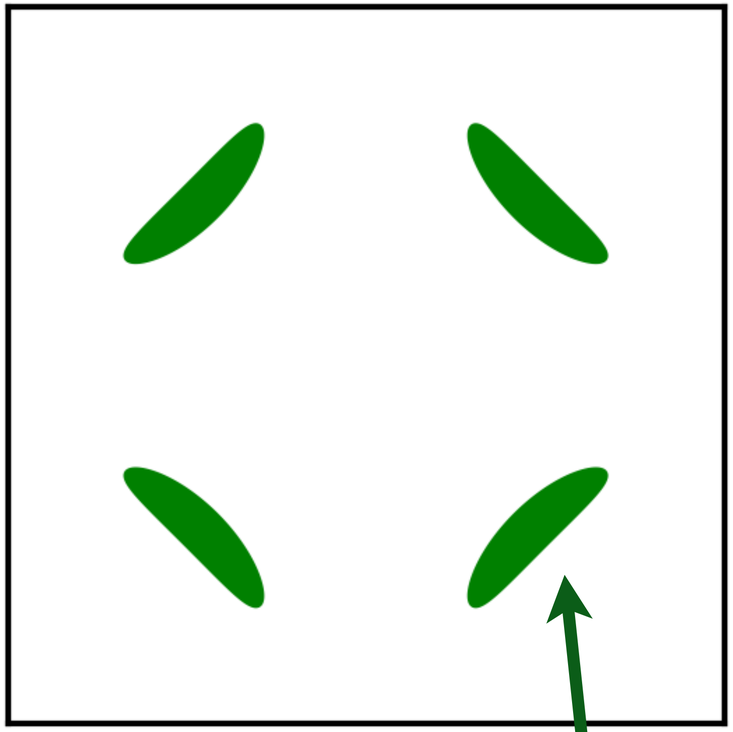

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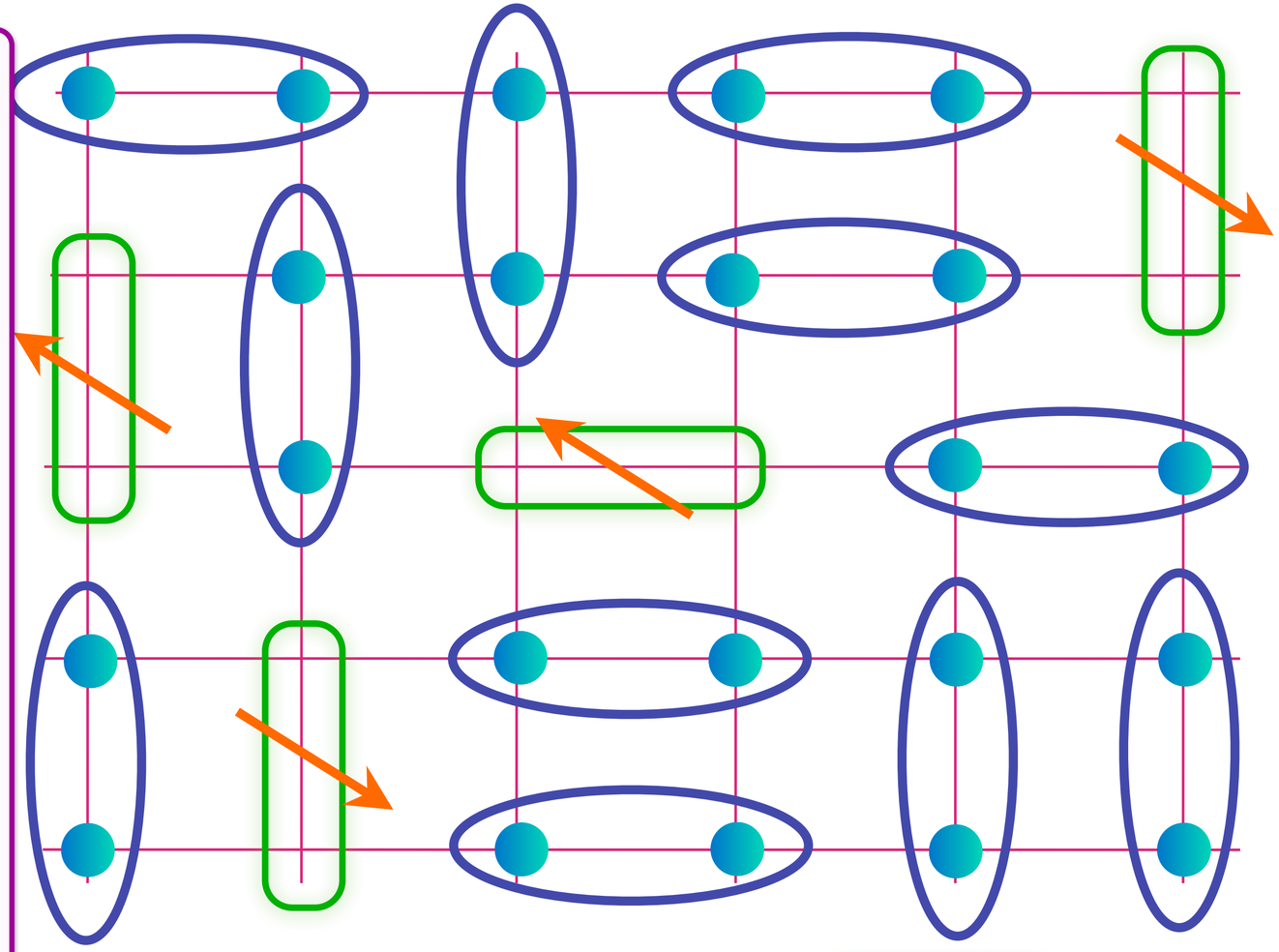

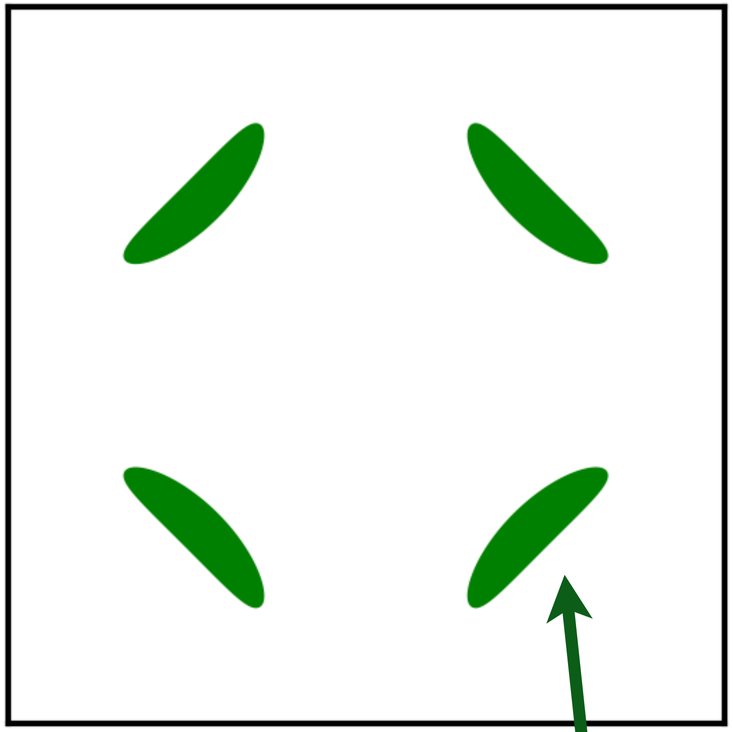

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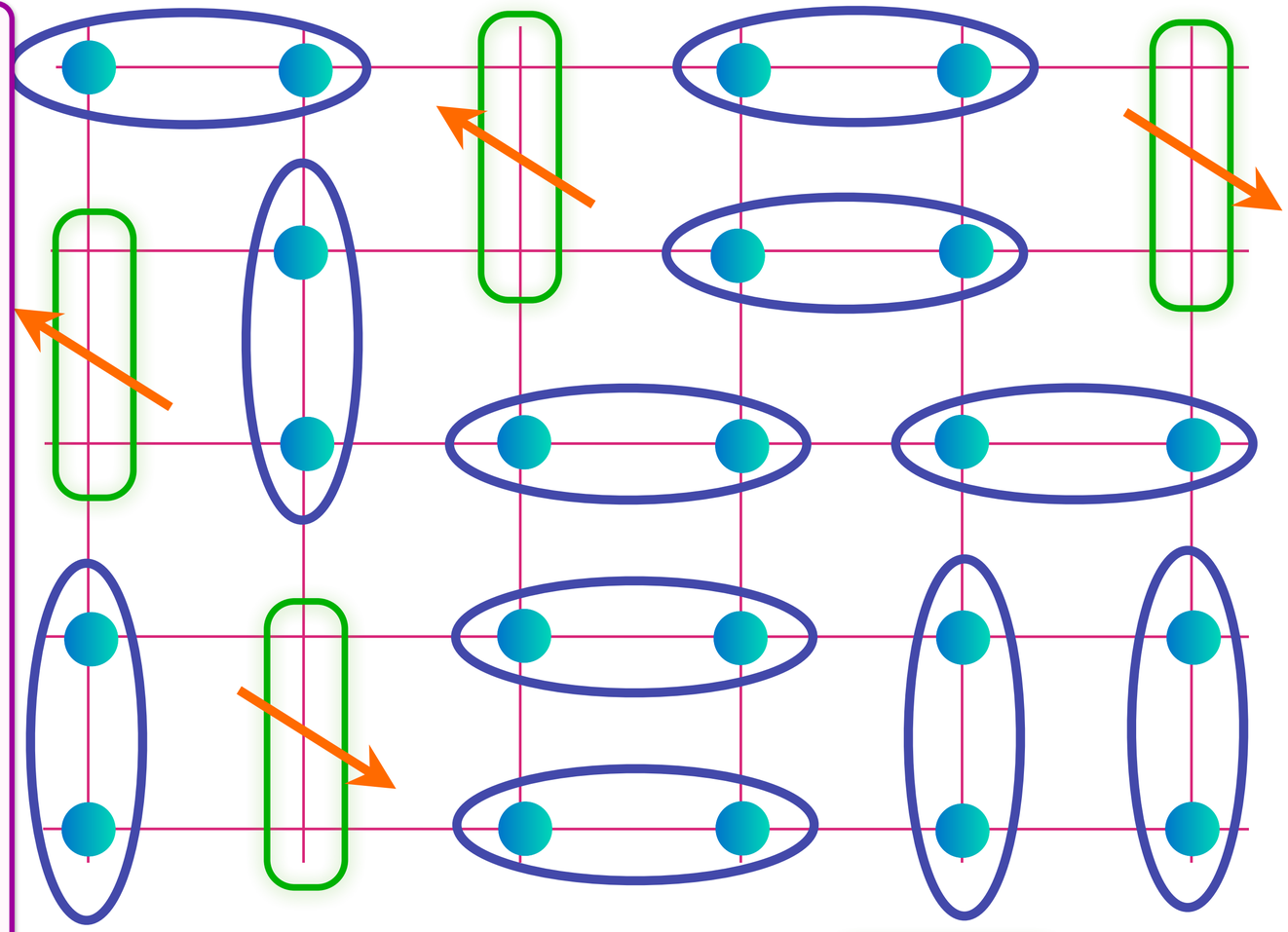

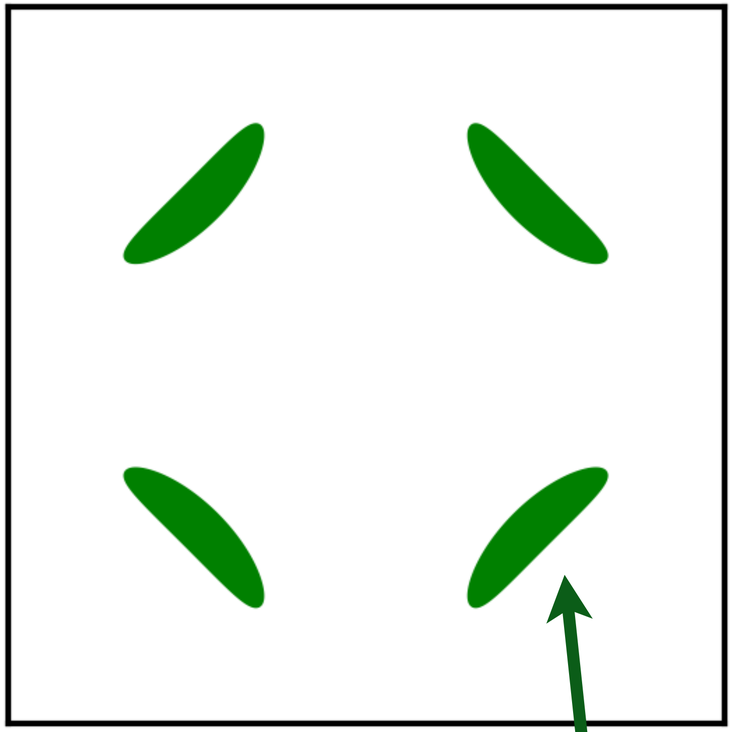

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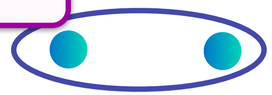

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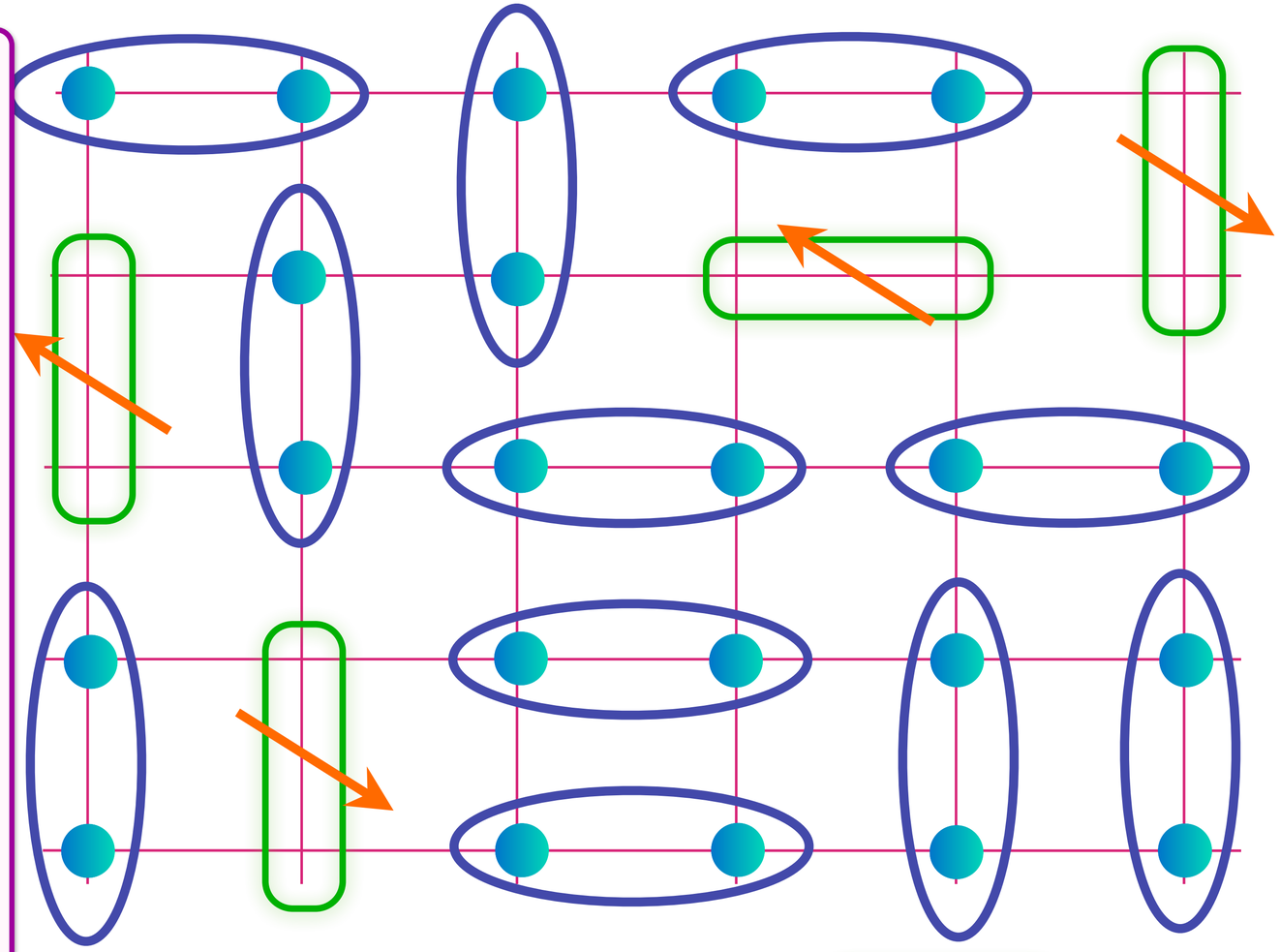

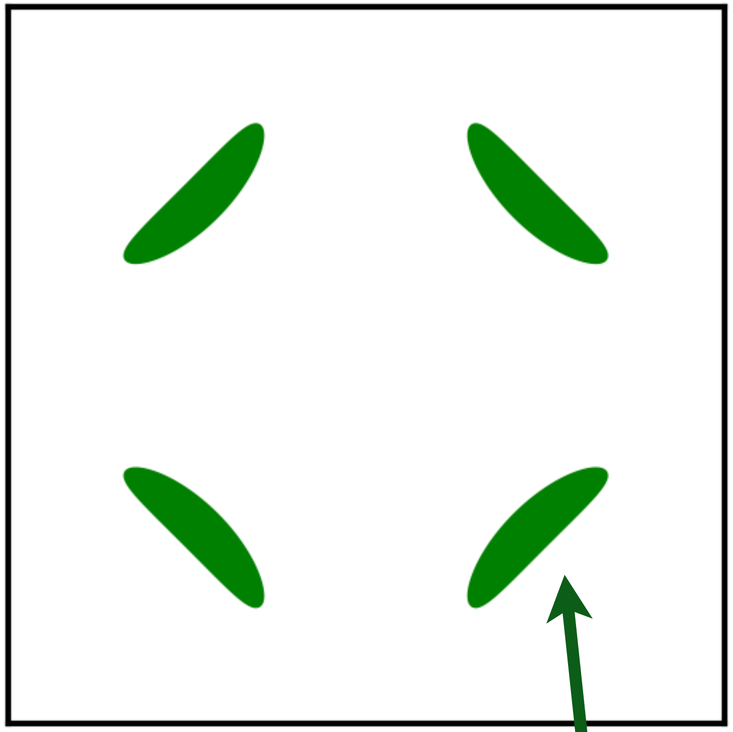

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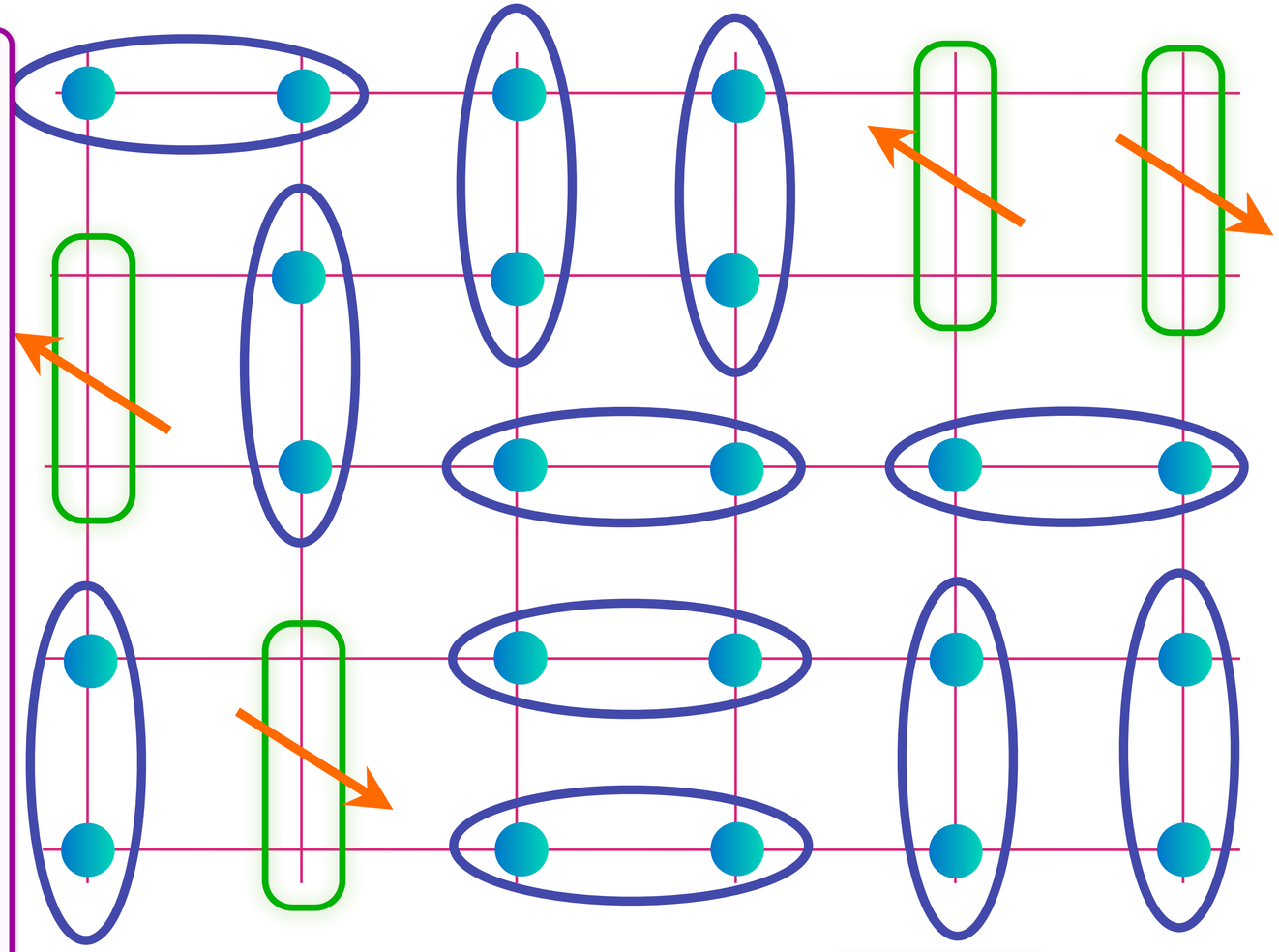

