

# Rejecting Small Gambles Under Expected Utility\*

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## Abstract

This paper contributes to an important recent debate around expected utility and risk aversion. Rejecting a gamble over a given range of wealth levels imposes a lower bound on risk aversion. Using this lower bound and empirical evidence on the range of the risk aversion coefficient, we calibrate the relationship between risk attitudes over small-stakes and large-stakes gambles. We find that rejecting small gambles is consistent with expected utility, contrary to a recent literature that concludes that expected utility is fundamentally unfit to explain decisions under uncertainty. Paradoxical behavior is only obtained when calibrations are made in a region of the parameter space that is not empirically relevant.

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“The report of my death was an exaggeration.”

Mark Twain (May 1897)

## 1 Introduction

Within expected utility theory, risk aversion is identified with the concavity of the Bernoulli utility function  $u$  on wealth  $w$  (Arrow (1971), Pratt (1964); see Mas-Colell et al. (1995, chapter 6)). The expected utility framework has been severely criticized in a recent literature that concludes that diminishing marginal utility is an utterly implausible explanation for appreciable risk aversion.<sup>1</sup> The basis of the criticism can be best illustrated in Rabin (2000), who calibrates the relationship between risk attitudes over small and large stakes gambles under expected utility. Using his results, it is possible to present striking statements of the following kind: if a decision maker is a risk-averse expected utility maximizer and if he rejects gambles involving small stakes over a large range of wealth levels, then he will also reject gambles involving large stakes, sometimes with infinite positive returns. For instance, “suppose that, from any initial wealth level, a person turns down gambles where she loses \$100 or gains \$110, each with 50% probability. Then she will turn down 50-50 bets of losing \$1,000 or gaining *any* sum of money,” or “suppose we knew a risk averse person turns down 50-50 lose \$100/ gain \$105 bets for any lifetime wealth level less than \$350,000 ... Then we know that from an initial wealth level of \$340,000 the person will turn down a 50-50 bet of losing \$4,000 and gaining \$635,670.” (Rabin (2000, p. 1282)).

From this paradoxical, even absurd, behavior towards large-stakes gambles, Rabin (2000) and other authors conclude that expected utility is fundamentally unfit to explain decisions under uncertainty. This paper challenges this conclusion. We show that the flaw identified in this literature has little empirical support.<sup>2</sup> In particular, we show that it is the assumption of rejecting small gambles over a large range of wealth levels, and

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<sup>1</sup>See, for example, Hansson (1988), Kandel and Stambaugh (1991), Rabin (2001), Rabin and Thaler (2001) and other references therein. Samuelson (1963), Machina (1982), Segal and Spivak (1990) and Epstein (1992) also study various issues that are related to this literature.

<sup>2</sup>See Watt (2002), who has independently obtained results related to ours.

*not* expected utility, that does not typically match real-world behavior. In articulating our response, it is more useful not to argue whether expected utility is literally true (we know that it is not, since many violations of its underpinning axioms have been exhibited). Rather, one should insist on the identification of a useful range of empirical applications where expected utility is a useful model to approximate, explain, and predict behavior.

To be more specific on Rabin’s criticism, let  $p$  be the proposition “agent  $a$  is a risk averse expected utility maximizer,” let  $q$  be “agent  $a$  turns down the modest-stakes gamble  $X$  for a given range of wealth levels,” and let  $r$  stand for “agent  $a$  turns down the large-stakes gamble  $Y$ .” Then, Rabin’s statements can be expressed as: if  $p$  and  $q$  hold, then so does  $r$ . He then convincingly argues that most individuals do not turn down  $Y$ . This amounts to saying that  $r$  does not hold. From here, Rabin (2000) jumps to the conclusion that  $p$  does not hold. But this conclusion is not warranted unless  $q$  is either tautological or at least empirically compelling.

