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ECONOMICS WORKING PAPERS

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development**

Marc Bourreau, Pinar Doğan, and Matthieu Manant

**Paper Number 104**

**July 23, 2014**

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# Size of RJDs with partial cooperation in product development\*

Marc Bourreau<sup>†</sup>, Pinar Dođan<sup>‡</sup> and Matthieu Manant<sup>§</sup>

July 23, 2014

## Abstract

In this paper, we consider Research Joint Ventures (RJDs) that are set up to develop product components. We propose an  $n$ -firm oligopoly model to study the effect of the degree of cooperation in product development (i.e., the number of jointly developed product components) on the size of the RJD (i.e., the number of RJD participants). We assume that a higher degree of cooperation in product development restricts the firms' ability to differentiate their products, and hence intensifies competition among the participating firms. We find that the equilibrium size of the RJD can vary non-monotonically with the degree of cooperation. If the degree of product differentiation is not very sensitive to the degree of cooperation, then the RJD is industry-wide.

*Keywords:* R&D Cooperation, RJD, Product Development.

*JEL Codes:* L1, O3.

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\*An earlier but substantially different version of this paper was circulated as "Size of RJDs and Degree of Cooperation in Product Development" (HKS Faculty Research Working Paper Series RWP10-047). We would like to thank Paul Belleflamme, Nisvan Erkal, Nicolas Houy and Patrick Waelbroeck for valuable comments and suggestions. We also thank Katie Naeve for her editorial assistance.

<sup>†</sup>Telecom ParisTech, Department of Economics and Social Sciences, and CREST-LEI, Paris. Email: marc.bourreau@telecom-paristech.fr.

<sup>‡</sup>John F. Kennedy School of Government, Harvard University, Cambridge, MA, and Visitor at the Institute for Advanced Study, Princeton, NJ (AY 2013-15). E-mail: pinar\_dogan@hks.harvard.edu

<sup>§</sup>Université Paris Sud XI. Email: matthieu.manant@u-psud.fr.

# 1 Introduction

The motivation of this paper stems from the observation that in many industries, firms team up to jointly develop platforms or product components, which are then used to manufacture competing final products. For example, it is common for rival automakers to form Research Joint Ventures (RJVs) to develop car components such as hybrid systems, fuel-cell systems, transmissions systems, or break systems. Most recently, in 2013, Daimler, Ford, and Nissan have formed an alliance to develop fuel-cell systems to be used in fuel-cell electric vehicles (FCEVs). In the consumer electronics industry, two major competing firms, Sony and Samsung, established an RJV (S-LCD) in 2005 to develop liquid crystal display panels to be used in their flat screen televisions. Software companies cooperate to design some components of software, for example, through an open-source project, and then develop other components of the software independently. A relatively well known example of this is the Apache Web server application that is developed jointly by independent developers as well as large corporations such as IBM and Sun, who then added proprietary components to the application to commercialize Web servers under their own brand name. In all these examples, cooperation among the competing firms is not "full" in the sense that firms do not develop an entire product jointly, but cooperate partially and jointly develop only some of the product components.<sup>1</sup>

Firms benefit from such alliances as they save from R&D expenditures. However, joint development of product components also entails a potential cost, which is associated with the post-R&D competition stage. When firms cooperate on more product components, the competition among the cooperating firms may be intensified due to a lower degree of (perceived) differentiation among their products. Such a competitive cost may be significant, particularly in markets where the end-consumers highly value product differentiation. Firms also seem to be aware of this potential drawback of cooperation, and often make public statements to underline how their end products will be "different" despite the cooperative agreement. For example, a press release from Nissan notes "The partners plan to develop a common fuel cell stack and fuel cell system that can be used by each company in the launch of highly differentiated, separately branded FCEVs." (Press release, Jan. 28, 2013, available at <http://nissannews.com/en-US/nissan/usa/releases>). A similar remark was also made by the management of BMW AG, following BMW Group's teaming up with DaimlerChrysler AG and General Motors Corporation to develop a two-mode hybrid drive system

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<sup>1</sup>There are also instances where firms team up to develop the entire end product together, such as "twin automobiles." See Lado et al. (2004) for examples of such identical cars that are then sold under different brand names.

for engines: “Because the technologies will be adapted to the individual vehicle models, the participating brands will retain their distinctive characters” (News and Issues, Sept. 9, 2005, available at <http://www.gm.com>).

To the extent that the degree of cooperation (i.e., the number of jointly developed product components) can determine the competitive cost incurred by the participating firms, it is also expected to determine the size of the RJV (i.e., the number of firms participating in the RJV). In this paper, we propose a model to study the effect of the degree of cooperation in product development on the size of an RJV, when jointly developed product components restrict the firms’ ability to differentiate their products. We consider an  $n$ -firm oligopoly, with a subset of firms forming and RJV to cooperate in product development.<sup>2</sup> We assume that a higher degree of cooperation (i.e., a larger number of product components developed jointly) leads to a lower degree of product differentiation, as it implies a larger number of common product components in participant firms’ end-products. We find that the equilibrium size of the RJV can vary non-monotonically with the degree of cooperation. While savings from the development costs operates as a source of "complementarity" between the size of the RJV and the degree of cooperation, the effect of reduced degree of differentiation on firms’ profits operates as a source of "substitutability" between the two. We find that if the degree of product differentiation is not sensitive to the degree of cooperation, then the RJV is industry-wide. We also find that the equilibrium size of the RJV is too small from the industry’s perspective (unless there is an industry-wide RJV), whereas it is too large from consumers’ point of view.