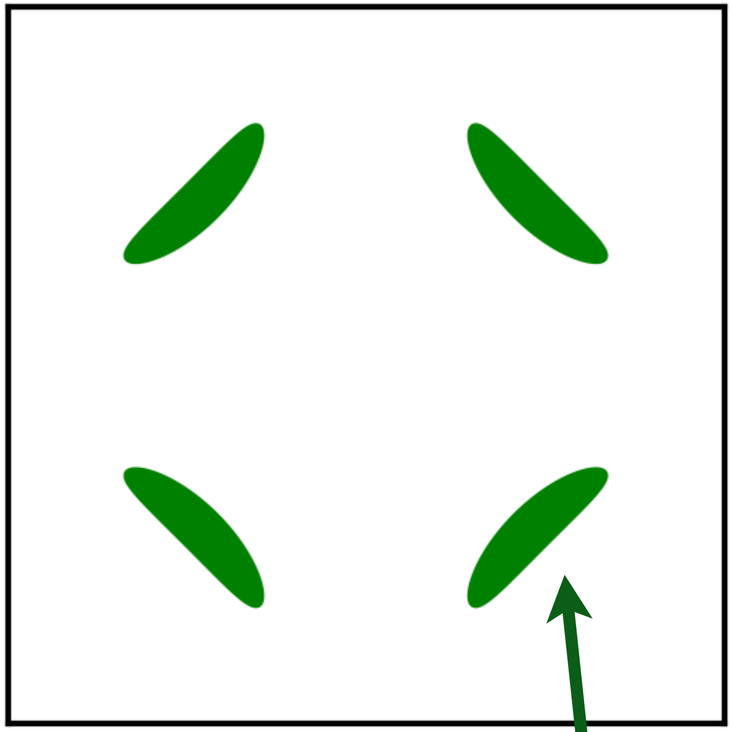

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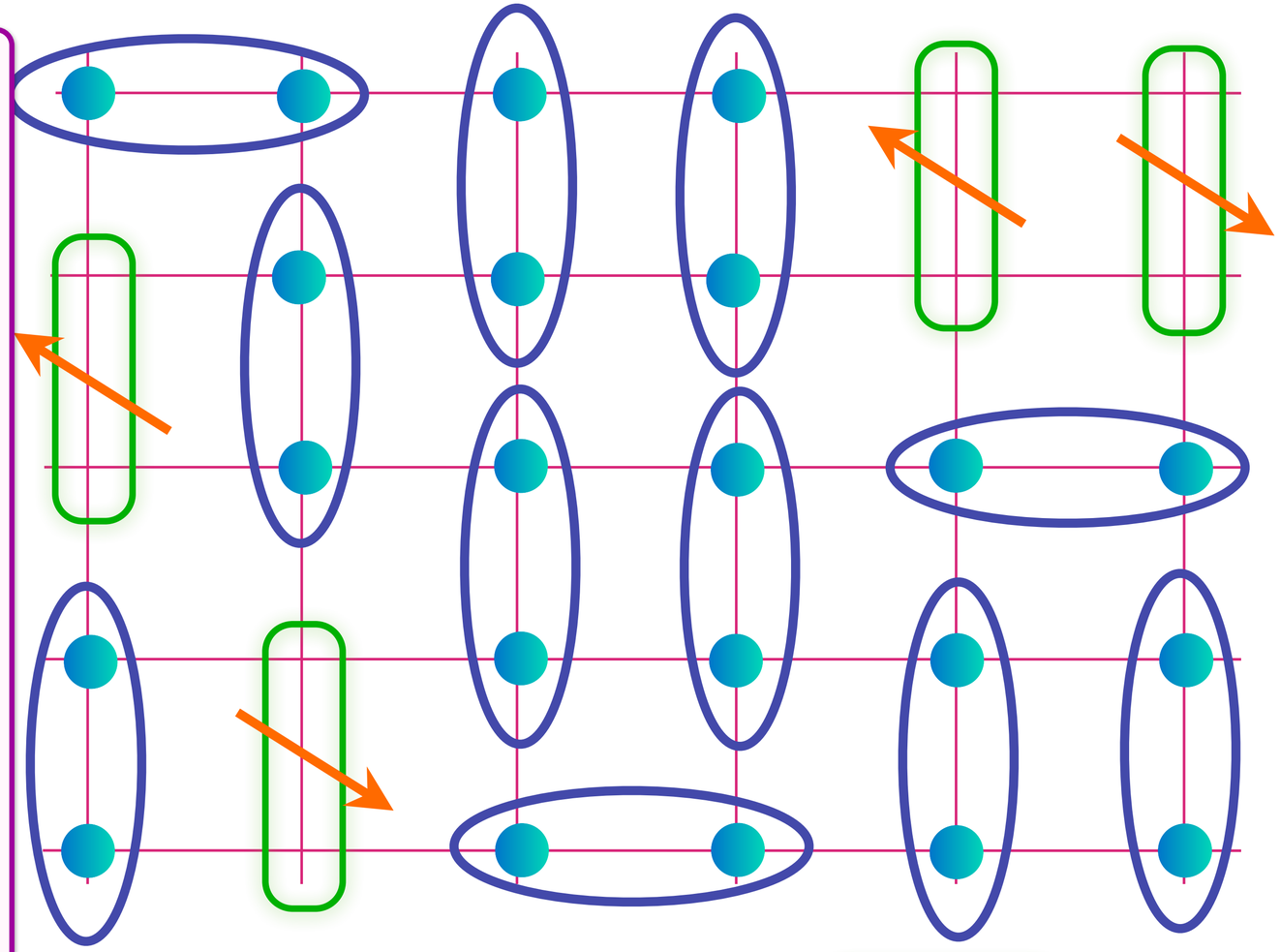

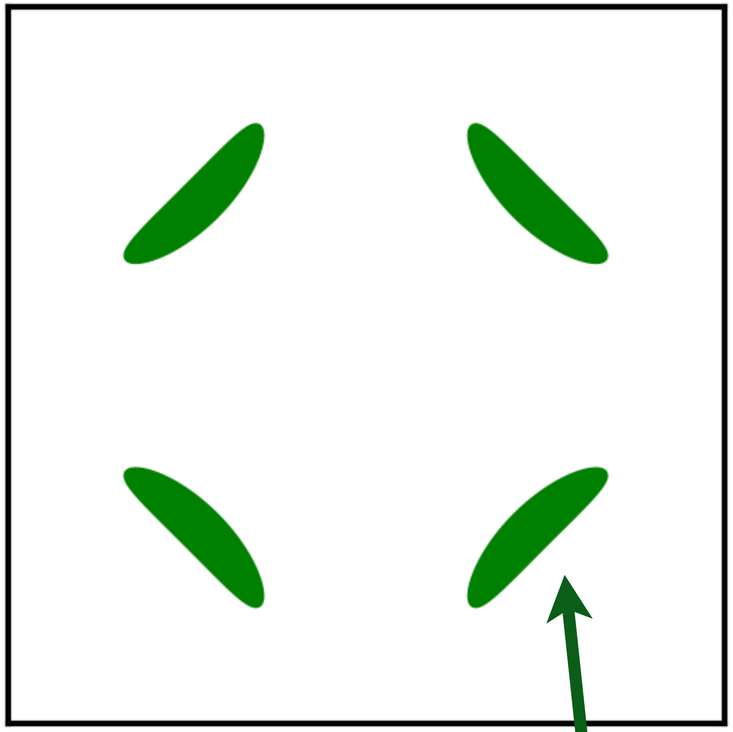

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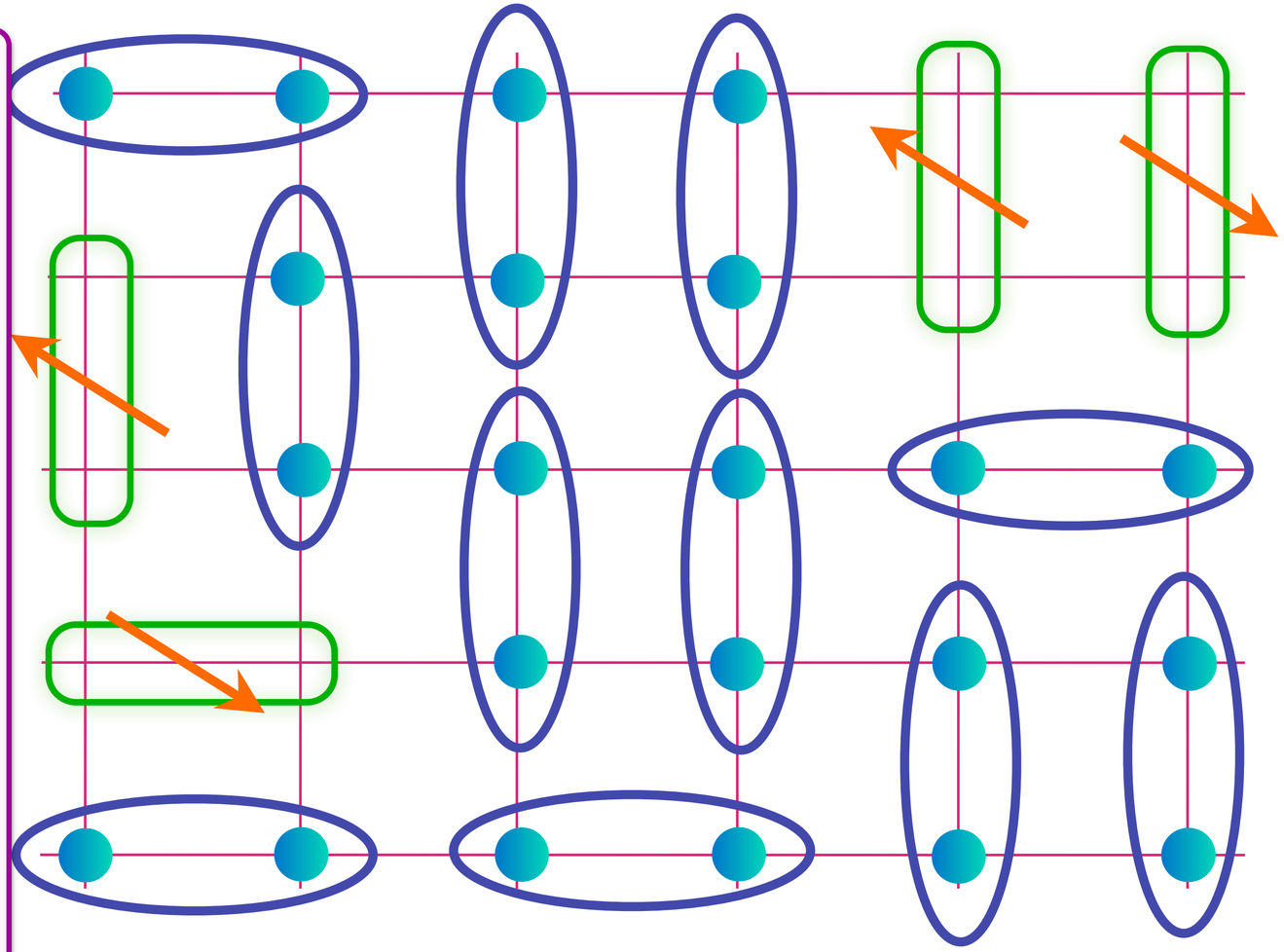

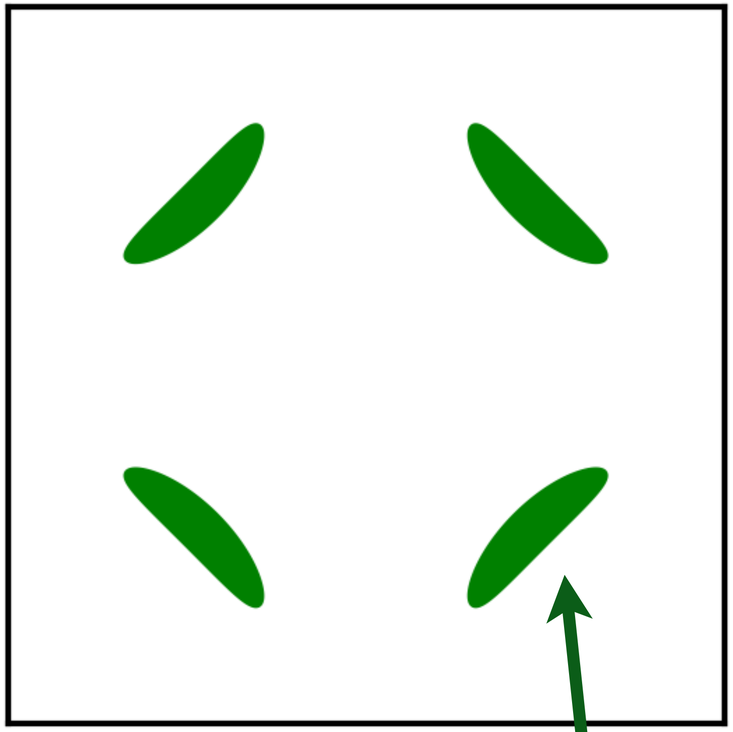

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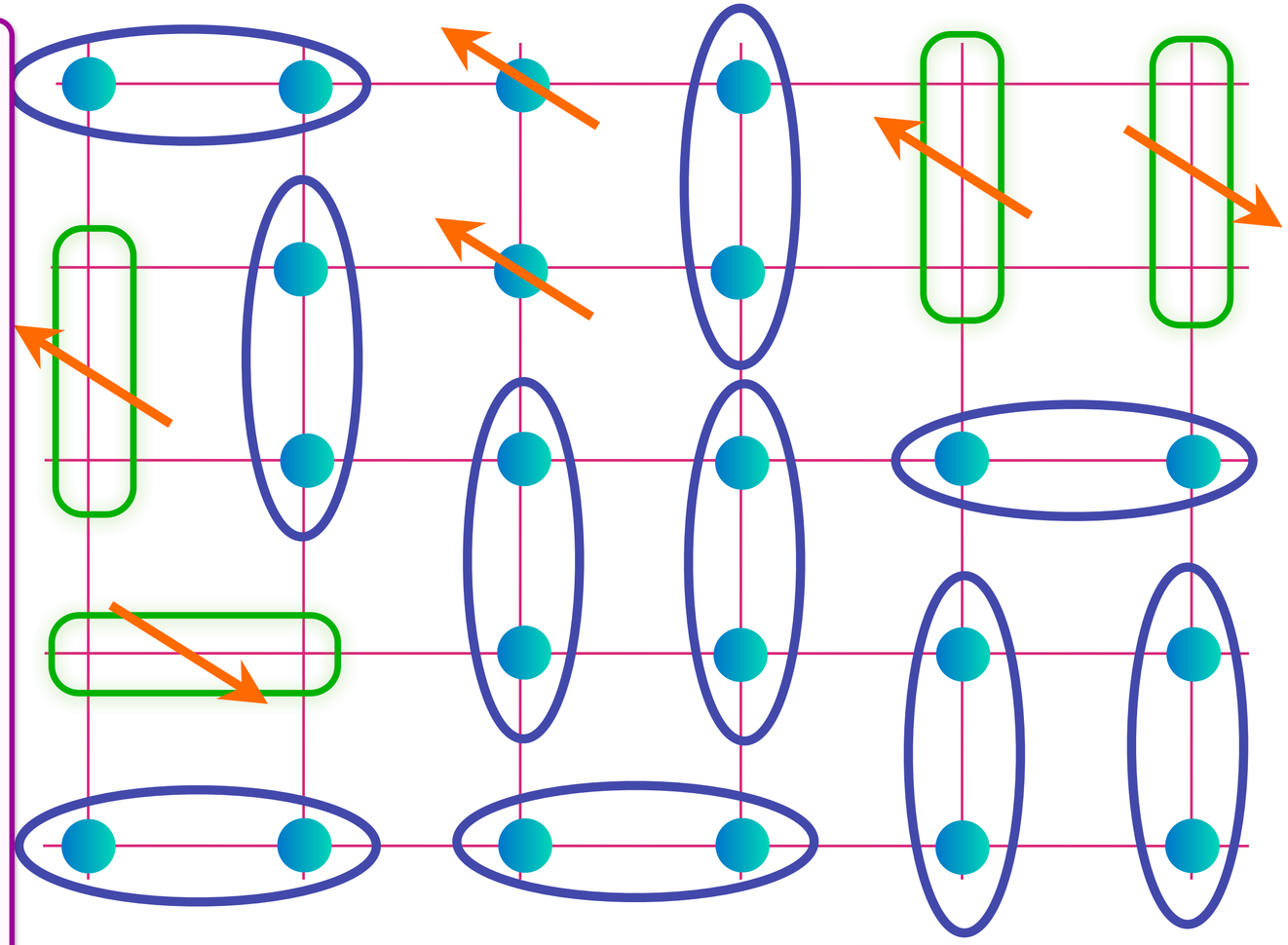

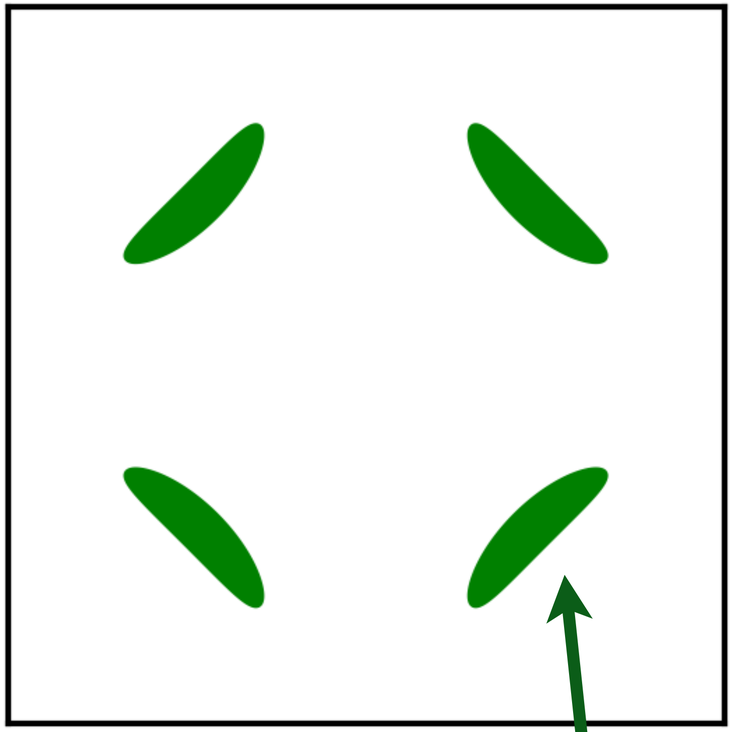

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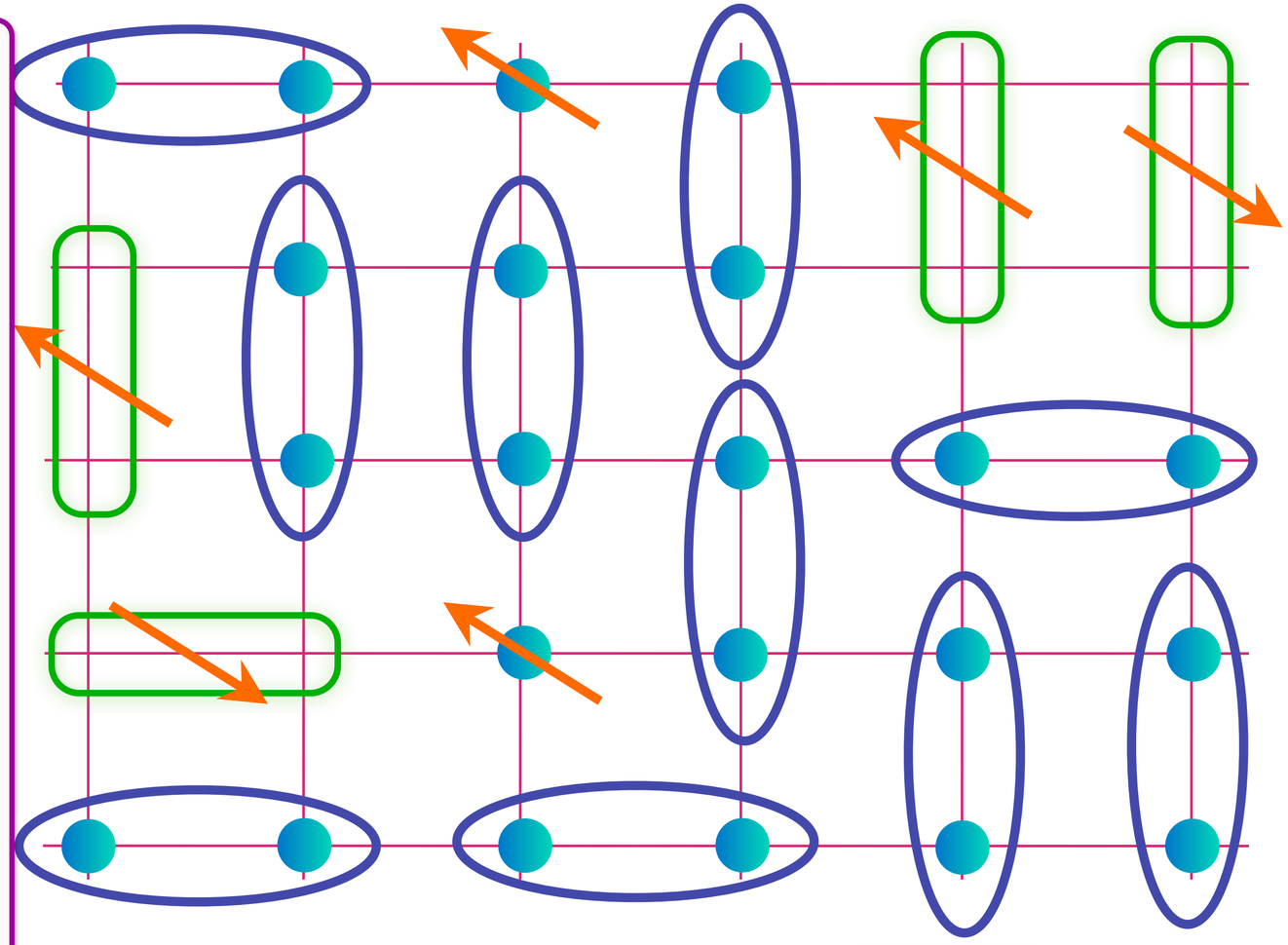