The plausibility of  $q$  as an assumption is argued in the literature purely by appealing to the reader’s introspection. Introspection, however, may sometimes be misleading: what people think they *would do* in a thought experiment may turn out to be quite different from what they *actually do* when confronted with a similar real-life situation. Indeed, we shall see that  $q$  is far from being empirically relevant. We report experimental evidence that supports our claim, but most of the evidence we wish to bring to bear is empirical. In doing so, we stress a methodological point. In assessing levels of risk aversion, it is important to study what real economic agents actually do when facing very different problems involving uncertainty. These data sets have been carefully analyzed by empirical economists, who have estimated the levels of the risk aversion coefficient over a wide range of problems.

Our methodology is the following. Rather than relying on introspection to validate  $q$ , we first investigate the theoretical implications of  $q$  on the preferences of the decision maker. We shall demonstrate that it implies a specific positive lower bound on the coefficient of absolute risk aversion, and show how it can be calculated exactly. Indeed, something beyond strict concavity is being assumed.<sup>3</sup>

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<sup>3</sup>This is contrary to what is claimed in the literature. For instance, “the calibration theorem is entirely

After obtaining this lower bound on risk aversion implied by  $q$ , the relevant issue is empirical, that is, the question is whether this bound is broadly consistent with the shape of the utility functions supported by empirical evidence. We argue that this bound is often unreasonably high. For instance, the assumption that a person turns down gambles where she loses \$100 or gains \$110 for *any* initial wealth level implies that the coefficient of relative risk aversion must go to infinity when wealth goes to infinity, while the assumption that a 50-50 lose \$100/ gain \$105 bets is turned down for any lifetime wealth level less than \$350,000 implies a value of the same coefficient no less than 166.6 at \$350,000. In contrast, a vast body of empirical evidence consistently indicates that the coefficient of relative risk aversion is estimated to be in the single-digit range.

With the single-digit range of empirically plausible coefficient values in hand, we calibrate the relationship between risk attitudes over small-stakes and large-stakes gambles. The important question here, far from obvious a priori, is whether one could still find paradoxes similar to the ones found in Rabin (2000). Our answer is in the negative: we do not find paradoxical rejections of large-stakes gambles. Thus, it follows that paradoxical behavior is only obtained when calibrations are made in a region of the parameter space that is not empirically relevant.

In conclusion, within expected utility, the relevance of the criticism when comparing behavior towards small-stakes and large-stakes gambles relies on the validation of assuming  $q$ . We take the debate to the empirical arena, and we argue that much empirical evidence contradicts the plausibility of Rabin's  $q$  in a variety of settings. We conclude that expected utility is often a useful model to explain decision making under uncertainty.

## 2 Rejecting Small Gambles: Evidence

As explained for example in Segal and Spivak (1990), any expected utility agent with a differentiable utility function must accept infinitesimal gambles of positive expected

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nonparametric, assuming *nothing* about the utility function except concavity" (Rabin (2000, p. 1282)). However, we show that there are large families of concave functions that are being ruled out by  $q$ .

value, because locally these preferences amount to risk neutrality. However, as soon as the gambles are no longer infinitesimal, but of any finite size, expected utility is compatible with both accepting and rejecting small gambles. It is then necessary to perform more powerful tests to examine the question of the appropriateness of expected utility preferences.

There is much work, both experimental and empirical, concerning the behavior of agents towards small-stakes gambles under expected utility. We briefly review it next.

## 2.1 Experimental Evidence

Camerer (1995) provides perhaps the most comprehensive survey of experimental evidence on individual decision making, virtually gathered in all cases from what may be considered small-stakes gambles. He finds substantial support for expected utility. In particular, he concludes that expected utility appears to be a helpful model to understand the reactions of individuals to risk observed in experimental data involving small gambles. For instance, after evaluating the relative performance of different theories, he concludes that “the result is an ‘efficient frontier’ of theories that are more accurate, best fitting, given the number of patterns they allow. A compilation of 23 data sets from a total of 2,000 subjects making 8,000 choices shows that the following theories are on the efficient frontier: mixed fanning, prospect theory, expected utility and expected value” (p. 638). He also concludes “that many subjects obey expected utility and that the lean functional form of expected utility is more statistically robust to estimation error than more complex functional forms” (p. 640), and that “more general theories appear to fit better than expected utility since they have more degrees of freedom, but are not better in predicting new choices” (p. 642).<sup>4</sup>