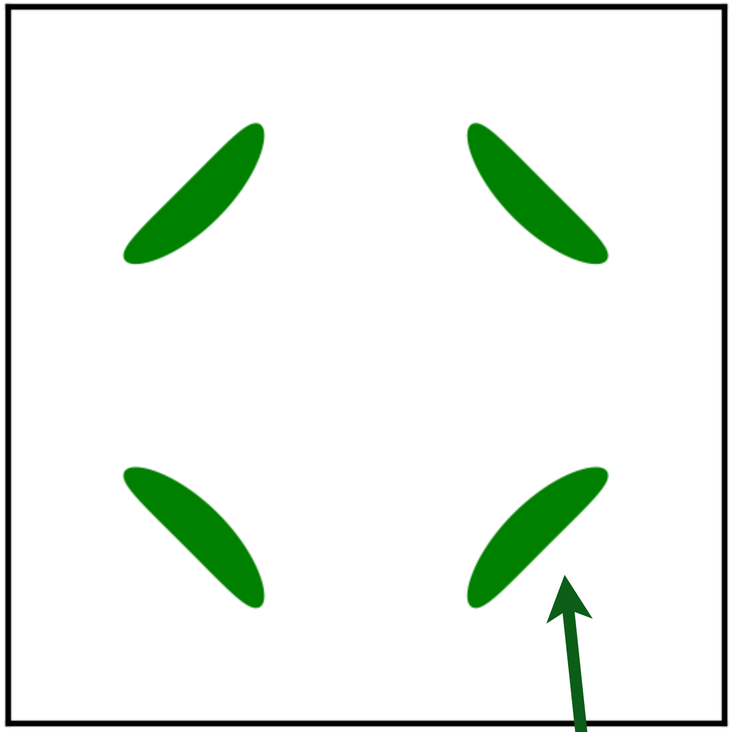

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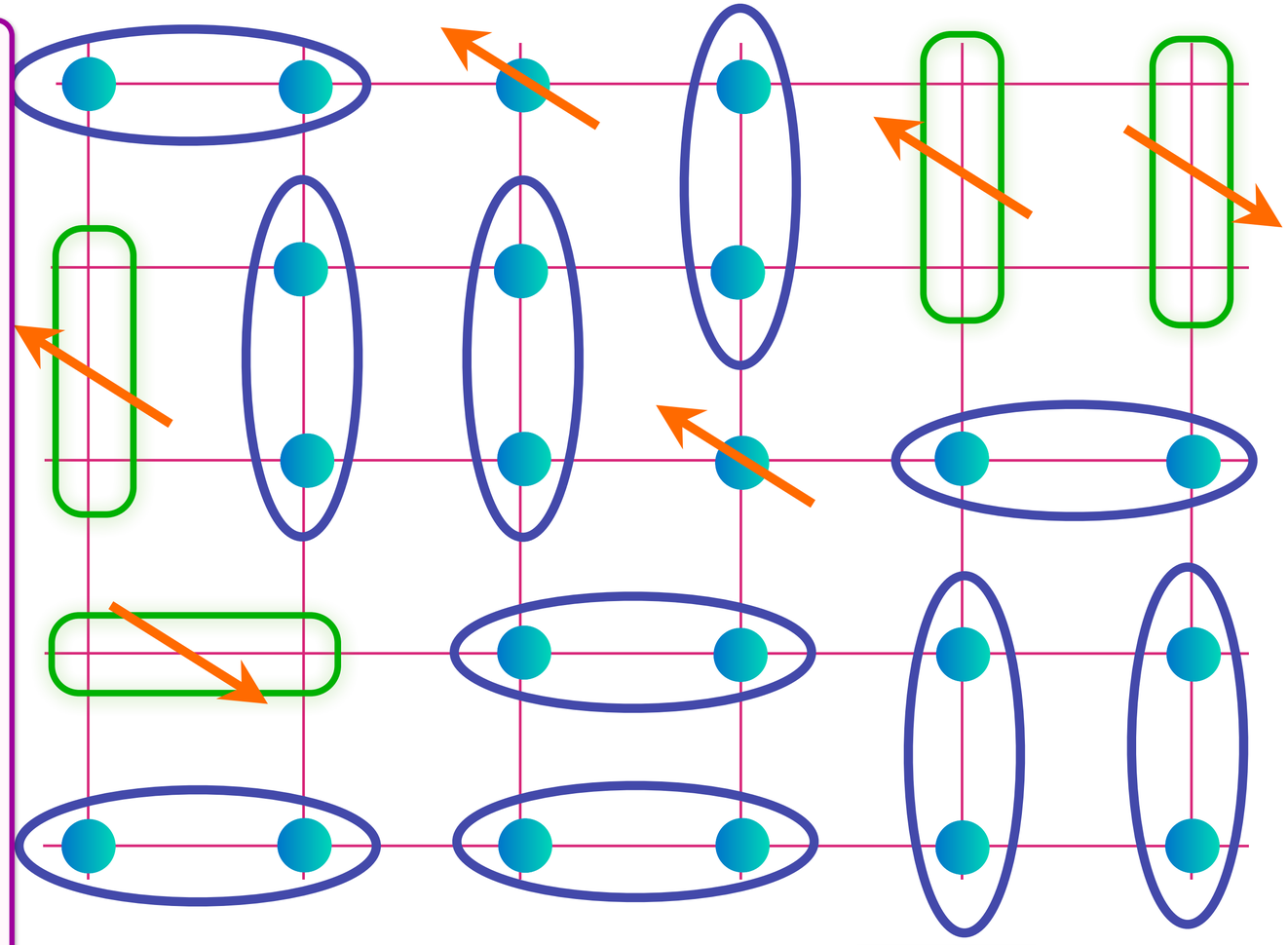

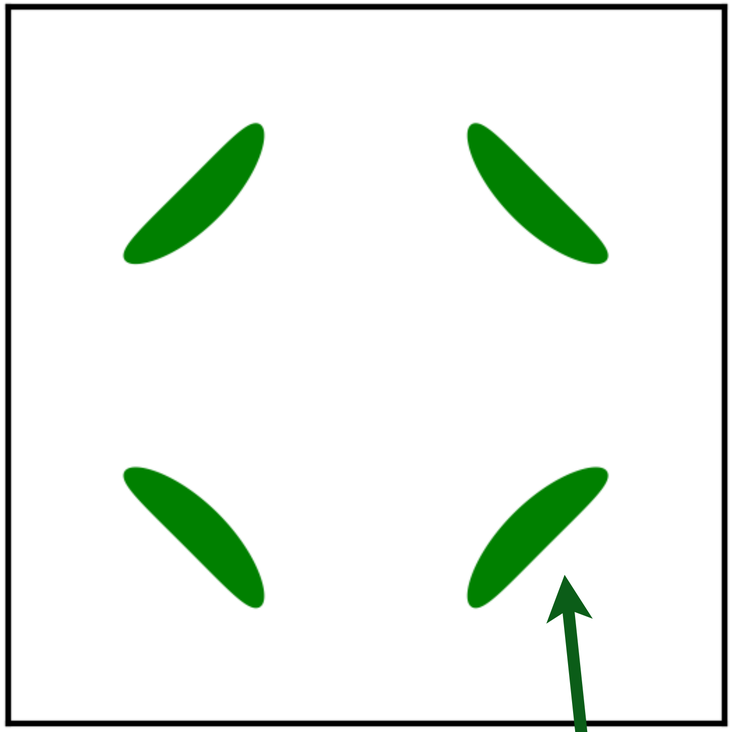

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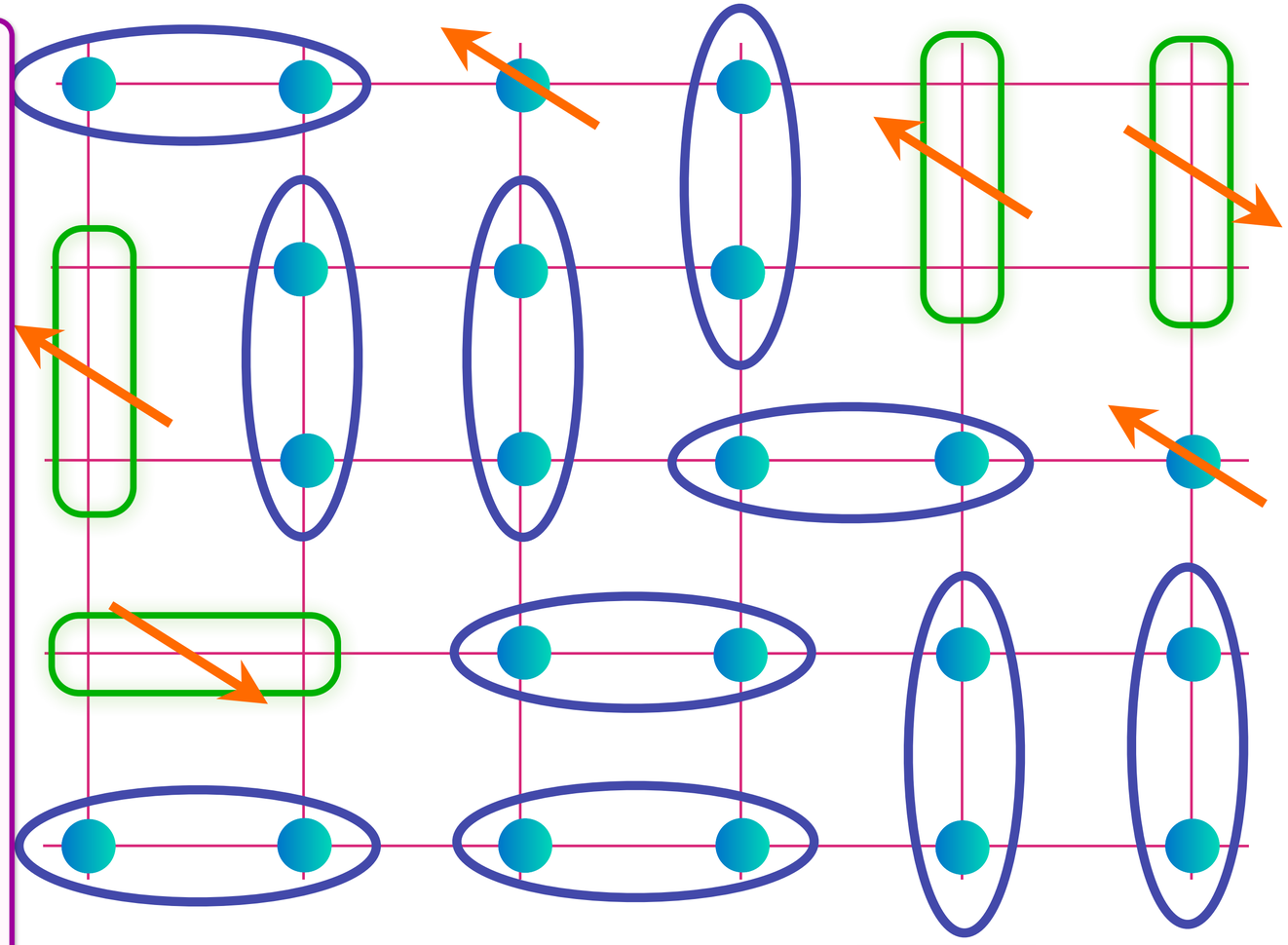

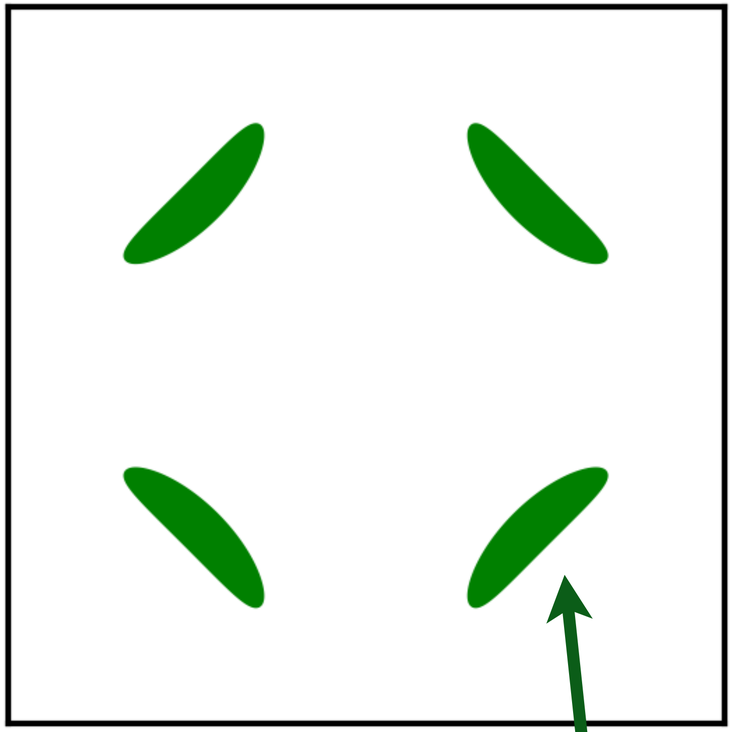

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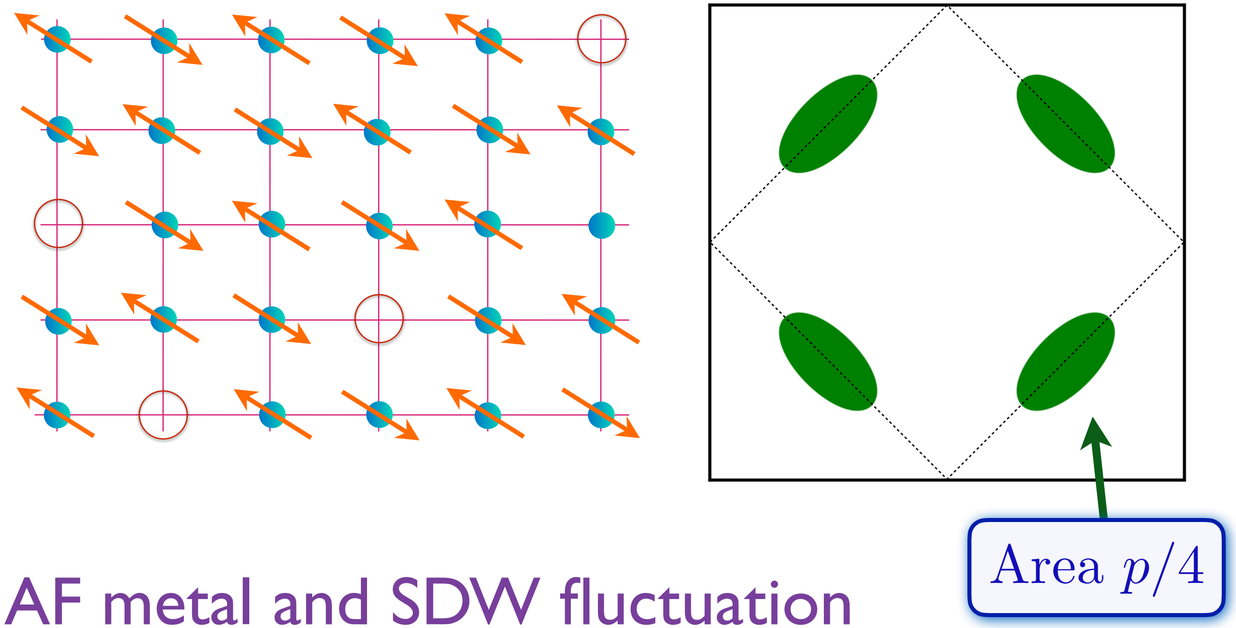
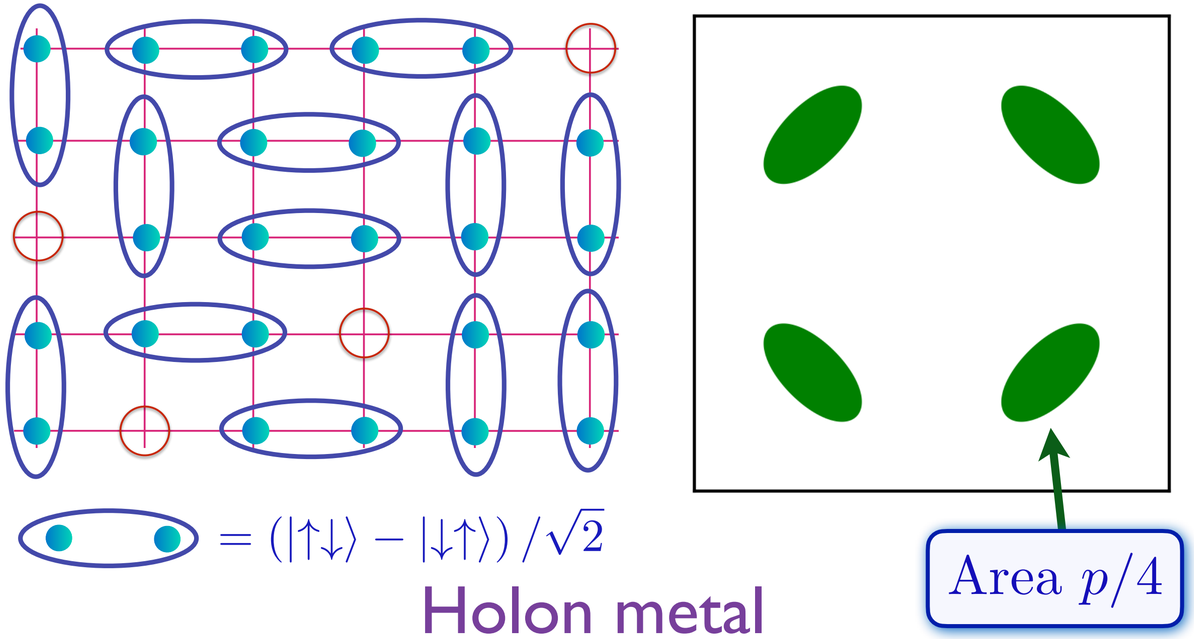


AF metal and SDW fluctuation



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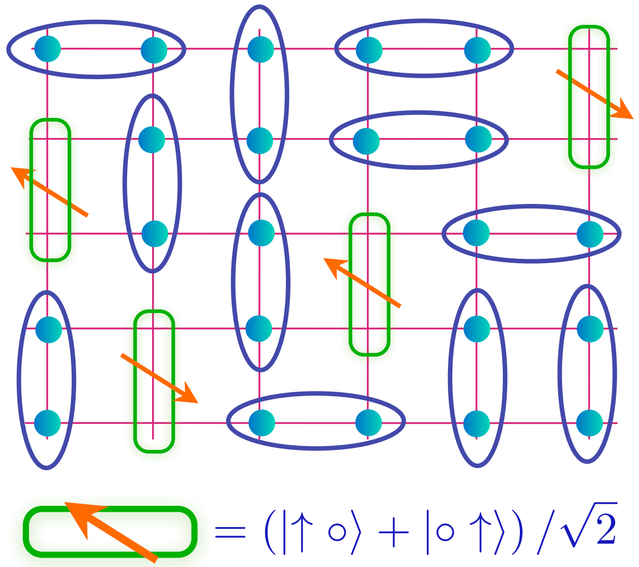
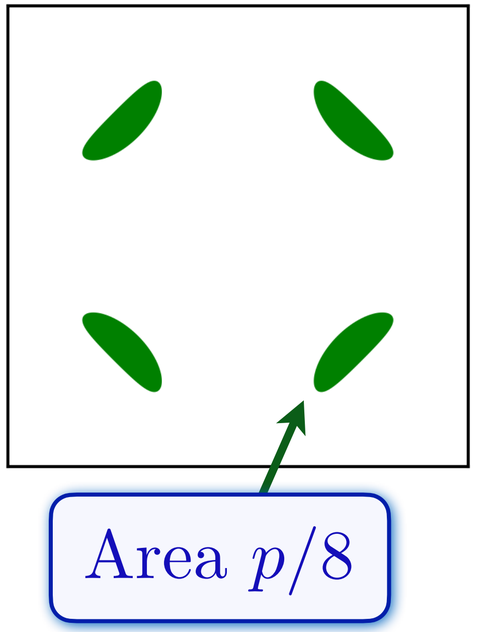


FL*

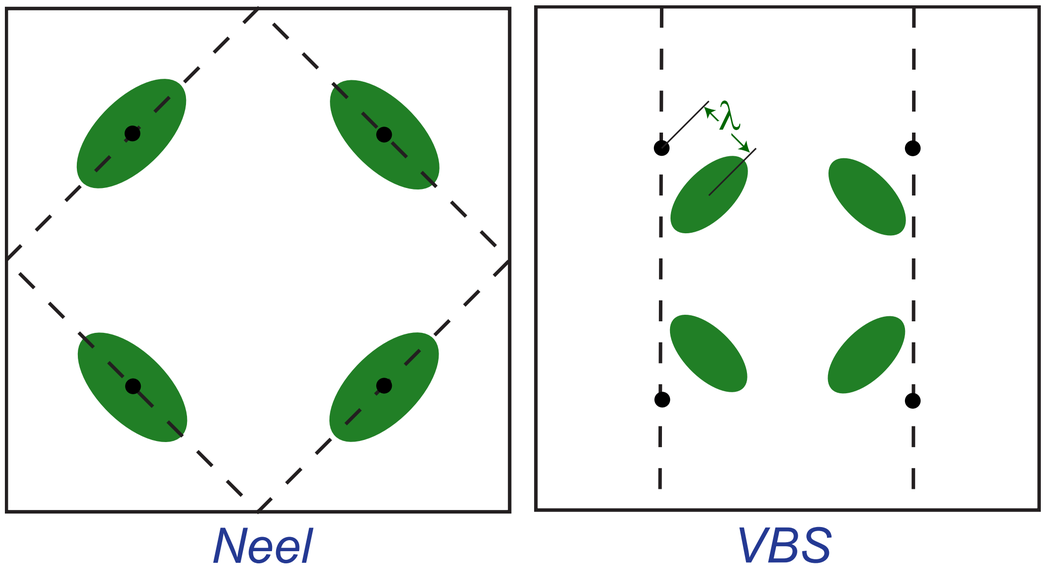


Quantization of spin liquid anomaly implies Fermi surface areas are also *quantized* and robust to all corrections.

PHYSICAL REVIEW B **75**, 235122 (2007)

Hole dynamics in an antiferromagnet across a deconfined quantum critical point

Ribhu K. Kaul,[1] Alexei Kolezhuk,[1,2] Michael Levin,[1] Subir Sachdev,[1] and T. Senthil[3,4]



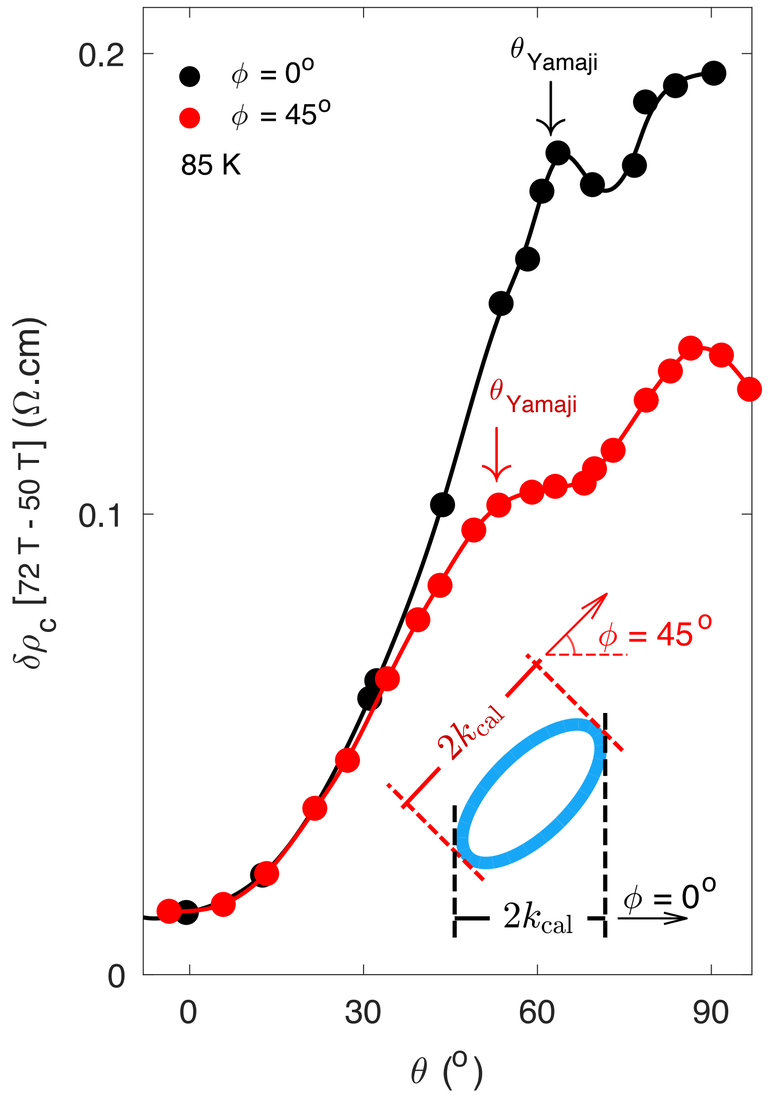
The dashed line in the Néel phase indicates the boundary of the magnetic Brillouin zone. Only the Fermi surfaces within this zone contribute to the Luttinger counting, and so the area of each ellipse is $\mathcal{A}_F = (2\pi)^2 \delta/4$. In the VBS phase, all four pockets are inequivalent, and so the area of each ellipse is $\mathcal{A}_F = (2\pi)^2 \delta/8$.

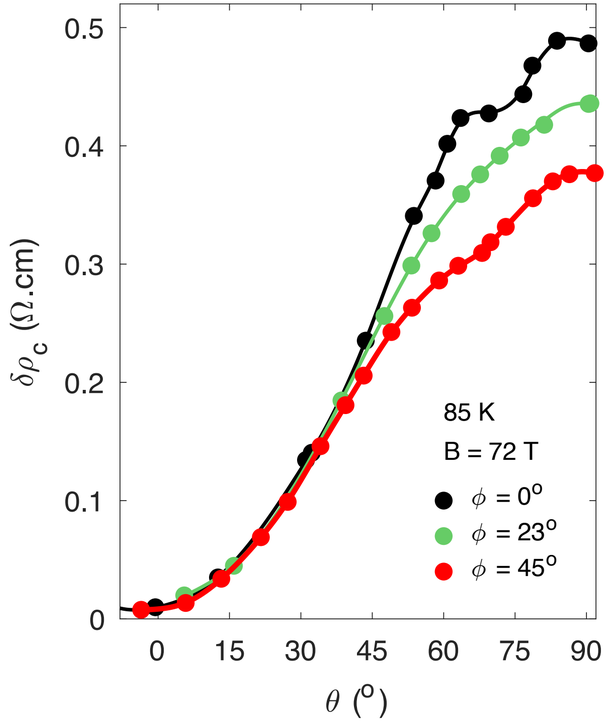
Factor of 2 between SDW fluctuation and FL*

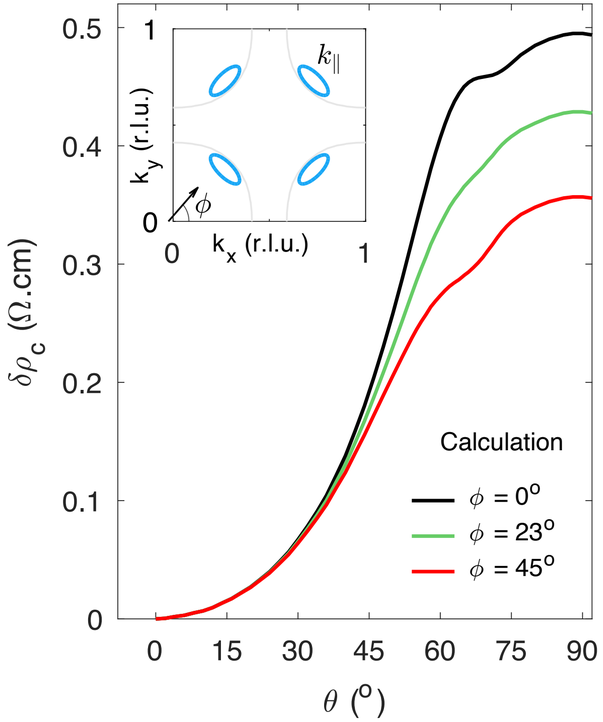
Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan[1], Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela[1], Eric D. Bauer[2], Arkady Shekhter[1] & Neil Harrison[1]

nature physics
arXiv:2411.10631
Published online: 16 September 2025







Doping
$p = 0.1$

"The small size of the pockets determined from the Yamaji effect is . . . approximately 1.3% of the Brillouin zone area"

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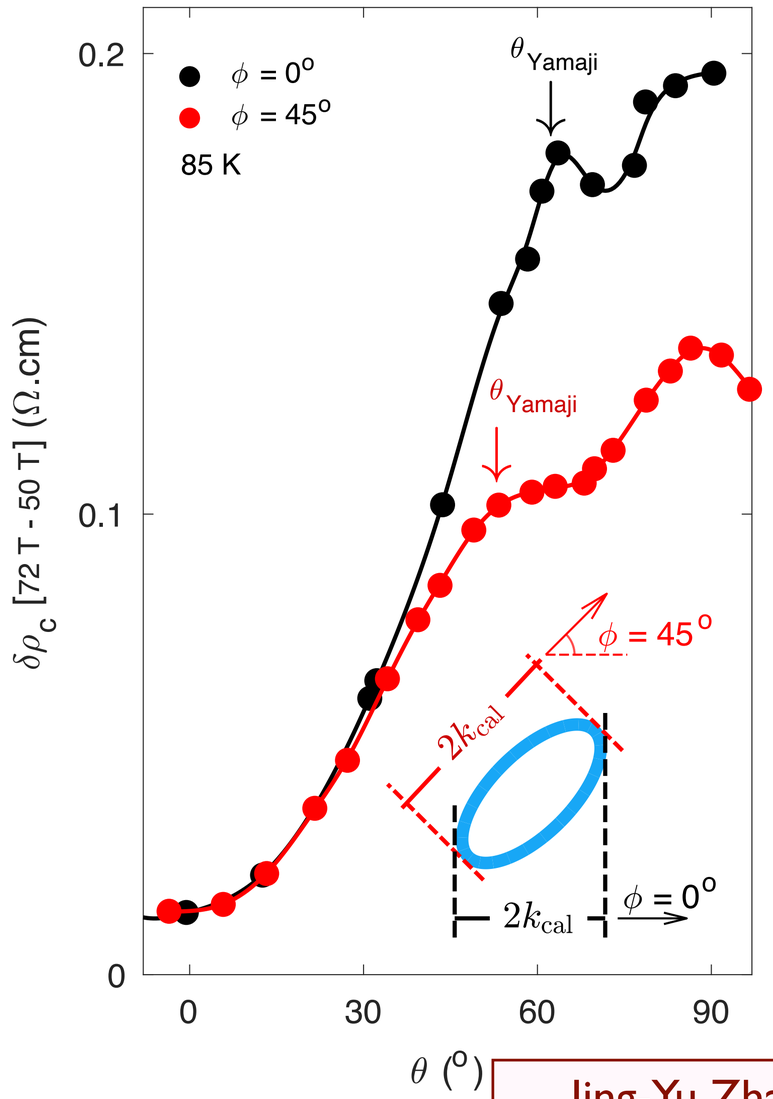

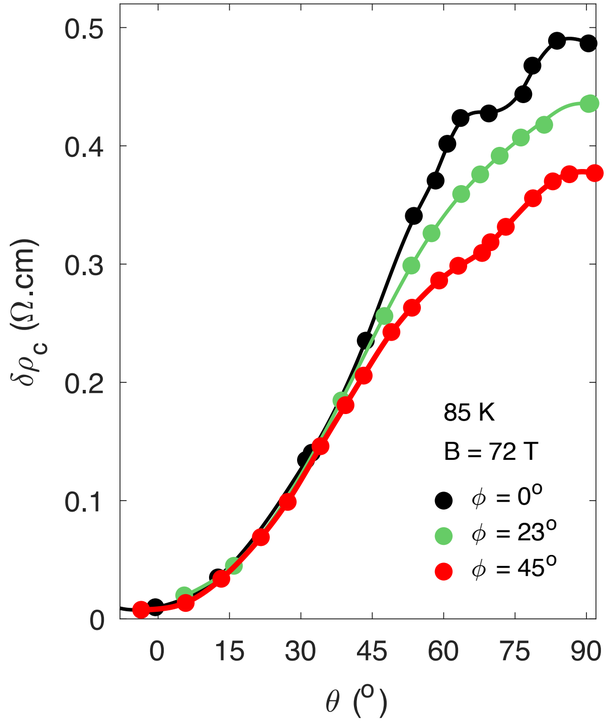

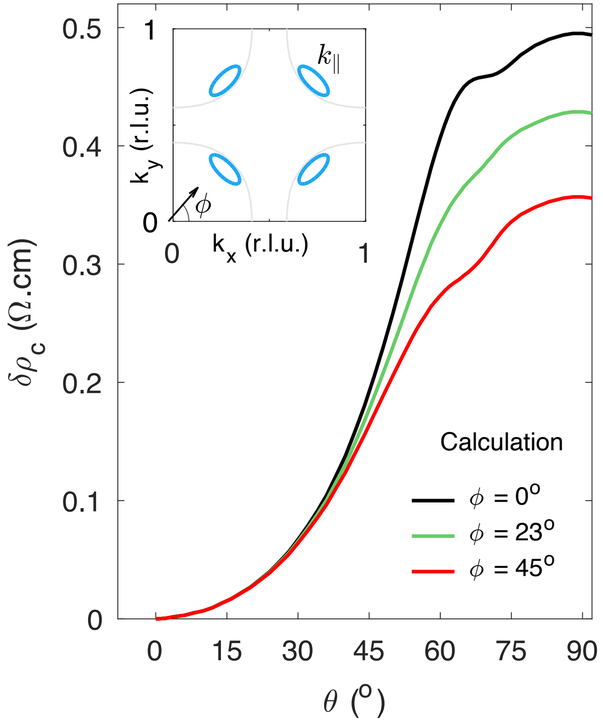

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FL* pocket fraction $= p/8 = 1.25\%$!
Fluctuating AF metal fraction $= p/4 = 2.5\%$.

Jing-Yu Zhao, S. Chatterjee, S. S., and Ya-Hui Zhang, arXiv:2510.13943

($p/8$ also in YRZ ansatz, Peter Johnson photoemission, and Jenny Hoffman and Seamus Davis STMs; Stanescu-Kotliar)

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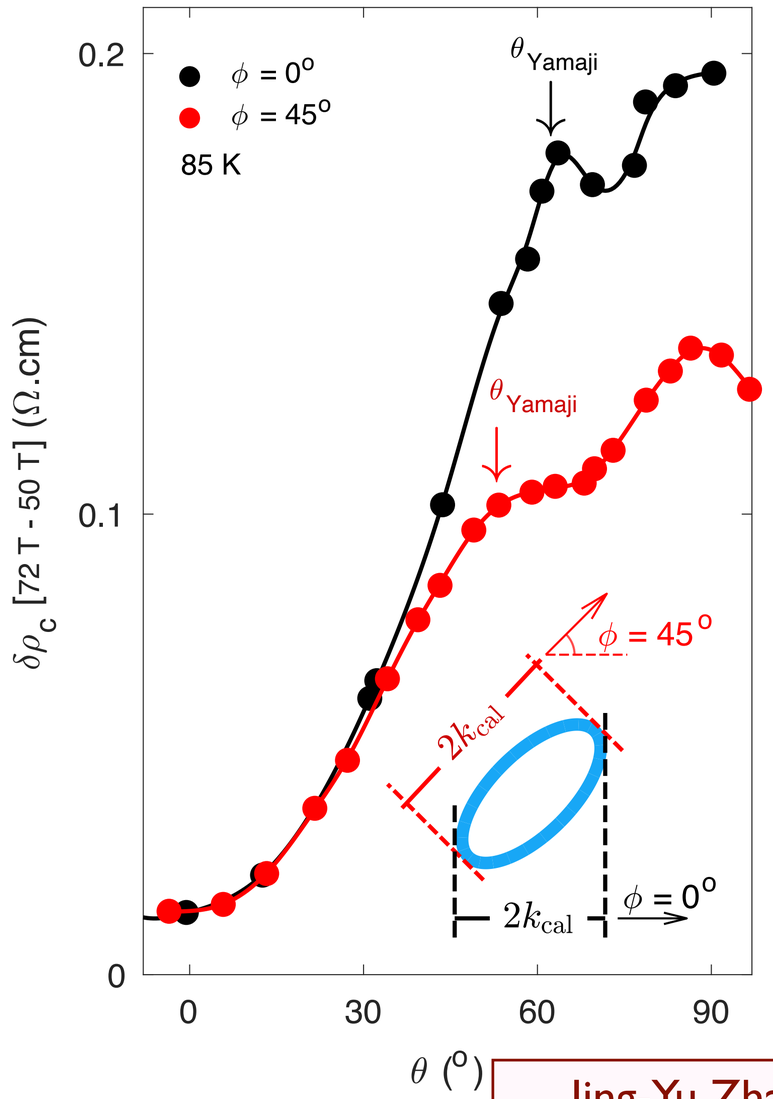

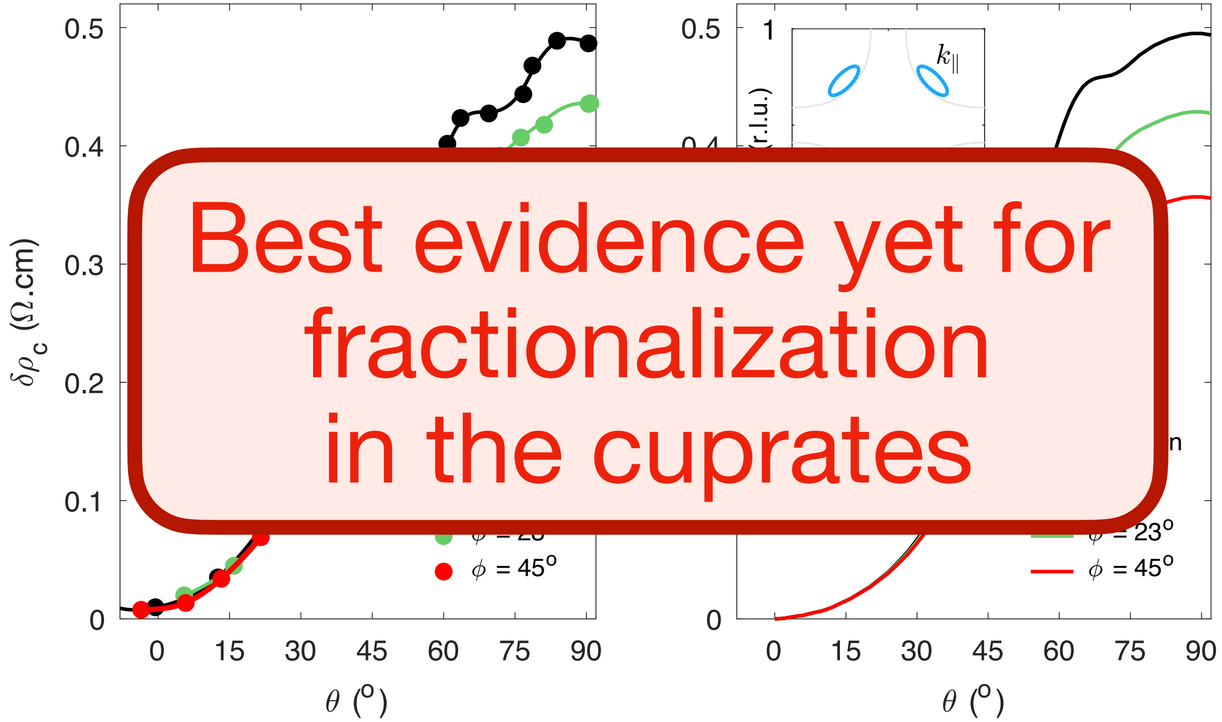

Best evidence yet for fractionalization in the cuprates

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($p/8$ also in Peter Johnson photoemission, and Jenny Hoffman and Seamus Davis STMs; Stanescu-Kotliar)