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<sup>4</sup>Camerer and Ho (1994) fit data from different studies using generalizations of expected utility that weaken the independence axiom and add only one parameter. They find that they often fit better and that violations of the independence axiom are less systematic when mixtures are presented in compound form. Loomes and Segal (1994) performs an experimental study that provides evidence of different orders of risk aversion, including local risk neutrality. Hey and Orme (1994) estimate a number of generalizations of the expected utility preference functional using experimentally generated data involving 100 pairwise choice questions repeated on two separate occasions. They use likelihood ratio tests and the Akaike information criterion to distinguish between the various generalizations and to explore their economic superiority. The paper concludes that, for many subjects, the superiority of several of these generalizations cannot be

As to the range of risk aversion estimates obtained in experimental studies, the evidence is quite robust. For instance, Holt and Laury (2002) present the results of simple lottery-choice experiments involving gambles that range from \$0.10-\$3.85 to a few hundred dollars. They find that regardless of the scale of risk the coefficient of relative risk aversion,  $r_R(w, u)$ , is *always* below 1.37, and typically between -0.97 and 1.37, in a utility for money specification. They also find that many subjects are risk neutral or risk loving for these small scale risks. In experimental studies of games the estimates of  $r_R(w, u)$  are also very low. For instance, Goeree, Holt, and Palfrey (2000, 2002) examine several asymmetric matching pennies games and private-values auction experiments, respectively. These experiments also involve very small gambles, and total payoffs after all rounds have been completed typically range from 5 to 20 dollars per individual subject. Their estimates of  $r_R(w, u)$  are virtually in all cases below 1, and highly significant across treatments and games: estimates typically range from 0.3 to 0.7, centered around 0.5. The value of 0.5 is also almost identical to that obtained in many experimental studies of similar nature that these authors cite.

## 2.2 Empirical Evidence

The empirical evidence on decisions under uncertainty in natural environments is vast, and not easy to summarize. However, the relevant conclusion for the purposes of our analysis is again noticeably robust: values of  $r_R(w, u)$  in the single-digit range appear to rationalize the reactions to risk in virtually all circumstances examined in the literature. Indeed, the evidence on the size of  $r_R(w, u)$  is remarkably consistent across hundreds of studies that examine the behavior of agents choosing among risky alternatives that greatly differ in the scale of risk in a broad variety of economic contexts.

We first discuss empirical evidence on small gambles in familiar real life situations. We again find that these studies yield low estimates of the risk aversion coefficient. As will be shown, the magnitudes of the gambles in these studies would certainly appear to be

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established.

negligible for *any* consumer. We later devote some attention to review studies that involve larger gambles where the risky alternatives substantially differ in size, circumstances, and economic environments.

First, as Friedman and Savage (1948) already noted, individuals often accept, rather than reject, small gambles and often do both. “People not only engage in fair games of chance, they engage freely and often eagerly in such unfair games as lotteries ... The empirical evidence for the willingness of individuals to purchase lottery tickets, or engage in similar forms of gambling, is extensive” (pp. 280, 286). While accepting, rather than rejecting, small gambles directly questions the assumptions in Rabin (2000), we are interested in the pattern of findings concerning the size of  $r_R(w, u)$ .

Cicchetti and Dubin (1994) investigate the decision of whether to purchase insurance against the risk of telephone line trouble in the home using a structural logit model. The price of this insurance was \$0.45 a month, while the average cost of repair in case of trouble was \$55 and the probability of line trouble was 0.005. A 42.9% of users in the sample chose not to buy insurance, so these users were taking a small gamble. Using data on monthly income, they find that observed behavior is consistent with expected utility in about 78% of the households and support a utility specification of the form  $u(w) = (w + 0.6)^{1-0.63}$ . The average income in the sample was 1.7 (in thousands). Correspondingly, the average value found for  $r_R(w, u)$  is  $\frac{1.7 \cdot 0.63}{1.7 + 0.6}$ , and it is lying between 0 and 1 for most individuals.