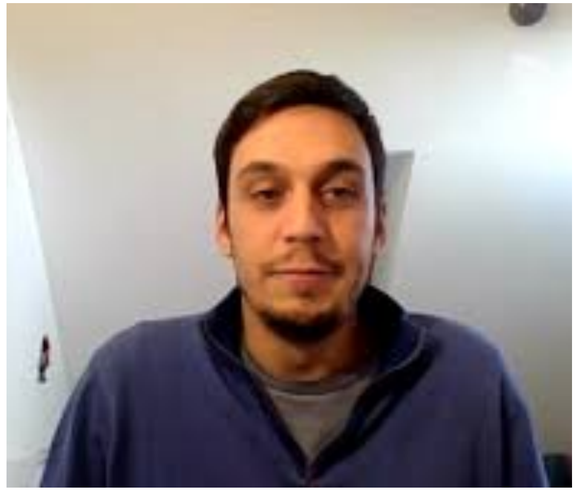
Coupling hole pockets to spin liquid

(the spin liquid is required to satisfy Oshikawa anomaly)

Thermal SU(2) lattice gauge theory of the cuprate pseudogap: reconciling Fermi arcs and hole pockets

H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, A. Nikolaenko, S. Sharma, S.S., arXiv:2507.05336

Maine Christos
Caltech



Pietro Bonetti

Alexander Nikolaenko

The Institute of Mathematical Sciences, Chennai

Harshit Pandey

Ravi Shanker

Sayantan Sharma

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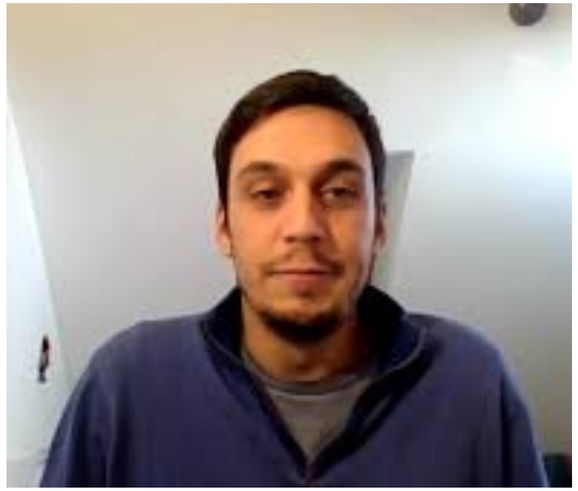


Pietro Bonetti

Alexander
Nikolaenko

Aavishkar Patel
ICTS, Bengaluru

arXiv > cond-mat > arXiv:2508.20164

Condensed Matter > Strongly Correlated Electrons

[Submitted on 27 Aug 2025]

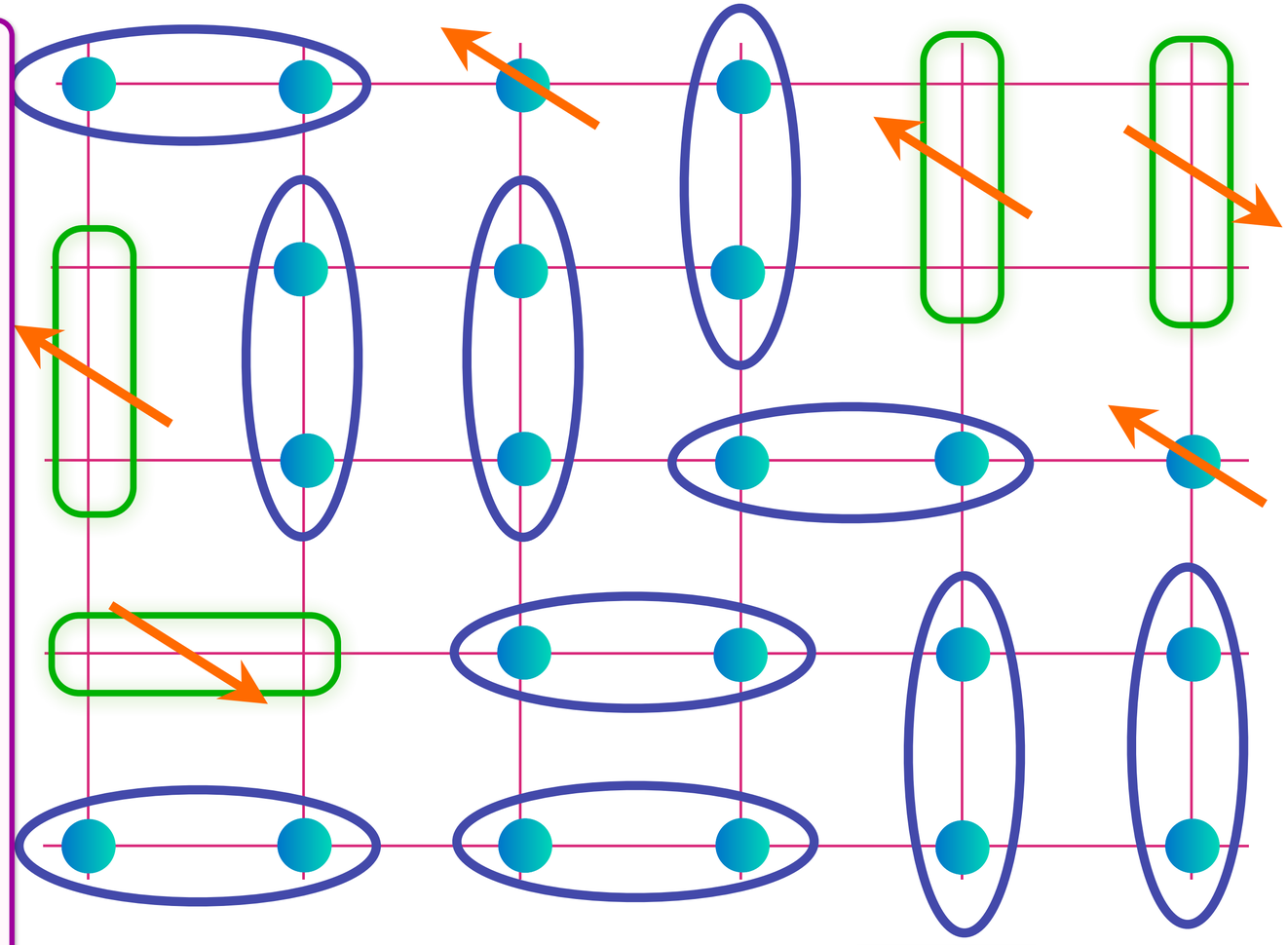
Critical quantum liquids and the cuprate high temperature superconductors

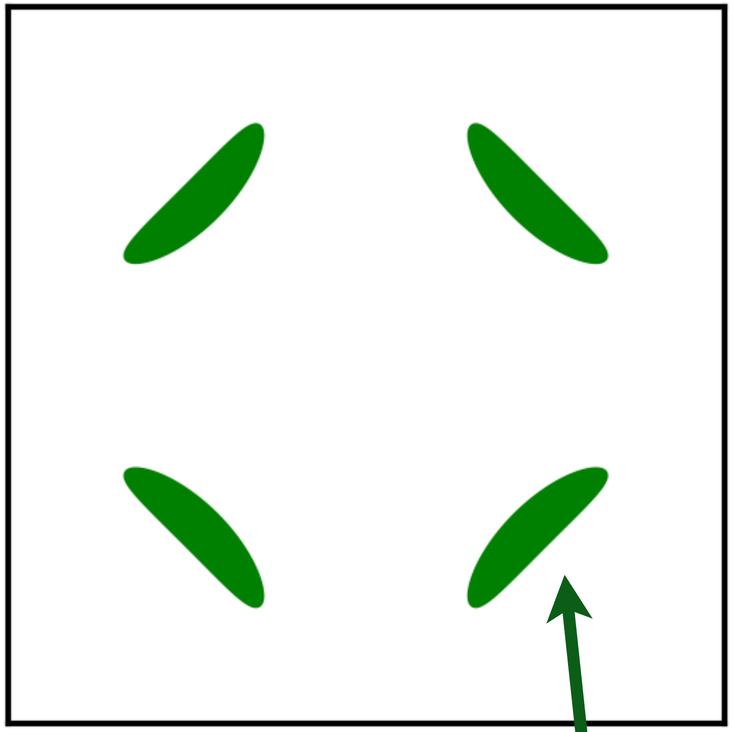
Doping an insulating antiferromagnet with holes of density p

FL*

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Metal with density p of spin-1/2, charge $+e$ 'holes' (or 'magnetic polarons') and charge 0, spin-1/2 'spinons'





Area $p/8$

$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

$= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

1. Dope spin liquid with *holes*, <u>not</u> holons.
   The Ancilla Layer Model (ALM) enables a theory of FL* with hole pockets and a general spin liquid.

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

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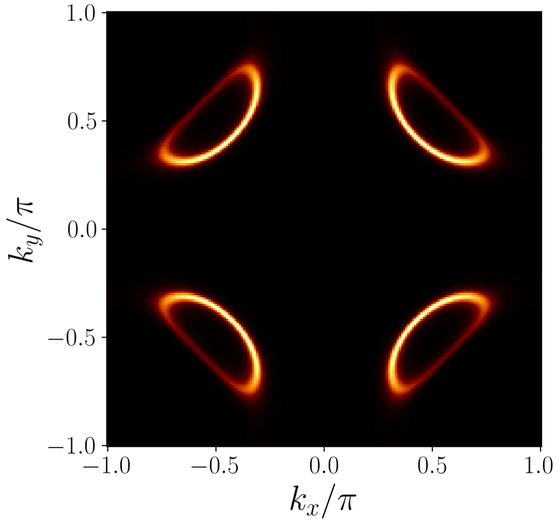

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang, PRB **73**, 174501 (2006)
Y. Qi and S. S., PRB **81**, 115129 (2010)

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I. Affleck and J.B. Marston, PRB **37**, 3774 (1988); N. Read and S. Sachdev, PRL **62**, 1694 (1989) C. Wang, A. Nahum, M.A. Metlitski, C. Xu, T. Senthil, PRX **7**, 031051 (2017); Zheng Zhou, Liangdong Hu, Wei Zhu, Yin-Chen He, PRX **14**, 021044 (2024)

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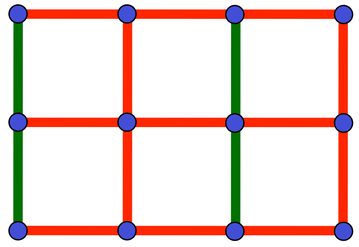

$e_{ij} = 1$ $\quad$ $e_{ij} = -1$

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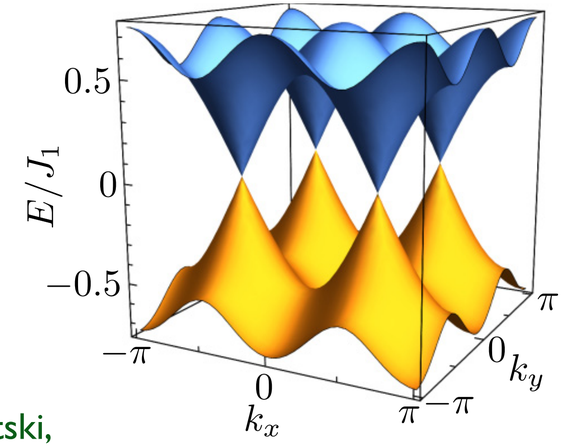
   (QCD is a SU(3) gauge theory with $N_f = 3$ massless Dirac quarks in 3+1 D.)



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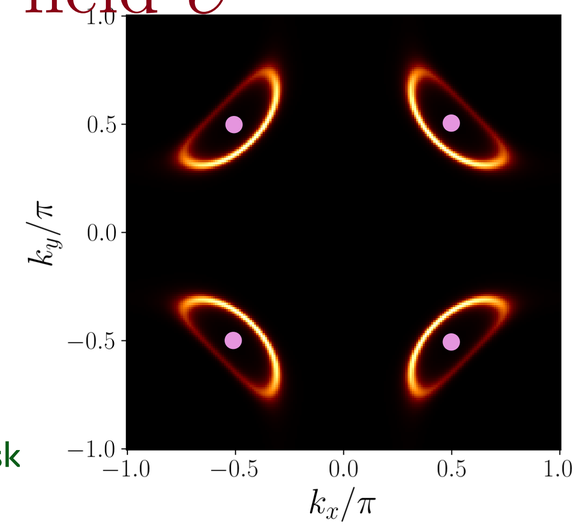

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4. Couple hole pockets coupled to critical spin liquid by a charge e, SU(2) fundamental Higgs field B
   (similar to Yukawa coupling of Higgs field in Weinberg-Salam theory).

- To create the electron operator from the spinon Ψ, we have to fuse it with a charge e, gauge SU(2) fundamental, spinless boson $B_{\boldsymbol{i}} = (B_{1\boldsymbol{i}}, B_{2,\boldsymbol{i}})$, whose symmetries follow from those of Ψ.
  This yields the quadratic and quartic energy functionals $\mathcal{E}_2[B,U] + \mathcal{E}_4[B,U]$

$$\mathcal{E}_2[B,U] = r \sum_{\boldsymbol{i}} B_{\boldsymbol{i}}^\dagger B_{\boldsymbol{i}} + iw \sum_{\langle \boldsymbol{ij} \rangle} e_{\boldsymbol{ij}} \left(B_{\boldsymbol{i}}^\dagger U_{\boldsymbol{ij}} B_{\boldsymbol{j}} - B_{\boldsymbol{j}}^\dagger U_{\boldsymbol{ji}} B_{\boldsymbol{i}} \right)$$

$$+ \kappa \sum_{\square} \left\{ 1 - \frac{1}{2} \mathrm{ReTr} \prod_{\boldsymbol{ij} \in \square} U_{\boldsymbol{ij}} \right\}$$

$$H_{\mathrm{Yukawa}} = ig \sum_i \left(B_{1i}^* f_{1i\sigma}^\dagger f_{i\sigma} + B_{2i}^* \varepsilon_{\sigma\sigma'} f_{1i\sigma}^\dagger f_{i\sigma'}^\dagger - \mathrm{H.c.} \right)$$

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)

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Hole pockets

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Spinons

$$\mathcal{E}[B,U] = r \sum_i B^\dagger_i B_i + iw \sum_{\langle ij \rangle} e_{ij} \left(B^\dagger_i U_{ij} B_j - B^\dagger_j U_{ji} B_i \right) + \mathcal{E}_4[B,U] + \kappa \sum_\square \left\{ 1 - \frac{1}{2} \mathrm{ReTr} \prod_{ij \in \square} U_{ij} \right\}$$

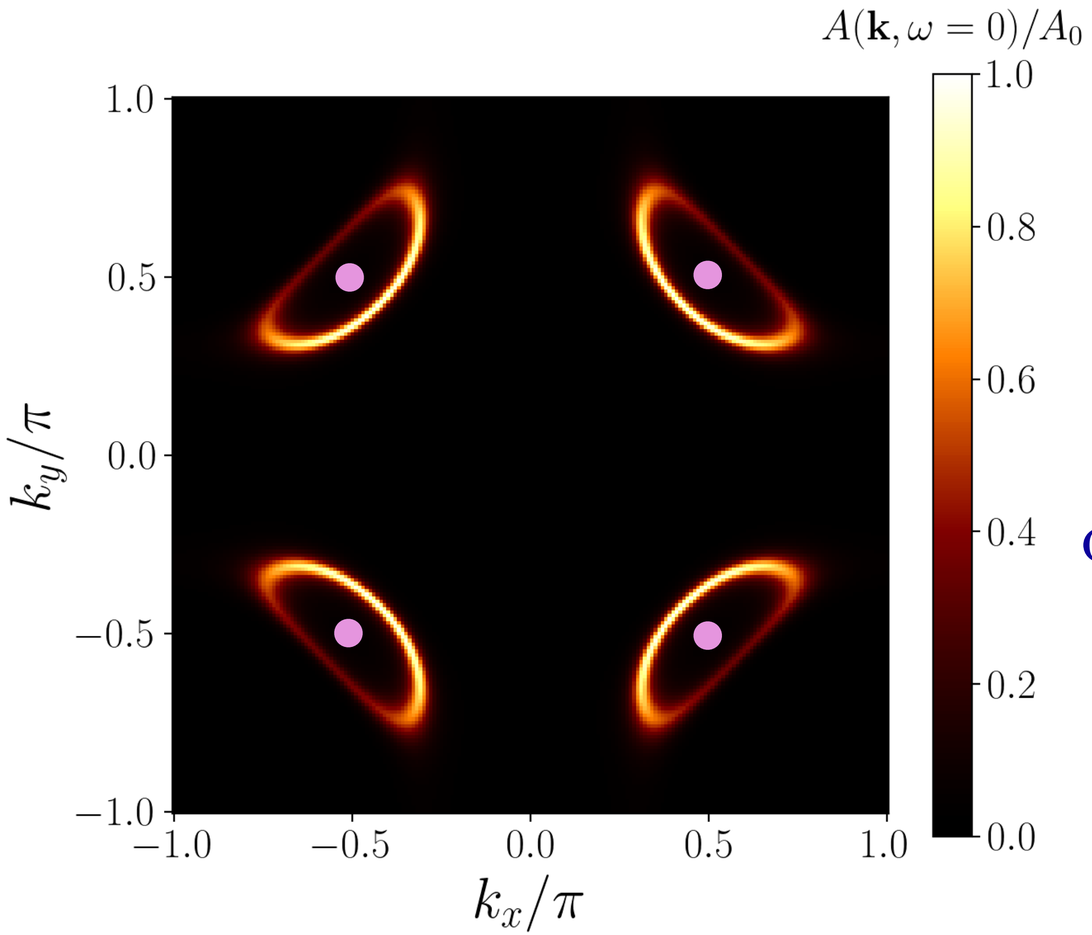
Higgs potential

$$H_{\text{Yukawa}} = ig \sum_i \left(B^*_{1i} f^\dagger_{1i\sigma} f_{i\sigma} + B^*_{2i} \varepsilon_{\sigma\sigma'} f^\dagger_{1i\sigma} f^\dagger_{i\sigma'} - \text{H.c.} \right)$$

Yukawa coupling

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)

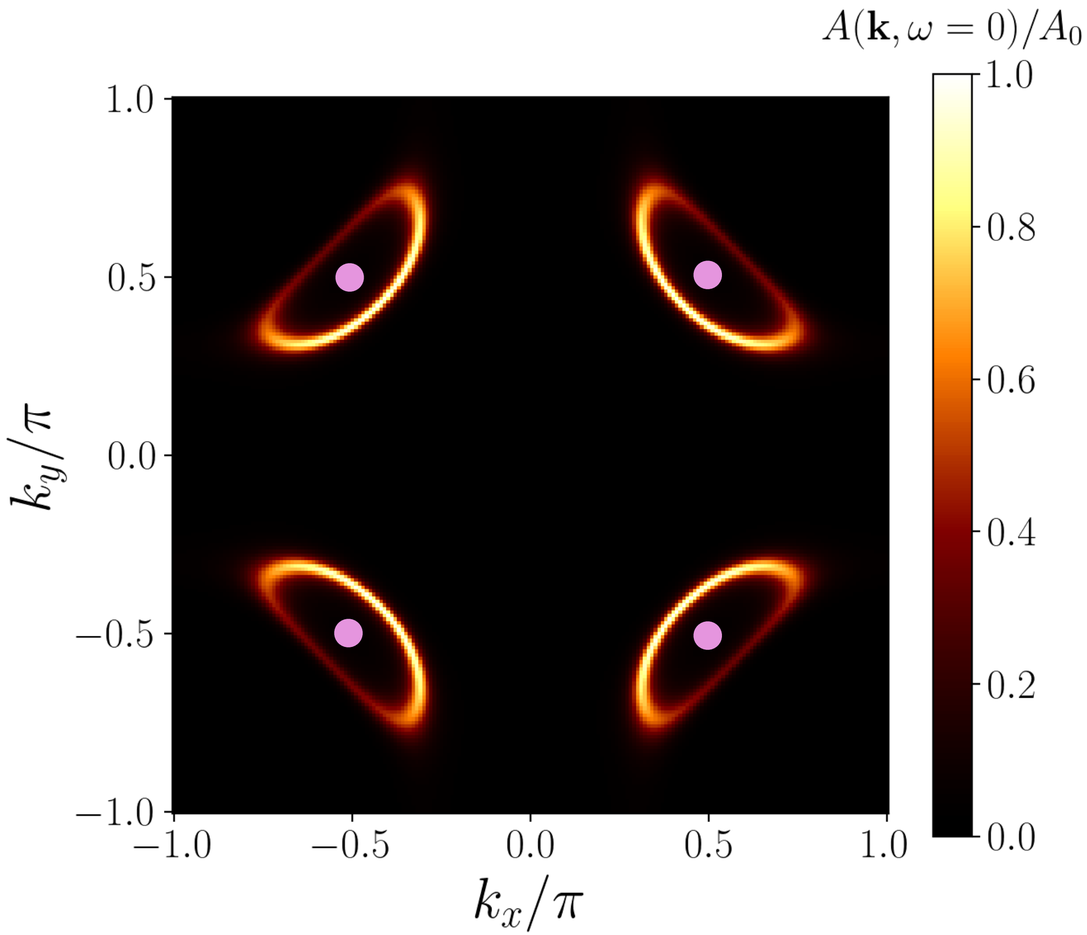
The pseudogap metal and its transition to d-wave superconductivity