Evans and Viscusi (1991) estimate state-dependent utility functions on risk-dollar tradeoffs for minor health hazards in the reported use of insecticides and toilet bowl cleaners. In the no-injury state the Bernoulli utility function that they specify is  $\ln(w)$ . With small probability ( $15n/10000$ , where  $n$  is the number of units of the product consumed annually, ranging between 1.5 and 6), the use of the products they consider in their sample may cause some minor health hazard. In these injury states, two alternative specifications for the Bernoulli utility function are postulated:  $\alpha \ln(w)$  and  $\ln(w - L)$ . The results of the analysis provide overwhelming support of the latter specification, and the authors interpret this finding as consumers identifying the minor health hazard with a loss in income. The average annual income in their sample is about \$30,000. Although given the state-

dependence of utility, it is not possible to come up with an overall value of  $r_R(w, u)$ , this is obviously 1 for risks within the no-injury state, and given their data, its average value in the unhealthy states is at most 12/11: for risks within these states,  $r_R(w, u) = w/(w - L)$  and the parameter  $L$  is estimated to be at most around 2,500.

Miravete (2001) examines the behavior of individuals choosing among alternative local telephone tariffs in Kentucky. Consumers must choose between two options: a flat tariff option with unlimited usage for a \$18.70 monthly fee, and a measured service option with a \$14.02 monthly fee, a \$5 allowance, and subsequent costs per call that depend on their time of the day, duration, and distance. Switching to a different tariff option can be made every month. In the sample, the difference between the ex ante choice and the ex-post right tariff option is on average about \$4.88 out of a representative \$1,600 per capita monthly income. Despite the fact that the magnitude of the difference in the costs of each alternative are very small, consumers appear to respond in the very short term to these small savings by switching options. Using panel data on individual actual and expected usage distributions and tariff choices, the results show that individuals who choose the flat tariff can be characterized as risk neutral and those who choose the measured tariff as risk lovers.

Likewise, evidence on risk attitudes from natural experiments involving television game shows estimate  $r_R(w, u)$  to be in the single-digit range and often cannot reject the null hypothesis that agents are risk neutral (see, for example, Metrick (1995)).

We next briefly review the evidence from larger gambles in order to see whether data sets dominated by modest-risk opportunities yield higher or lower estimates of risk aversion than those dominated by larger-risk opportunities.

Mehra and Prescott (1985) quote several microeconomic studies of aggregate fluctuations of consumption and investment, the response of small countries' balance of trade to shocks, the behavior of farmers, portfolio holdings, and the study of life-cycle savings behavior that obtain values of  $r_R(w, u)$  between 0 and 2. These authors consider that the studies they cite "constitute an a priori justification for restricting the value of this coefficient to be a maximum of ten," as they do in their study. Kocherlakotta (1996), in

reviewing the research on the equity premium puzzle, quotes estimates obtained from financial market data and summarizes the current state of professional opinion in this field: “A vast majority of economists believe that values above 10 (or for that matter, above 5) imply highly implausible behavior on the part of individuals.” Luttmer (1996) shows how single-digit values of  $r_R(w, u)$  can explain the size of the equity premium taking into account actual capital market frictions. Barsky et al. (1997) report the results of survey questions designed to elicit measures of risk tolerance, time preference and intertemporal substitution as part of the Health and Retirement Study. They obtain single-digit values of  $r_R(w, u)$ . Rosenzweig and Wolpin (1993) estimate a structural dynamic model of agricultural investment behavior that incorporates income uncertainty and credit market constraints using longitudinal household data on farm profits, bullock stocks, and pump sets. The value of  $r_R(w, u)$  is precisely estimated to be 0.96. Binswager’s (1980) is also a classic study with low-income farmers in Bangladesh that finds similar estimates. Many other references with similar findings on the size of this coefficient can be found in these and other papers.