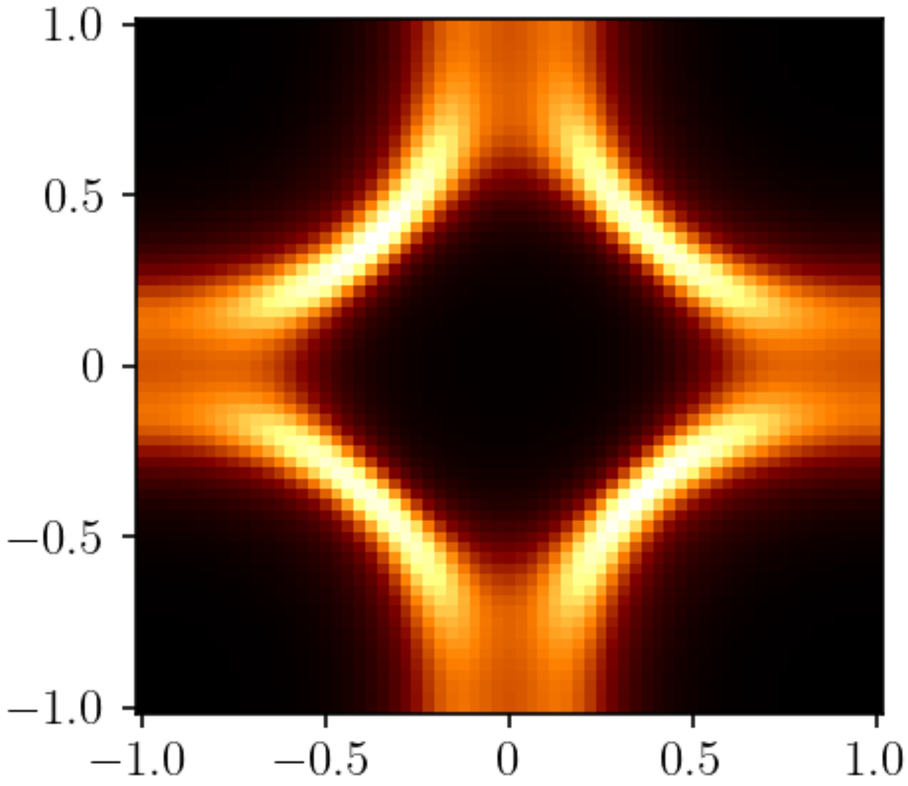


Hole pockets and spin liquid.
If the spin liquid is stable at $T = 0$,
hole pockets remain intact
despite Yukawa coupling to spin liquid.

E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

- Born-Oppenheimer theory of thermal fluctuations:
  Classical thermal ensemble of $\mathcal{E}_2[B,U] + \mathcal{E}_4[B,U]$ at a temperature $T > T_c$.
  Diagonalize Hamiltonian for electrons and spinons for each realization of B, U.





H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, A. Nikolaenko, S. Sharma, S.S., arXiv:2507.05336

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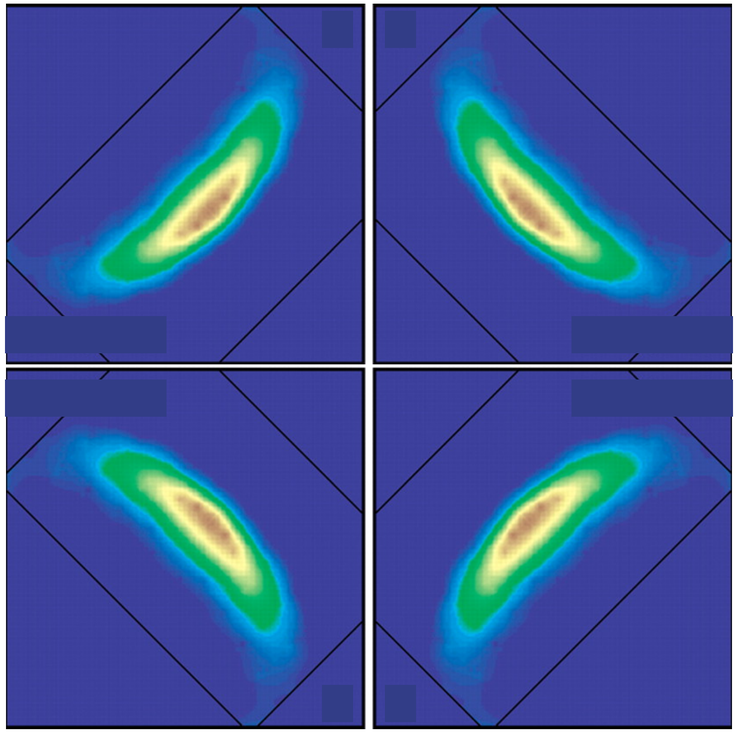
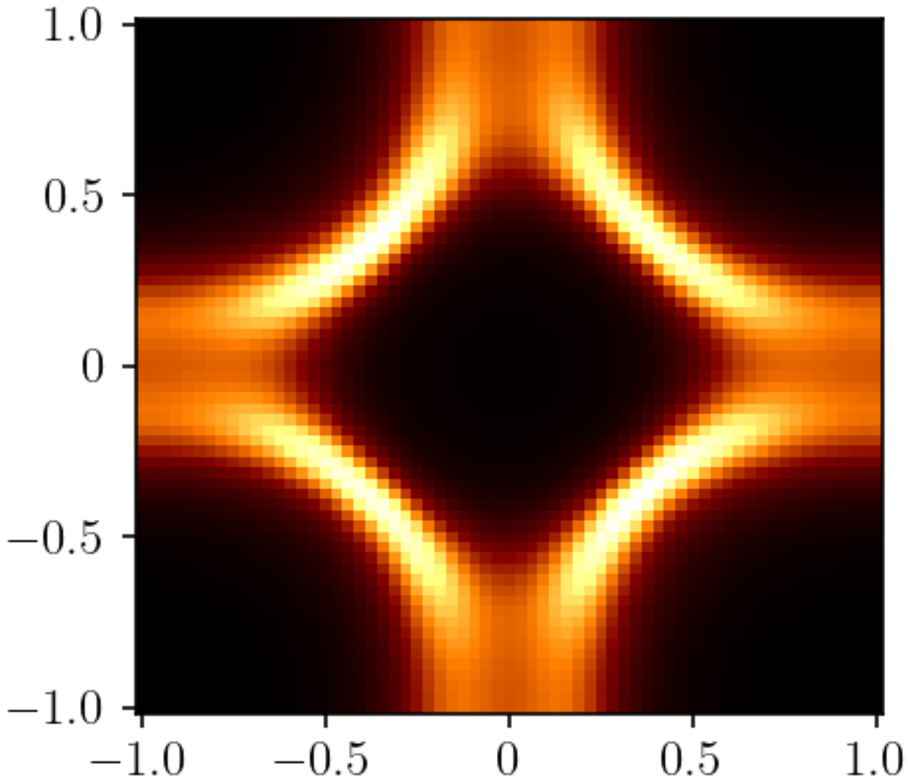


Kyle M. Shen, ... Z.-X. Shen, Science **307**, 901 (2005)



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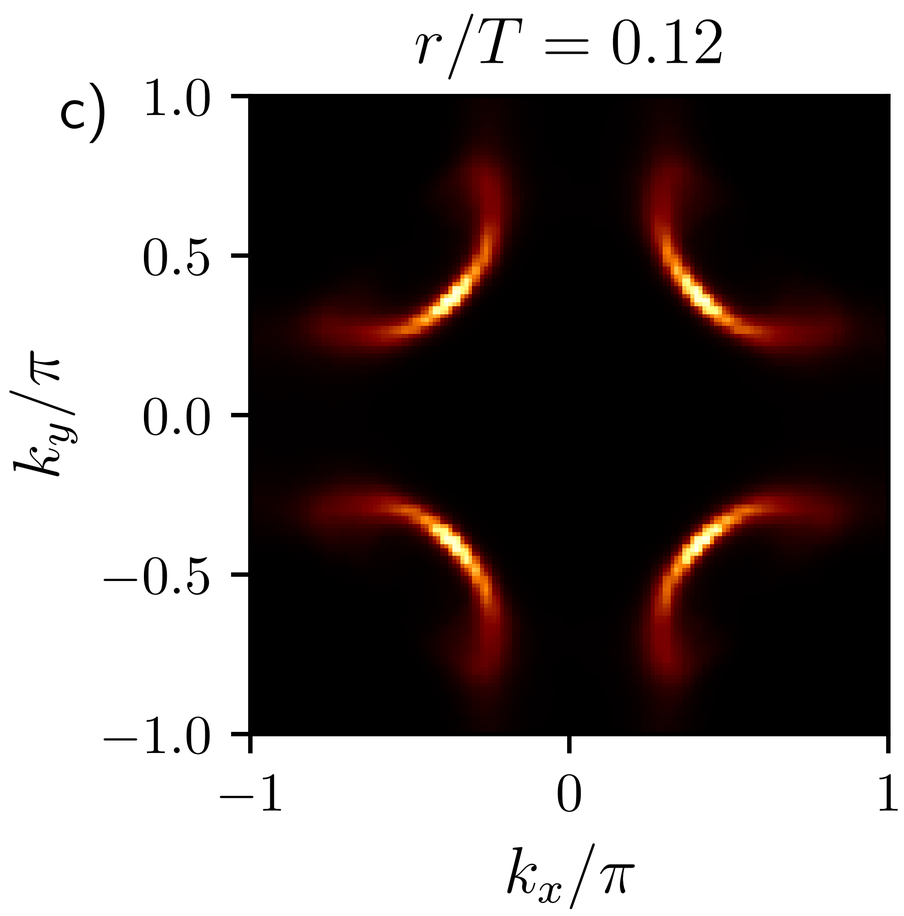
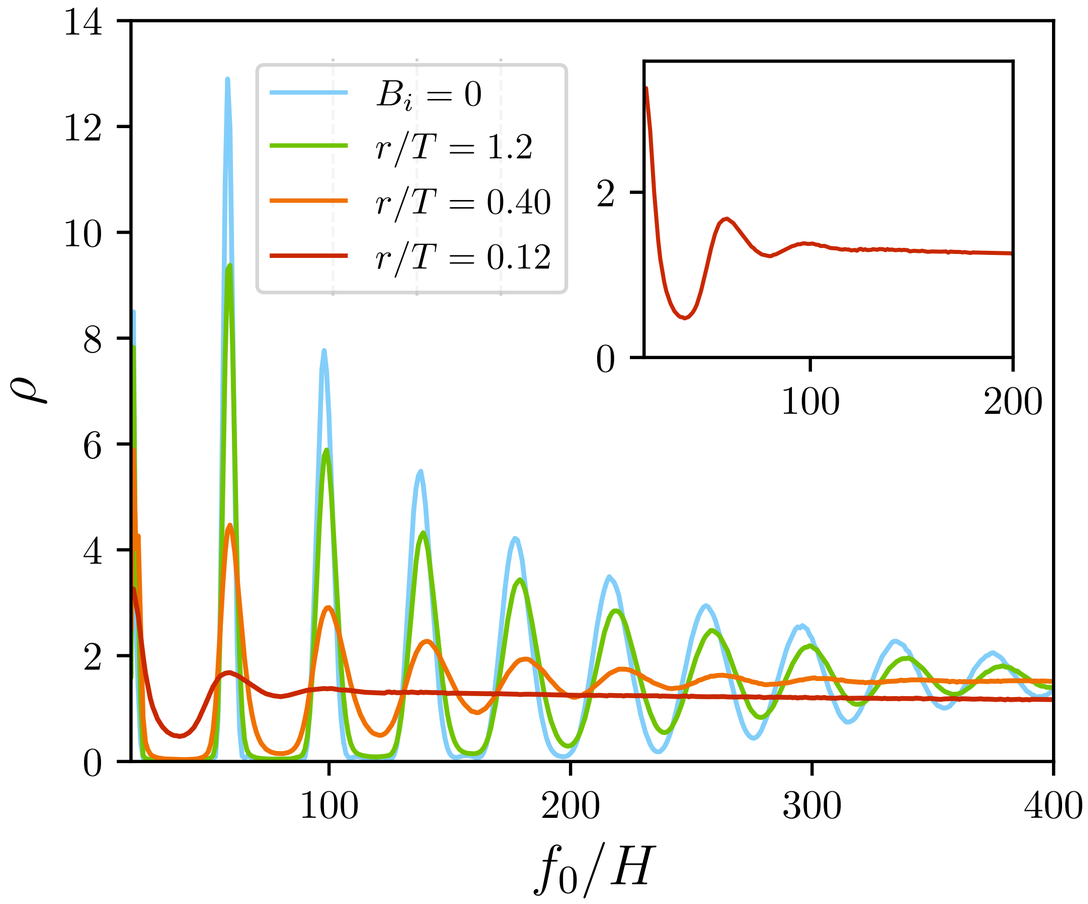


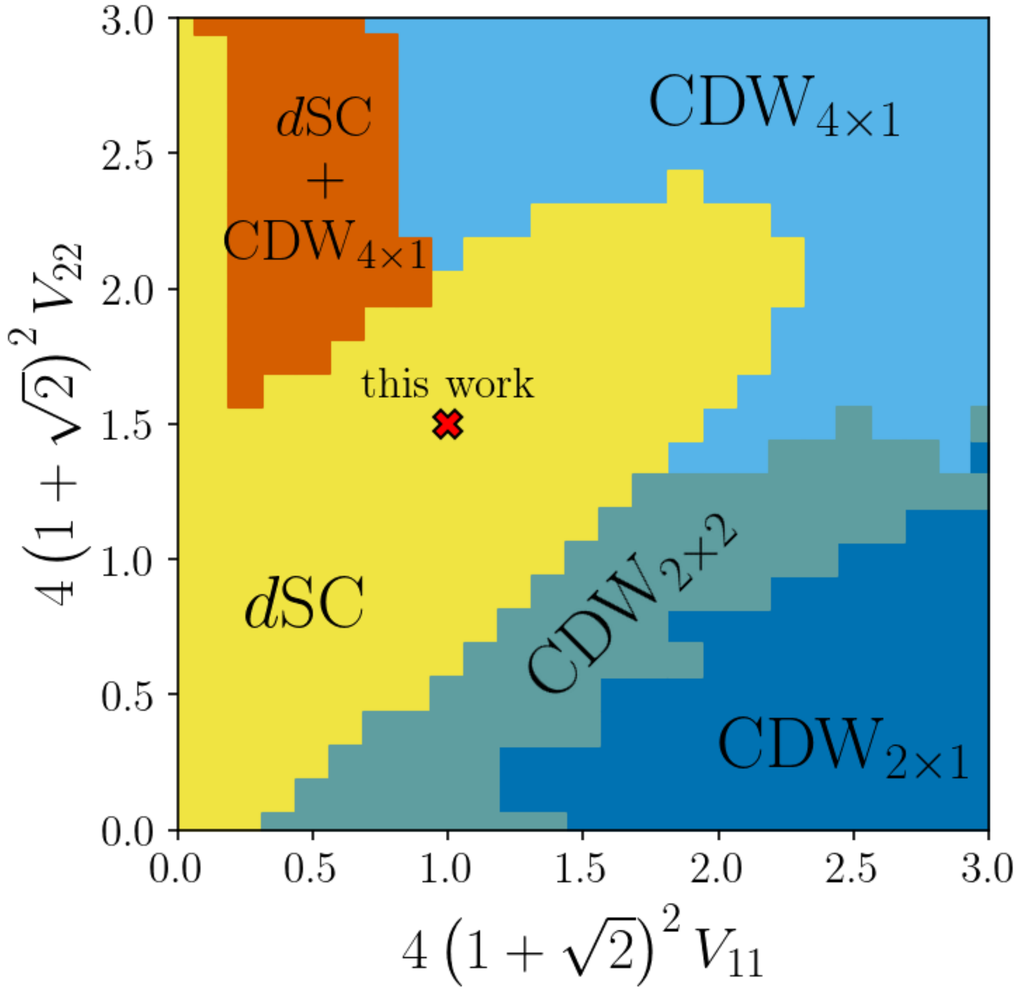


Fermi arcs co-exist with quantum oscillations of hole pockets of area $p/8$.

H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, A. Nikolaenko, S. Sharma, S.S., arXiv:2507.05336

- Born-Oppenheimer theory of thermal fluctuations:
  Classical thermal ensemble of $\mathcal{E}_2[B,U] + \mathcal{E}_4[B,U]$ at a temperature $T \to 0$.

V_{Higgs} chosen so that the ground state is a d-wave superconductor, and second best state is a period-4 stripe.



$T = 0$ phase diagram

H. Pandey,
M. Christos,
P.M. Bonetti,
R. Shanker,
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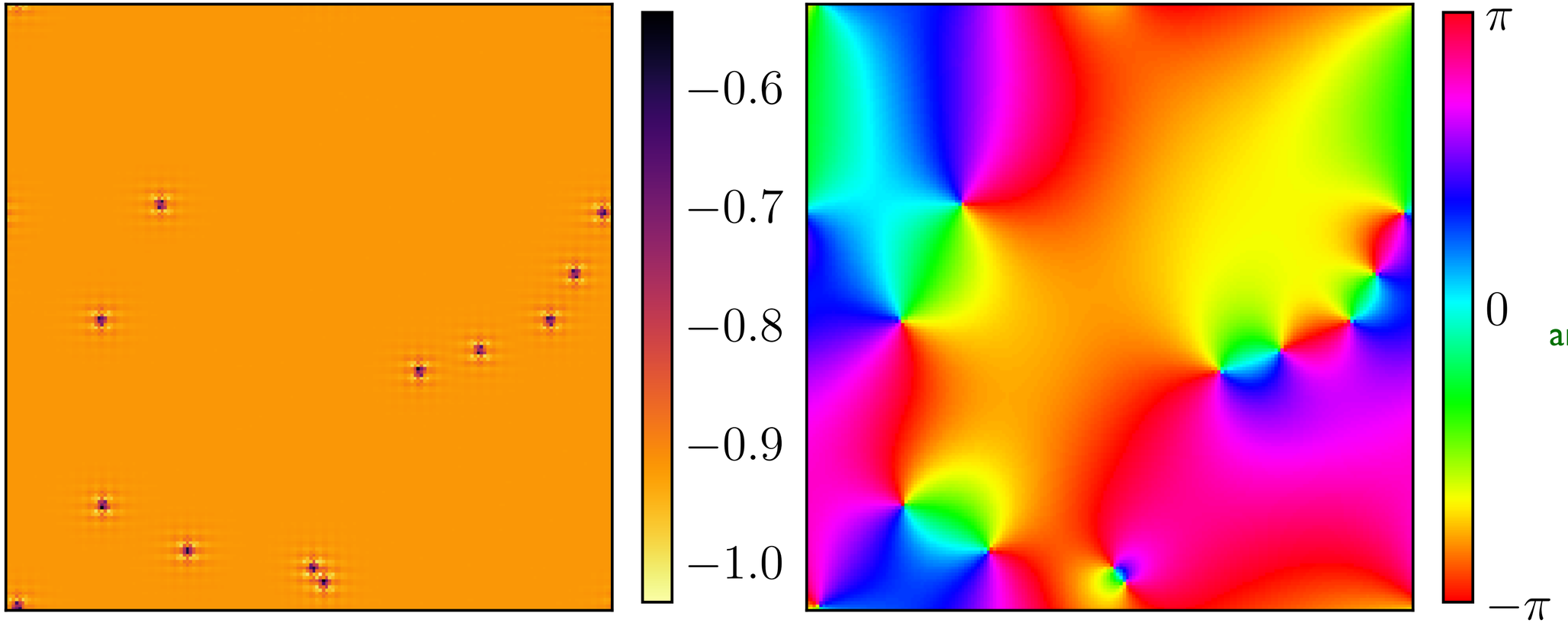
S. Sharma,
S.S.,
arXiv:2507.05336

Minimize V_{Higgs} w.r.t. $\left\langle \begin{pmatrix} B_{1i} \\ B_{2i} \end{pmatrix} \right\rangle$

Set $U_{ij} = 1$.