We thus consider that the experimental and empirical evidence conclusively indicates the range of parameter values where it would be empirically relevant to calibrate attitudes towards risk for gambles of different size. We can safely take that range of preferences to be those for which  $r_R(w, u)$  is in the single-digit range.<sup>5</sup>

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<sup>5</sup>A final observation concerning the evidence is in order. While the theory usually focuses on life-time wealth, empirical studies typically use per period data (e.g., monthly income, yearly consumption, and so on). Under the standard assumption of time stationarity of preferences, one can easily show that estimates of the preference parameters for the per period utility function correspond to those of the utility function over life-time wealth. Moreover, as Rubinstein (2001) notes, “nothing in the von Neumann-Morgenstern (vNM) axioms dictates use of final wealth levels ... vNM are silent about the definition of prizes ... The definition of prizes as final wealth levels is no less crucial to Rabin’s argument than the expected utility assumption.” As a referee noted, it is certainly possible that subjects may view lab choice situations and “short-term” problems in isolation, rather than in conjunction with other sources of income. This is Rubinstein’s (2001) response to Rabin (2000).

### 3 Rejecting Small Gambles: Testable Implication

Rabin (2000) shows that if an individual is a risk averse expected utility maximizer and rejects a given gamble of equally likely gain  $g$  and loss  $l$ ,  $g > l$ , over a given range of wealth levels, then he will reject correspondingly larger gambles of gain  $G$  and loss  $L$ . We investigate now the implication of the “rejecting small gambles” assumption.

For a decision maker with wealth level  $w$  and twice continuously differentiable Bernoulli utility function  $u$ , denote the Arrow-Pratt coefficient of absolute risk aversion by  $r_A(w, u) = -\frac{u''(w)}{u'(w)}$ , with  $r_R(w, u) = w \cdot r_A(w, u)$ . We next show that the assumption that an expected utility maximizer turns down a given gamble with gain  $g$  and loss  $l$  for a given range of wealth levels implies that there exists a positive lower bound on  $r_A(w, u)$ . In fact, this positive lower bound can be calculated exactly and, therefore, provides a testable implication of the assumption.

**Proposition.** Let  $u$  satisfy non-increasing absolute risk aversion. Let  $I$  be an interval in the positive real line. If for every  $w \in I$ ,

$$\frac{1}{2}u(w + g) + \frac{1}{2}u(w - l) < u(w),$$

there exists  $a^* > 0$  such that the absolute risk aversion coefficient  $r_A(w, u)$  is greater than  $a^*$  for all  $w \in I$ . Moreover, the highest such  $a^*$  is the solution to the equation

$$f(a) = e^{al} + e^{-ag} - 2 = 0.$$

**Proof:** Suppose not. Then, for every  $a > 0$  there exists  $w \in I$  such that  $r_A(w, u) < a$ . In particular, this holds for the unique  $a^* > 0$  solving the equation  $f(a) = 0$ . (By the intermediate value theorem, note that  $a^*$  is well defined because  $f$  is continuous,  $f(0) = 0$ ,  $f(\infty) = \infty$ ,  $f'(0) < 0$ ,  $f'(a) = le^{al} - ge^{-ag}$  is first negative, vanishes at a single point and is positive thereafter).

Consider the constant absolute risk aversion utility function  $v(w) = -e^{-aw}$  for  $a < a^*$ .

For such a choice of  $a$ ,  $f(a) < 0$ , i.e.,

$$e^{al} + e^{-ag} < 2,$$

or

$$\frac{1}{2} \left( -e^{-a(w-l)} \right) + \frac{1}{2} \left( -e^{-a(w+g)} \right) > -e^{-aw}.$$

Thus, an individual with utility function  $v$  would agree to play the small-stakes lottery with gain  $g$  and loss  $l$  starting from any wealth level  $w$ .

Denote by  $w' \in I$  the wealth level for which  $r_A(w', u) = a$ . By non-increasing absolute risk aversion, for  $w \in I$ ,  $w \geq w'$ , the individual with utility function  $v$  is at least as risk averse as the one with utility function  $u$ . Therefore, using the well-known characterization of comparisons of risk attitudes across individuals in Rothschild and Stiglitz (1970), it follows that

$$\frac{1}{2}u(w+g) + \frac{1}{2}u(w-l) > u(w)$$

for every  $w \in I$ ,  $w \geq w'$ , which is a contradiction. ■

The hypothesis of non-increasing absolute risk aversion is generally accepted. Note, however, that it is not essential to the argument in the above proof. We use it only in the last step to assert that the range of wealth levels over which the “rejecting the small-stakes lottery” assumption is violated constitutes an interval of the form  $[w', \infty)$ , something stronger than we need. In the absence of the non-increasing absolute risk aversion assumption, continuity of the utility function would suffice to obtain the same result over some arbitrary interval. This would also be enough for our purposes.

Hence, contrary to several statements in Rabin (2000, 2001) and Rabin and Thaler (2001), the conclusion to be drawn from this proposition is that the assumption of rejecting the small-stakes gamble does go beyond the assumption of concavity of the Bernoulli utility function. A positive lower bound on risk aversion is also assumed, and this bound is independent of the interval  $I$  over which the assumption is made. This lower bound on the coefficient of relative risk aversion clearly increases when for a given small-stakes gamble

we enlarge the interval  $I$  over which it should be rejected. This means for example that the assumption that a given gamble is rejected for all wealth levels is incompatible with the agent becoming risk neutral at some sufficiently high level of wealth, feature shared by a large class of concave utility functions. The proposition implies that  $r_R(w, u)$  must go to infinity as wealth goes to infinity.

## 4 Rejecting Small Gambles: Some New Calibrations

In this section we calibrate attitudes towards risk for gambles of different size in the region where the combination of prizes (which we will continue referring to as wealth) and the curvature of the utility function yields empirically plausible parameter values. That is, having turned the assumption of rejecting small gambles into an empirically testable proposition we examine whether or not rejecting small gambles, when the coefficient  $r_R(w, u)$  is in the single-digit range, induces paradoxical behavior. Although the point we are raising is general, only for computational simplicity, we shall work with the class of CRRA (constant relative risk aversion) Bernoulli utility functions  $u(w) = \frac{w^{1-\gamma}}{1-\gamma}$  for  $\gamma \geq 0$ . For this utility function,  $r_R(w, u) = \gamma$ . In order to facilitate comparison with the literature we next consider calibrations based on gambles similar to Rabin (2000). This is the only reason to use these gambles, in the absence of a clear definition of what it is a “small gamble.” One can certainly argue that there is a great deal of subjectivity in evaluating the magnitude of a gamble.<sup>6</sup>

The assumption of rejecting small-stakes gambles is generally made over a given range of wealth levels. In Table I we calculate, for two small-stakes lotteries and for different values of  $\gamma$ , the largest wealth level at which an individual rejects the lotteries.

[Table I here]

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<sup>6</sup>If one is to assume the rejection of an initial small gamble, this should hold with independence of how it is presented (e.g., as the choice between telephone calling plans, as a minor health hazard, or as a choice in a first-price auction or a matching pennies experiment). As pointed out by a referee, the fact that it is difficult to say what “small” or “large” stakes are suggests a possible role for reference points.

Note that the values of these wealth levels are extremely small. Therefore, the empirical relevance of the assumption, for decision makers with  $\gamma$  in the single digits, would seem to be quite limited.

Continuing with the specification of CRRA utility, the next question we examine is how high is the bound  $a^*$  associated with the given small-stakes lottery of gain  $g$  and loss  $l$ . On the basis of the same lotteries used in Rabin's (2000) Tables I and II, we calculate in Table II their corresponding values of  $a^*$ , as defined in the proposition above. The table also shows, for wealth levels of \$300,000 and \$30,000, the induced values of  $\gamma$ .