- Born-Oppenheimer theory of thermal fluctuations:
  Classical thermal ensemble of $\mathcal{E}_2[B,U] + \mathcal{E}_4[B,U]$ at a temperature $T \to 0$.



Bond density

Phase of pairing amplitude

H. Pandey,
M. Christos,
P.M. Bonetti,
R. Shanker,
A. Nikolaenko,
S. Sharma,
S.S.,
arXiv:2507.05336

See also
Jia-Xin Zhang
and S. S.,
PRB **110**,
235120
(2024)

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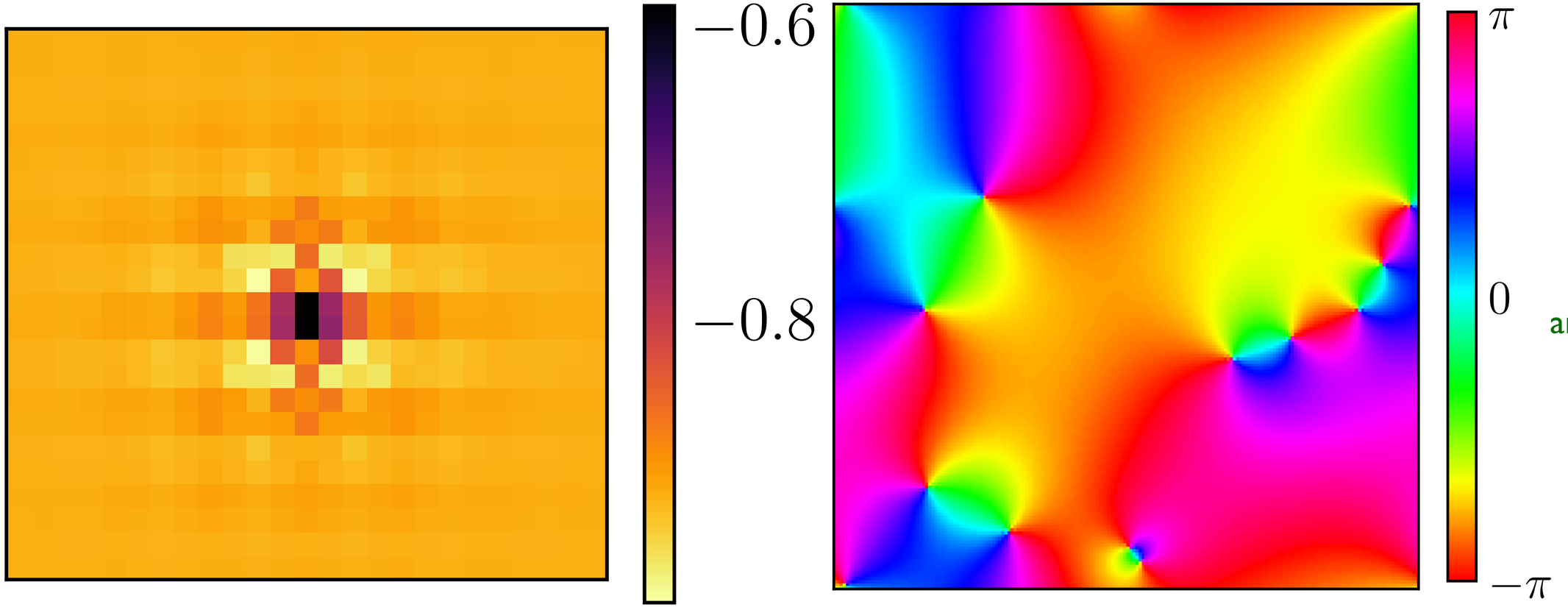

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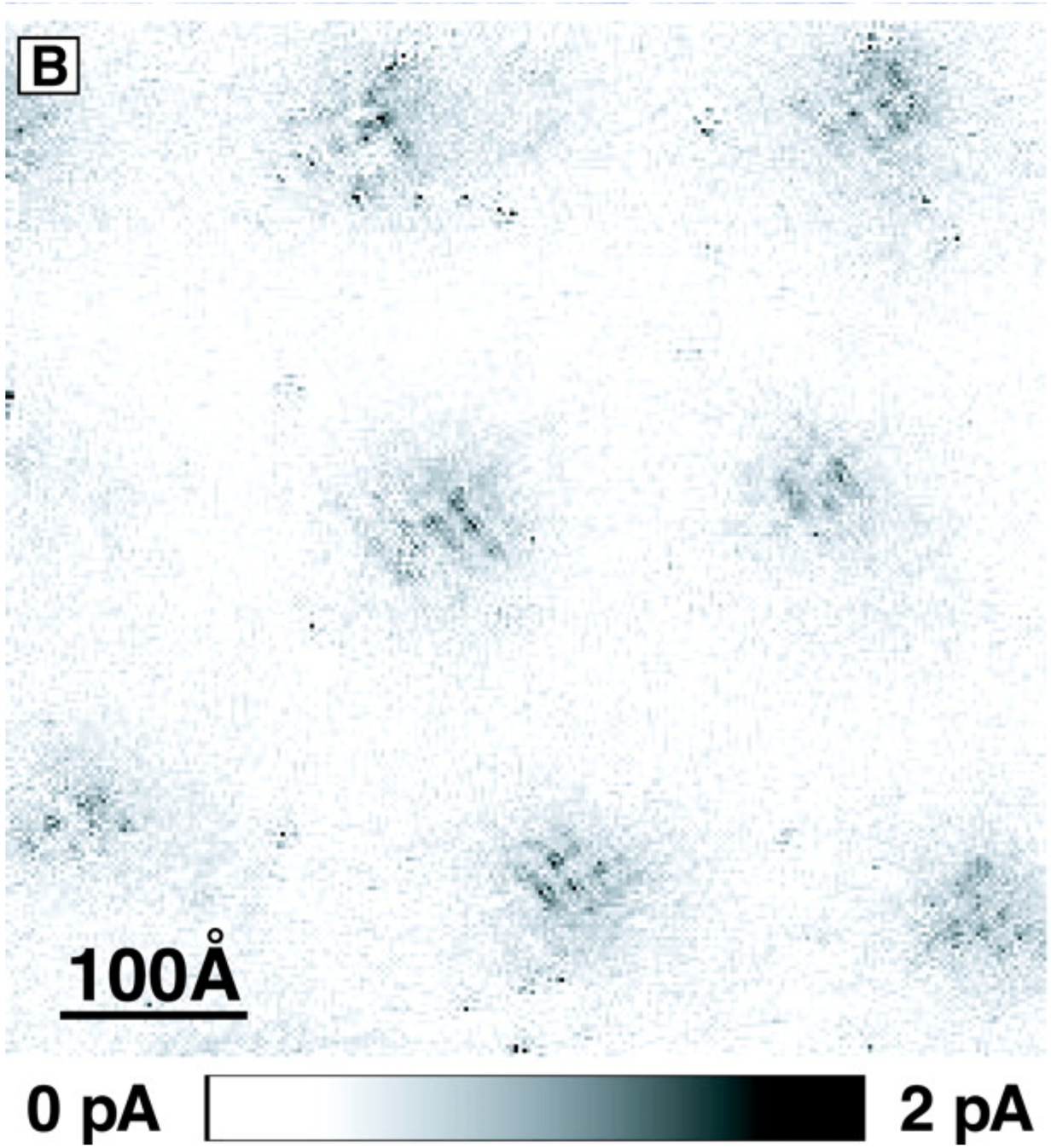


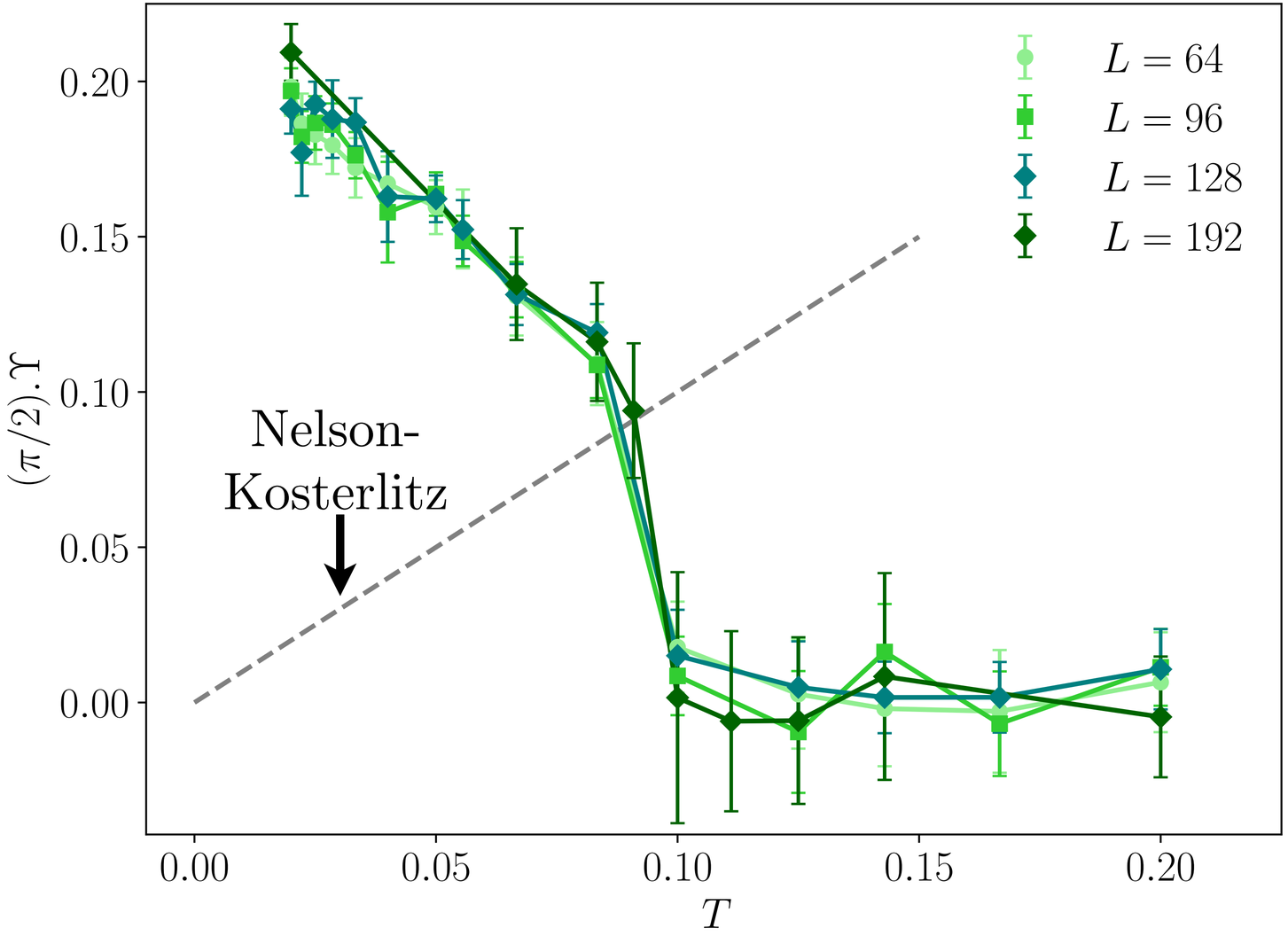
A Four Unit Cell Periodic Pattern of Quasi-Particle States Surrounding Vortex Cores in $Bi_2Sr_2CaCu_2O_{8+\delta}$

J. E. Hoffman, E. W. Hudson,
K. M. Lang, V. Madhavan,
H. Eisaki, S. Uchida, J.C. Davis

Science **295**, 466 (2002)

- Born-Oppenheimer theory of thermal fluctuations:
  Classical thermal ensemble of $\mathcal{E}_2[B,U] + \mathcal{E}_4[B,U]$ at a temperature $T \lessgtr T_{KT}$.

$\Upsilon =$
Helicity
Modulus



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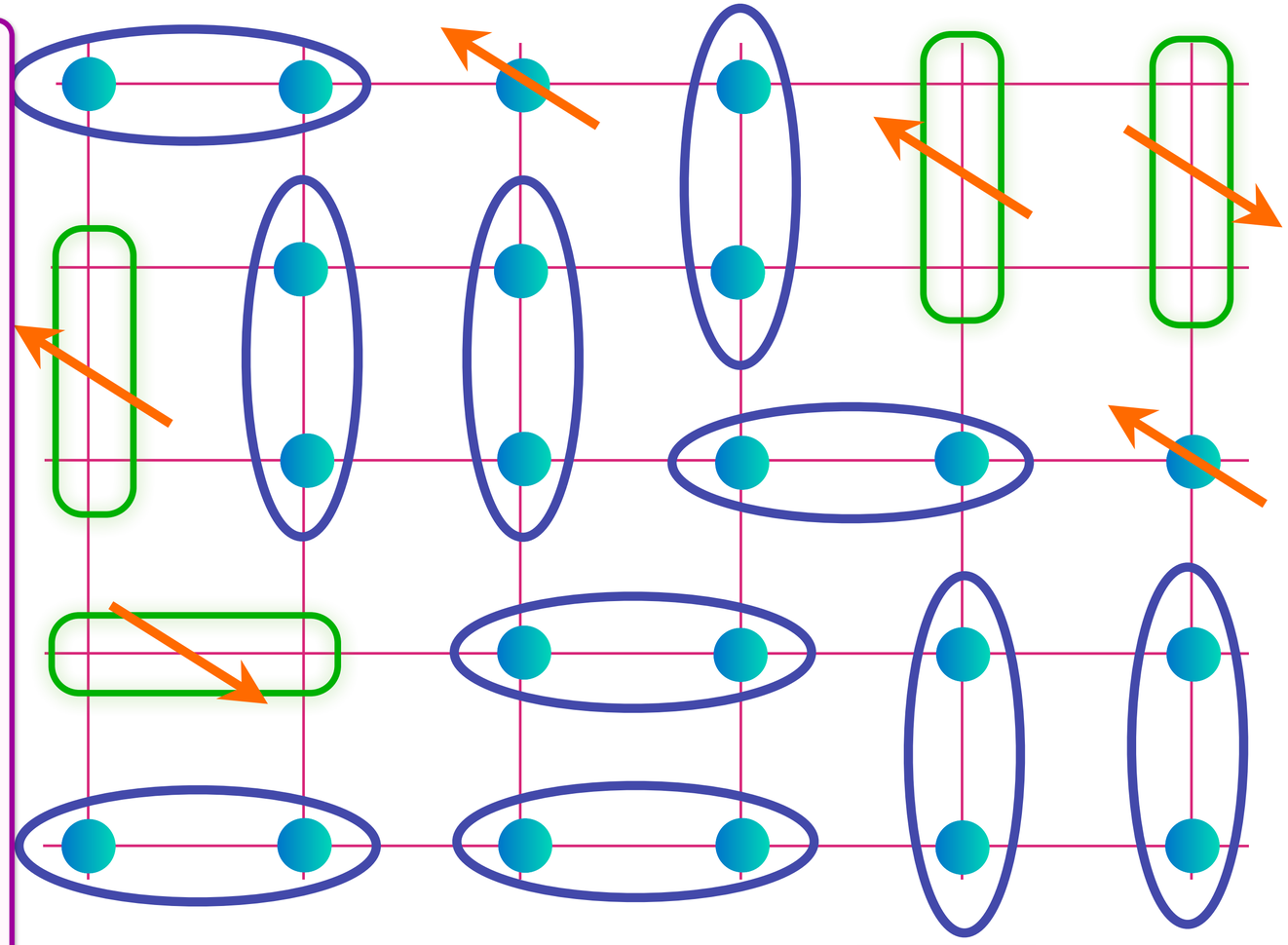
Doping an insulating antiferromagnet with holes of density p

FL*

Metal with density p of spin-1/2, charge $+e$ 'holes' (or 'magnetic polarons') and charge 0, spin-1/2 'spinons'



The d-wave superconductor is obtained by confining the SU(2) gauge field, and this exploits the pre-existing pairing in the blue dimers. Such a state is smoothly connected to the BCS d-wave superconductor.

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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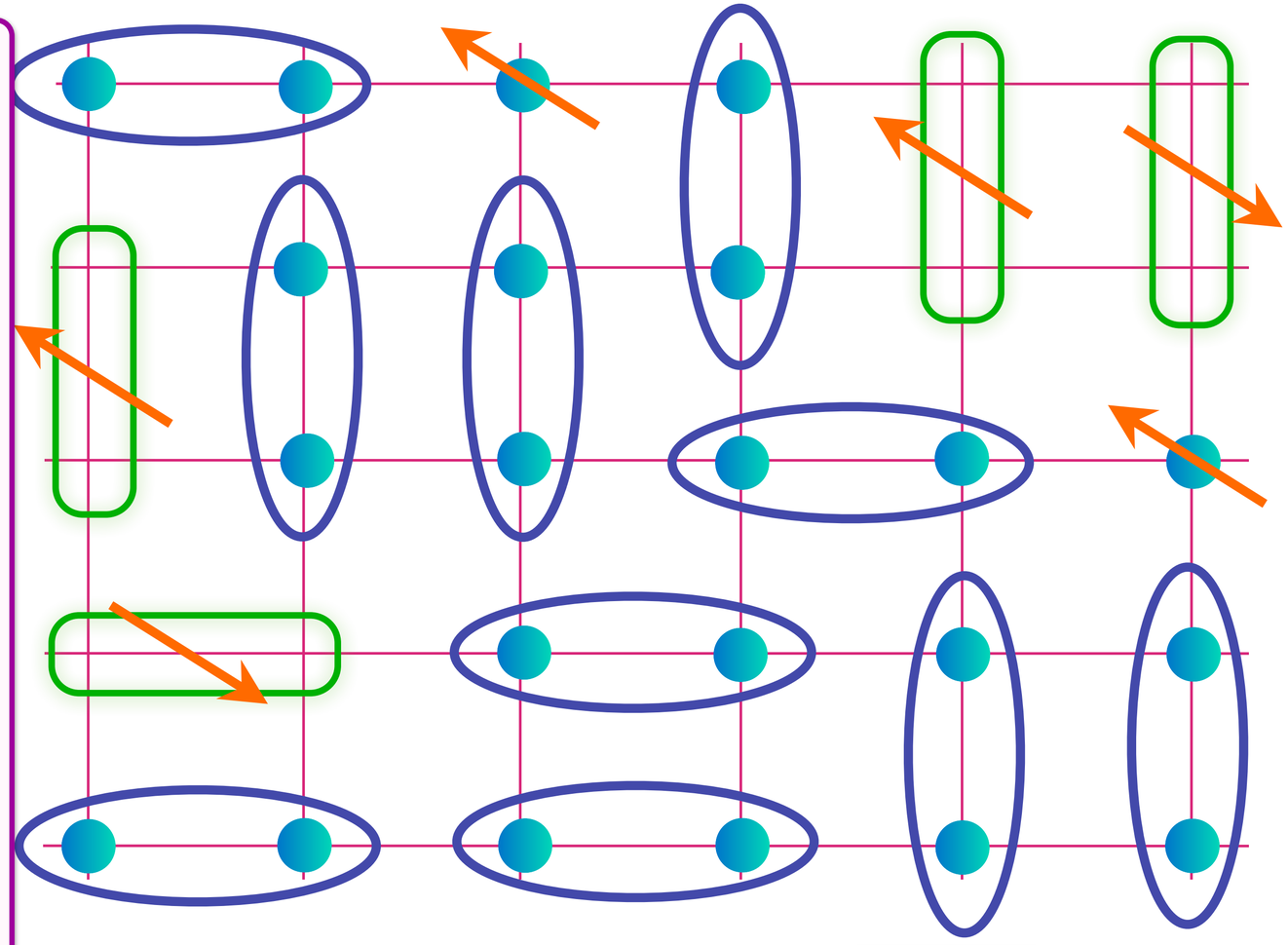

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Applying BCS pairing to the hole pockets (green dimers) yields a distinct SC* state, which is *not* smoothly connected to the BCS d-wave state.

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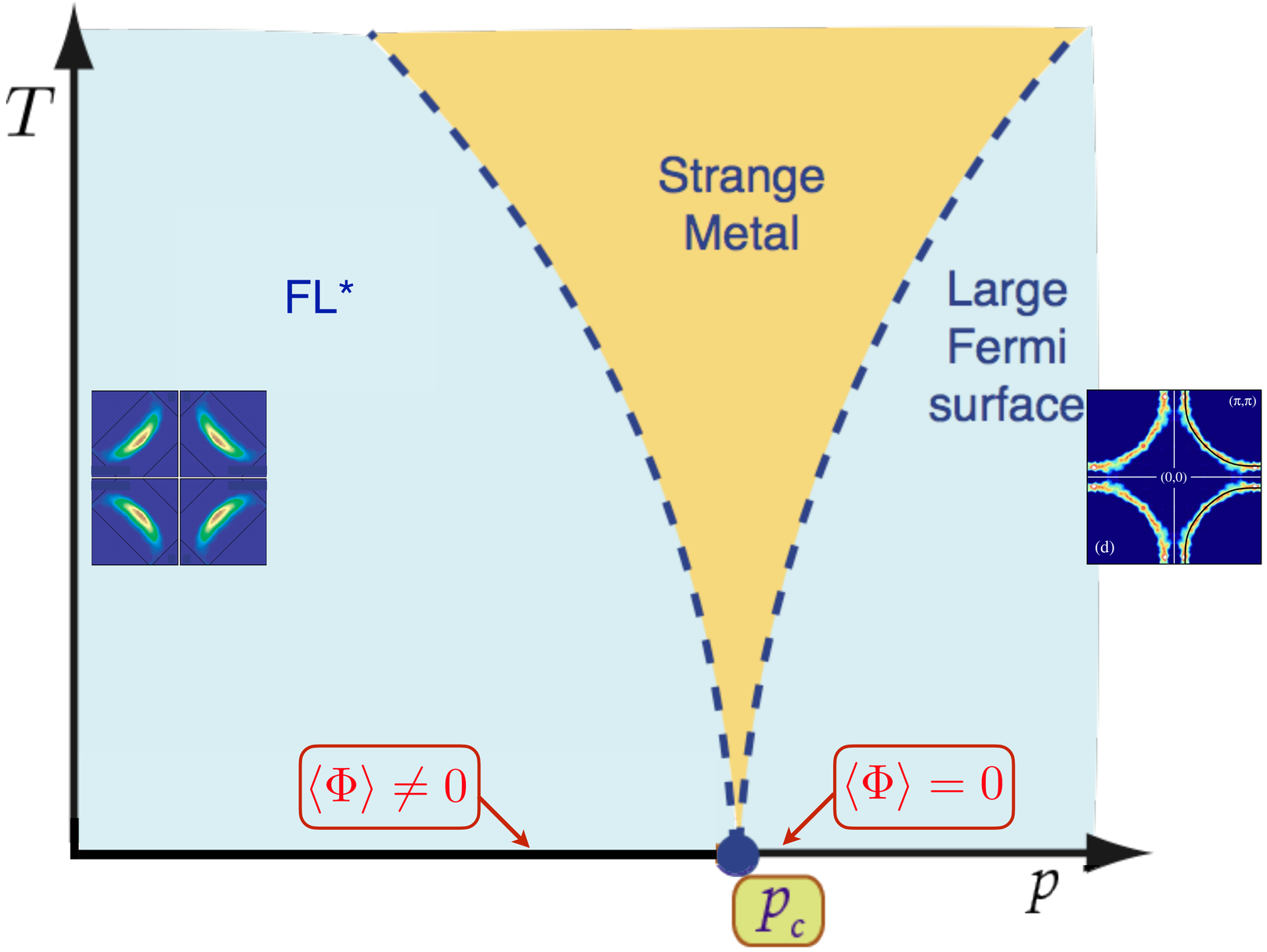
$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

The cuprate phase diagram



Quantum-criticality
of a quantum phase transition
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(FL* and FL) at $p = p_c$,
with no symmetry breaking.
Φ was the hybridization
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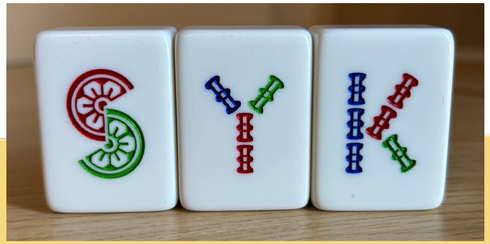
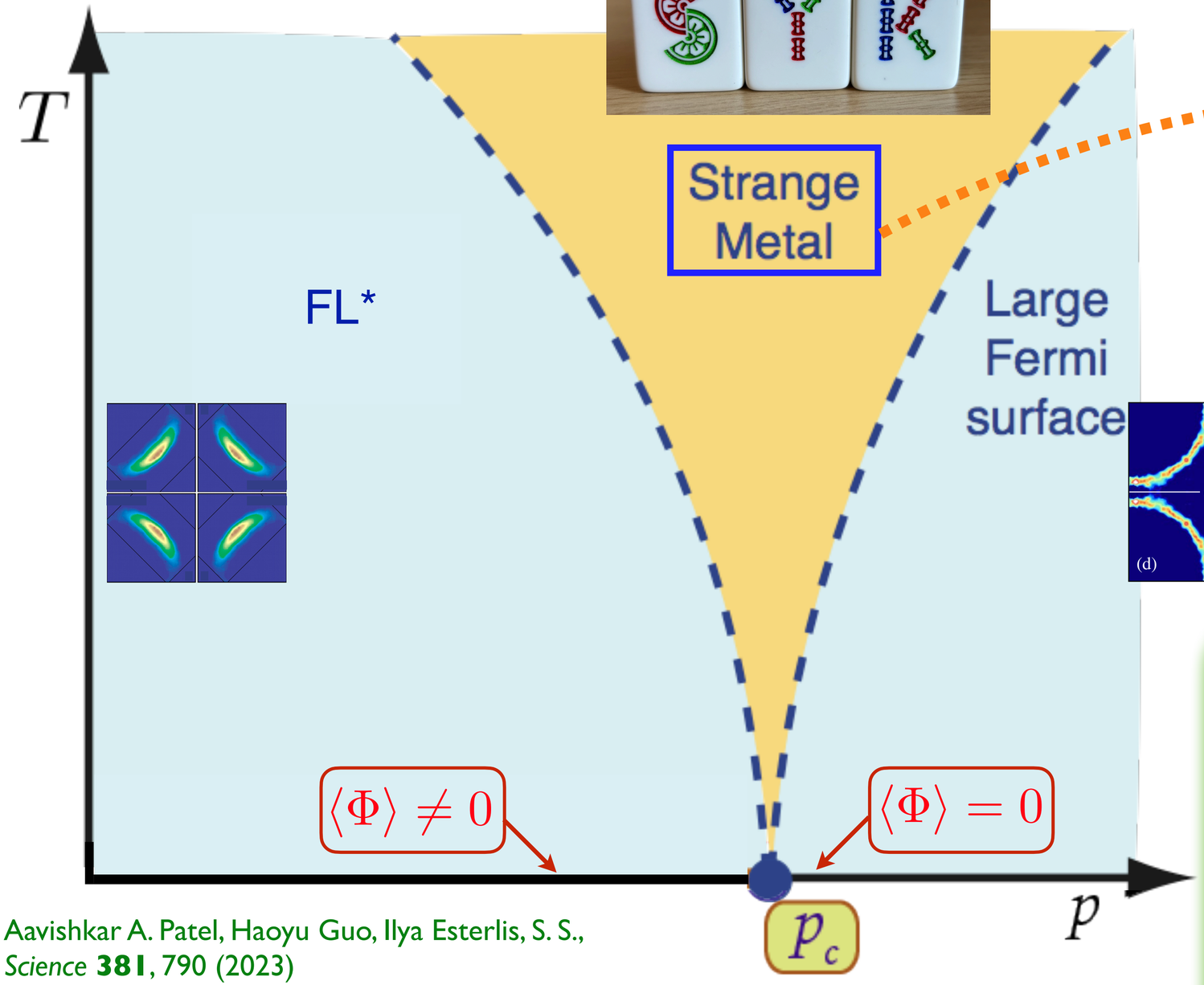
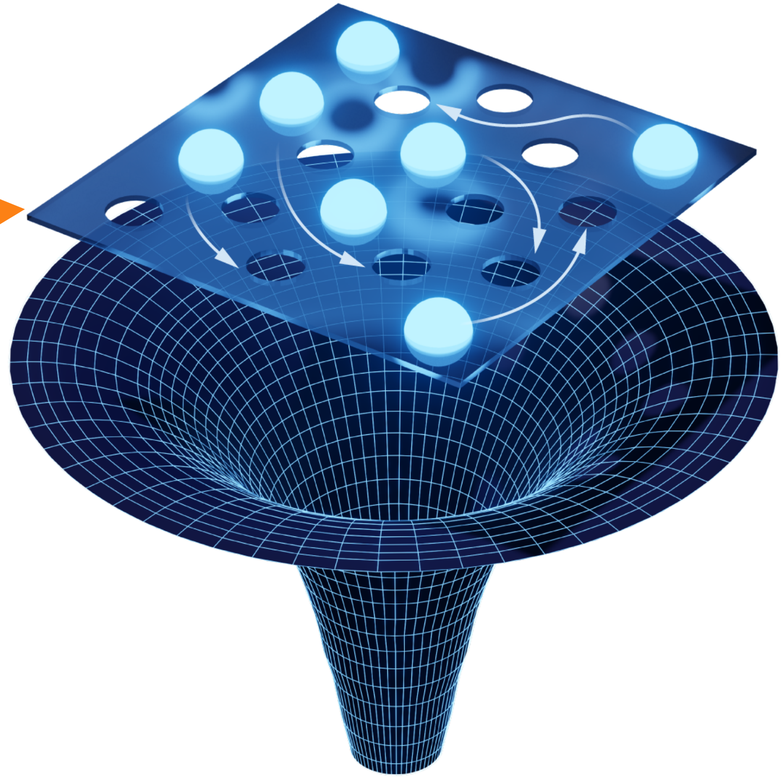
hole pockets— it is
now a Higgs field.

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

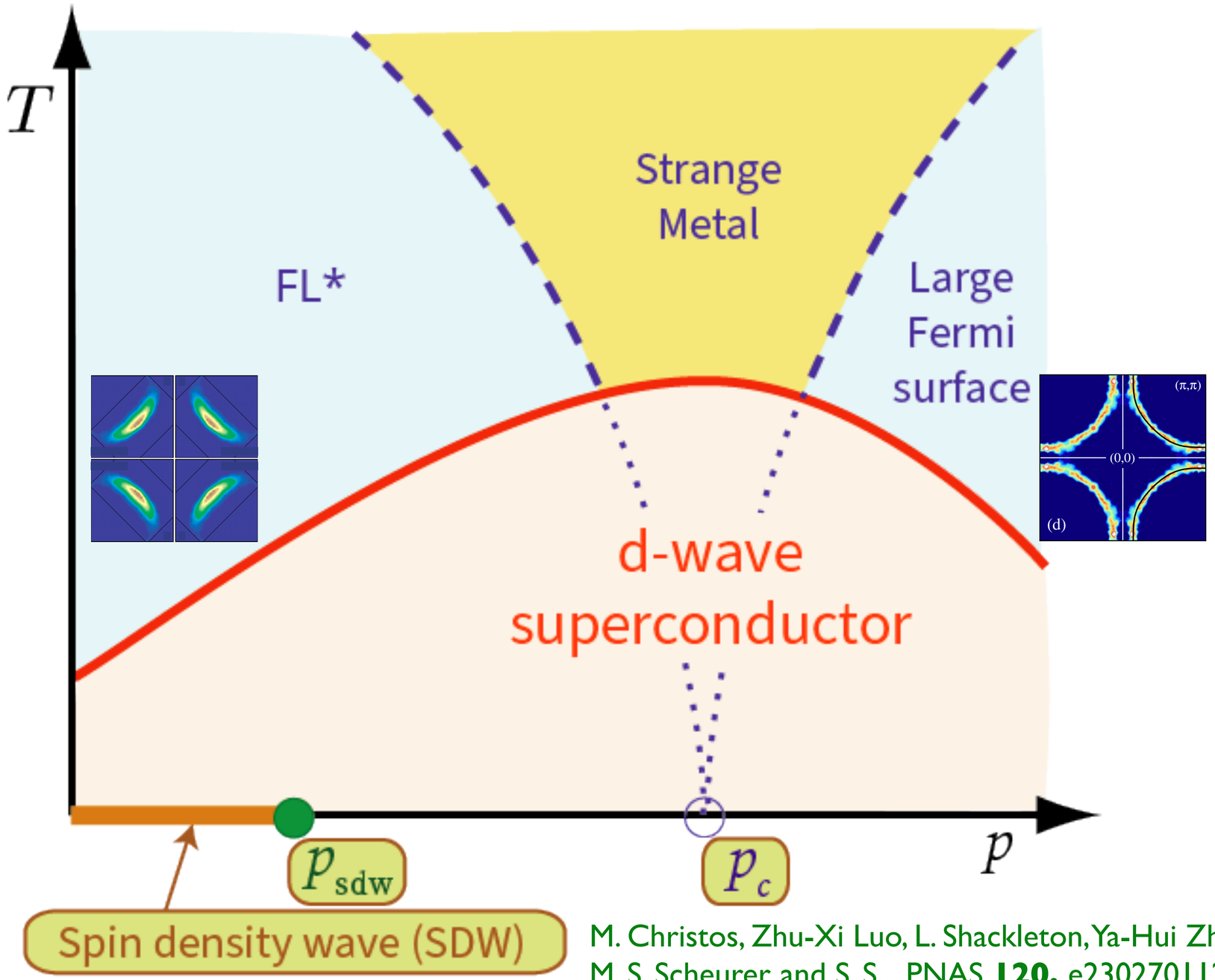






Explain with a local, two-dimensional extension of the Sachdev-Ye-Kitaev (SYK) model of mobile electrons, the 2D-Yukawa-SYK model with a spatially random Yukawa coupling between the large Fermi surface and Φ: a critical charge liquid

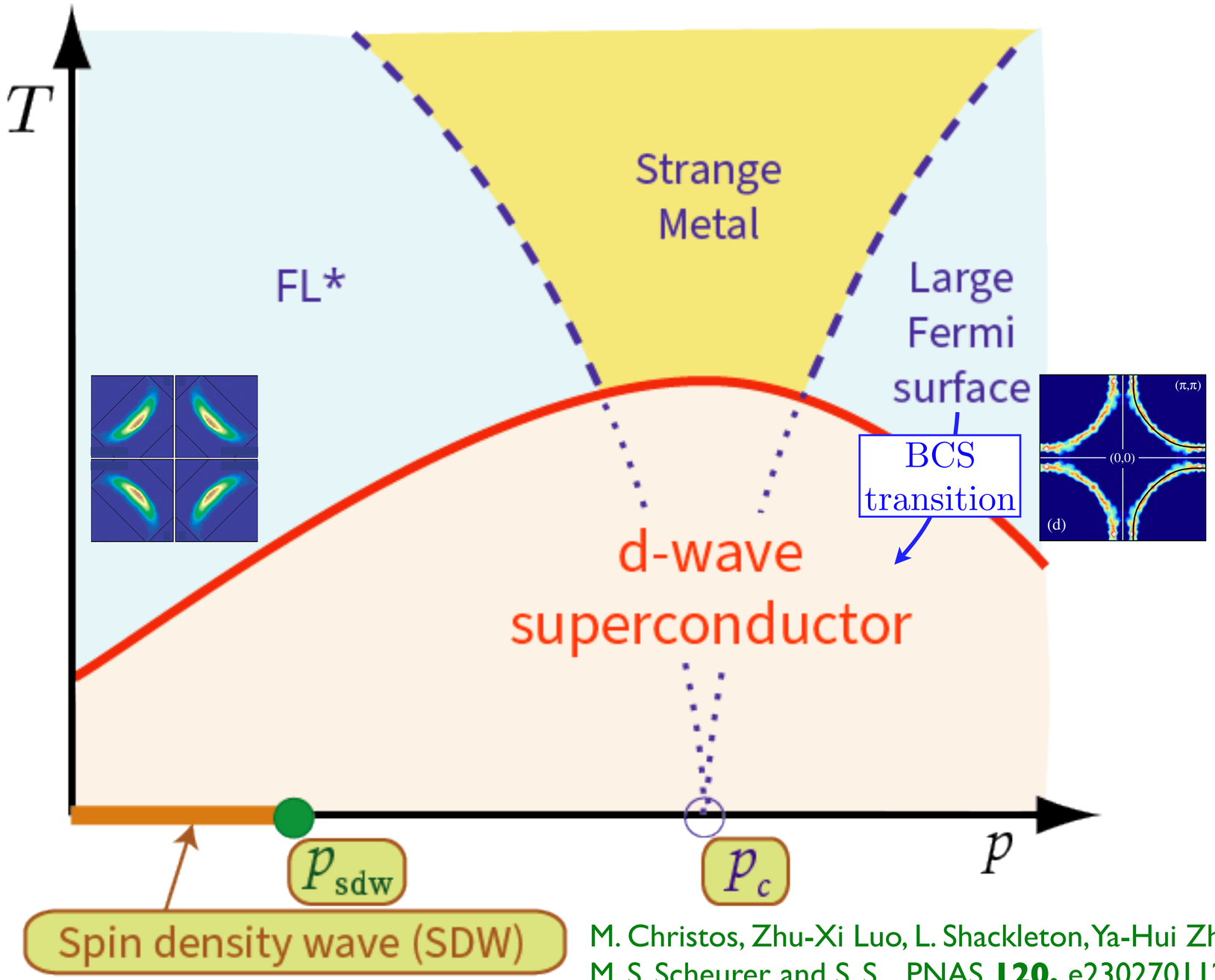
Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)
Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)



Quantum-criticality of a quantum phase transition between two metals (FL* and FL) at $p = p_c$, with no symmetry breaking. Φ was the hybridization between c and f_1 in theory of hole pockets— it is now a Higgs field.

Both metals lead to the same d-wave superconductor at lower temperatures, and so there is transition at $p = p_c$ within the superconducting state.

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)



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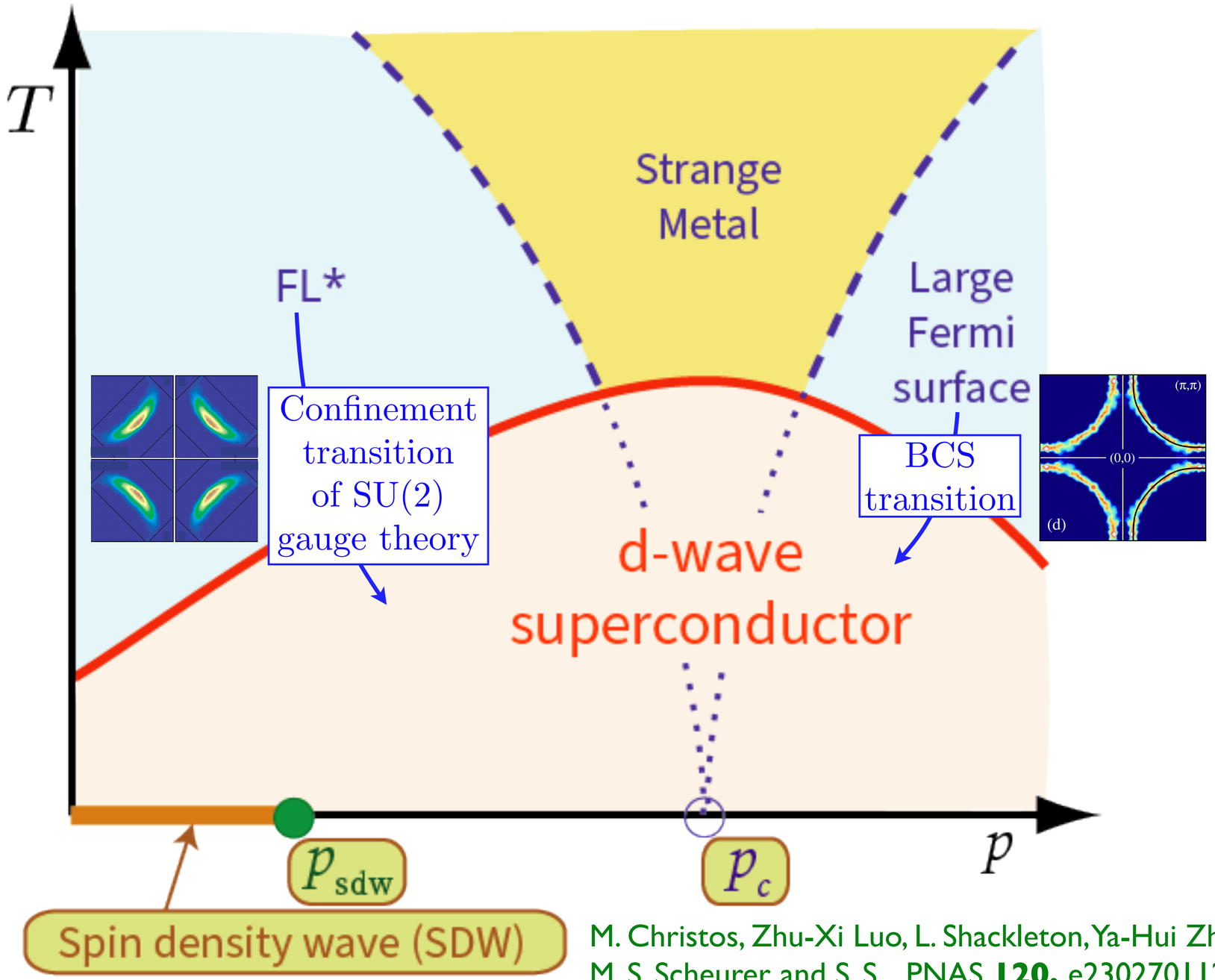

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