[Table II here]

It is first worth noting that for the wealth level of \$300,000 very few values of  $\gamma$  are in the single-digit range, or even in the teens. No single-digit value arises when the gamble involves losing \$100 or \$1,000. Only when the rejected gamble involves losing  $l = \$10,000$ , which would not appear to be a small-stakes gamble, such low values start to arise consistently. In an attempt to generate more  $\gamma$  coefficients in the single-digit range, we examine the same lotteries for a wealth level as extremely low as \$30,000. In this case, single-digit coefficients arise for some gambles where  $l = \$1,000$ , and for all gambles where  $l = \$10,000$ , which are hardly small-stake gambles for an individual with that wealth level. For the lowest stakes gambles involving  $l = \$100$  a single-digit  $\gamma$  is only found when  $g = 101$ . We conclude from Table II that empirically plausible, single-digit values of  $\gamma$  are compatible with the assumption only when the loss  $l$  in the gamble is a significant proportion of the individual's wealth. We thus learn that the assumption of rejecting truly small gambles does not hold, when applied to all the decision makers that are behind the experimental and empirical evidence mentioned above.

Finally, for various lotteries in Table II that yield values of  $\gamma$  in the single-digit range, Table III displays the best large-stakes lottery with gain  $G$  and loss  $L$  that the decision maker would reject.

[Table III here]

It is apparent that these rejections are no longer paradoxical. For instance, for a wealth of \$300,000 the agent turns down gambles involving losses  $L$  ranging from 5 to 15 percent of his wealth and gains  $G$  that appear reasonable. The same can be said for a wealth of \$30,000. In this case, note that relative to wealth these values of  $L$  are ten times greater than those in Rabin (2000). Thus, not even for these much larger gambles paradoxical behavior is obtained. Finally, it is worth stressing that gambles with  $G = \infty$  are turned down only when potential losses  $L$  represent a significantly great proportion of the individual's wealth.

The reasonable behavior described in these large-stakes gambles contrasts with the paradoxes in Rabin (2000) and in other authors in the literature, and indeed may be viewed as a further confirmation of the empirical soundness of single-digit values for  $r_R(w, u)$ .

These results refute assertions such as “paradoxical implications are not restricted to particular contexts or particular utility functions,” or “within the expected-utility framework, for *any* concave utility function, even very little risk aversion over modest stakes implies an absurd degree of risk aversion over large stakes” (Rabin (2001, p. 203)). That is, much more than “very little risk aversion over modest stakes” is needed to generate paradoxical behavior. Indeed, this is only obtained when calibrations are made in a region of the parameter space that is not empirically relevant.

Lastly, it is important to note that a rather straightforward empirical implication of the calibrations made in Rabin (2000), under his belief that the assumption of rejecting small gambles holds, is that “when measuring risk attitudes maintaining the expected-utility hypothesis ... data sets dominated by modest-risk investment opportunities are likely to yield much higher estimates of risk aversion than data sets dominated by larger investment opportunities” (Rabin (2000, p. 1287)). But as already discussed in Section 2, contrary to this implication, the empirical evidence gathered from many different studies consistently obtains estimates of  $r_R(w, u)$  that vary very little across a wide heterogeneity of scales of risk, as these estimates are narrowly confined to the single-digit range.

## 5 Concluding Remarks

Using a problem posed to one of his colleagues as a starting point, Samuelson (1963) argues that, under expected utility, the rejection of a given single gamble for all wealth levels implies the rejection of the compound lottery consisting of the single gamble being repeated an arbitrary number of times. Samuelson's exercise sheds light on the fact that some decision makers may be misapplying the law of large numbers when accepting a compound lottery (the colleague's response was that he would reject the single lottery, but accept its compound version). However, Samuelson was clearly aware of the crucial importance of the assumption of rejecting the single lottery for *all* wealth levels or a large range thereof: "I should warn against undue extrapolation of my theorem. It does not say that one must always refuse a sequence if one refuses a single venture: if, at higher income levels the single tosses become acceptable, and at lower levels the penalty of losses does not become infinite, there might well be a long sequence that it is optimal" (p. 112). Indeed, it may very well be the case that Samuelson's colleague was not fooled by any fallacy of large numbers. He simply violated the assumption of rejecting the given small-stakes lottery for all wealth levels or large range thereof.

The main advantage of expected utility is its simplicity and its usefulness in the analysis of economic problems involving uncertainty. As often argued in the literature, its predictions sometimes conflict with people's behavior. This has led economists to develop various non-expected utility models which can often accommodate actual behavior better. The non-expected utility research agenda is an important one, and there is no question that we should continue to pursue it. However, expected utility should not be accused when it is not at fault. The analysis in this paper shows how certain paradoxical examples in the literature are many times counterfactuals. Paradoxical behavior is only obtained when calibrations are made in a region of the parameter space that is not empirically relevant. In a more recent paper, Rabin and Thaler (2001) continue to drive home the theme of the demise of expected utility and compare expected utility to a dead parrot from a Monty Python show. To the extent that all their arguments are based on the calibrations in Rabin

(2000), the expected utility parrot may well be saying that “the report of my death was an exaggeration.”

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**TABLE I**

Wealth levels at which an individual with CRRA ( $\gamma$ ) utility function stops rejecting a 50-50 lose \$100 / gain \$g lottery.

$\gamma$	$g$	
	125	110
2	400	1,000
3	1,501.3	3,300.5
4	2,003.1	4,401.2
5	2,504.9	5,502
6	3,006.9	6,602.7
7	3,508.8	7,703.5
8	4,010.8	8,804.3
9	4,512.8	9,905.1
10	5,014.9	11,006
11	5,516.9	12,106.8
12	6,018.9	13,207.6
20	10,035.4	22,014.2
30	15,056	33,022.5
40	20,076.7	44,030.8
50	25,097.3	55,039.2

**TABLE II**

Lower bounds on the coefficient of absolute risk aversion for an individual that rejects a 50-50 lottery lose \$ $l$  / gain \$ $g$  ( $a^*$ ) for any range of wealth levels, and associated lower bound on the coefficient of relative risk aversion for wealth levels \$300,000 and \$30,000.

$l/g$	$a^*$	$\gamma = 300,000a^*$	$\gamma = 30,000a^*$
100 / 101	.0000990	29.7	2.9
100 / 105	.0004760	142.8	14.2
100 / 110	.0009084	272.5	27.2
100 / 125	.0019917	597.5	59.7
100 / 150	.0032886	986.5	98.6
1,000 / 1,050	.0000476	14.2	1.4
1,000 / 1,100	.0000908	27.2	2.7
1,000 / 1,200	.0001662	49.8	4.9
1,000 / 1,500	.0003288	98.6	9.8
1,000 / 2,000	.0004812	144.3	14.4
10,000 / 11,000	.0000090	2.7	0.2
10,000 / 12,000	.0000166	4.9	0.4
10,000 / 15,000	.0000328	9.8	0.9
10,000 / 20,000	.0000481	14.4	1.4

**TABLE III**

If averse to 50-50 lose  $\$l$  / gain  $\$g$  for wealth levels  $\$300,000$  and  $\$30,000$  with CRRA utility and coefficient of relative risk aversion  $\gamma$ , will also turn down a 50-50 lose  $L$  / gain  $G$  bet;  $G$ 's entered in Table.

	Wealth: \$30,000				Wealth: \$300,000	
$l/g:$	100/101	1,000/1,050	1,000/1,100	$l/g:$	10,000/11,000	10,000/12,000
$\gamma:$	2.9	1.4	2.7	$\gamma:$	2.7	4.9
$L$	<hr/>			$L$	<hr/>	
400	416	---	---	15,000	17,341	19,887
600	636	---	---	17,000	20,072	23,572
800	867	---	---	20,000	24,393	29,792
1,000	1,107	---	---	22,000	27,435	34,490
2,000	2,479	2,205	2,439	25,000	32,266	42,574
4,000	6,538	4,917	6,259	30,000	41,116	59,870
6,000	14,538	8,329	13,168	40,000	62,594	126,890
8,000	40,489	12,749	30,239	50,000	91,268	$\infty$
10,000	$\infty$	18,686	495,319	75,000	239,089	$\infty$