Our paper contributes to the theoretical literature on cooperative R&D and formation of RJVs. The existing literature addresses a large set of interesting questions, ranging from how private and social incentives for cooperation compare in different settings (e.g., in the presence of uncertainty, synergies, endogenous as well as exogenous spillovers, and with and without an innovation race)<sup>3</sup> to how cooperation may affect incentives to maintain post-innovation collusion.<sup>4</sup> The main distinguishing feature of our paper relates to the nature of the RJV, which entails partial cooperation, and focuses on product R&D. With very few exceptions, the literature deals with process R&D (i.e., R&D efforts for cost reduction). Four notable exceptions are Motta (1992), Lin and Saggi (2002), Lambertini et al. (2002), and Bourreau and Doğan (2010). The first three studies consider

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<sup>2</sup>In this paper we use RJV as a broader term that includes any type of cooperative alliances aimed at joint product development. We do not consider RJVs that aim at cost reduction.

<sup>3</sup>Among others, see Suzumura (1992), Choi (1993), Kamien et al (1992), and Kamien and Zang (2000).

<sup>4</sup>See Martin (1996), Cabral (2000), and Lambertini et al (2002).

cooperation in product development, but in a duopoly setting, and with either full or no cooperation. While Bourreau and Doğan (2010) introduce partial cooperation in product development, they also remain in a duopolistic setting, and study the interaction between cooperation in product and process R&D.<sup>5</sup> Partial cooperation in product development is somewhat analogous to "hybrid R&D"<sup>6</sup> in process R&D. While firms develop some product components jointly and some others individually in the former case, they allocate their resources between joint and private cost-reducing R&D efforts in the latter. Similar to the (limited) product R&D literature, however, the vast majority of studies in the process R&D literature considers full cooperation.

Another distinguishing feature of our paper relates with the size of the RJV. Most of the studies that adopt an oligopolistic setting—albeit with process R&D—consider an industry-wide RJV.<sup>7</sup> Studies that consider the possibility of an RJV with only a subset of the firms are Katz (1986), Poyago-Theotoky (1995), and Falvey et al. (2013). Katz (1986) analyzes cooperation for process innovation with spillovers, where the number of participants of the RJV can range from two to the total number of firms in the industry. Poyago-Theotoky (1995) analyzes the optimal size of a research joint venture, in an oligopolistic setting with homogeneous goods, process R&D and spillovers. Falvey et al. (2013) adopt a similar setting, and study the impact of coordination costs on the performance of the RJV.

Finally, with very few exceptions, the literature ignores any economic cost of cooperation, and therefore, considers cooperation as socially desirable (at least in a weak sense).<sup>8</sup> In this aspect, the closest paper to ours is Lambertini et al. (2002); in a duopoly setting, they assume that firms produce an identical product when they cooperate (and hence, competition is tough), whereas they produce horizontally or vertically differentiated products when they do not cooperate (and hence, competition is softer).

The remainder of the paper is organized as follows. In Section 2, we set up our oligopoly model of RJV in product development. In Section 3, we study the relationship between the size of the

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<sup>5</sup>The study by Goyal et al. (2008) can also be viewed as one on partial cooperation, as they consider joint R&D projects as well as in-house projects, and analyze the interplay between the two.

<sup>6</sup>The term is coined by Goyal et al. (2008). See also Atallah (2004), who considers the allocation of firms' resources to cooperative and noncooperative R&D.

<sup>7</sup>See, for example, Motta (1992), Suzumura (1992), Yi (1995), and Kamien et al. (1992). Suzumura (1992) extends the model of d'Aspremont and Jacquemin (1988) to oligopolies. Kamien et al. (1992) analyze  $n$ -firm oligopolies and compare four scenarios of R&D activity in terms of social welfare.

<sup>8</sup>Vilasuso and Frascatore (2000) consider an exogenous fixed cost of forming an RJV, which can be attributed to its management or auditing. Falvey et al. (2013) introduce coordination costs to run an RJV in an oligopolistic setting. The coordination costs are modeled through the marginal cost of process R&D; the authors assume that the marginal cost of process R&D increases with the number of participants in the RJV. Coordination costs tend to decrease the R&D of members of the RJV, as well as reduce the joint profits of the RJV.

RJV and the degree of cooperation. In Section 4, we extend our analysis to discuss Bertrand competition and the case where the degree of cooperation is also endogenous. Finally, we conclude.

## 2 A model of RJV for product development

Consider a market with  $n$  ex-ante identical firms, where the final product is composed of a continuum of product components normalized to 1. We study an RJV formed by  $k$  firms, with  $2 \leq k \leq n$ , for joint development of product components. Without loss of generality, we order the firms so that firms  $i = 1, \dots, k$  are the RJV members and firms  $l = k + 1, \dots, n$  are the non-member firms.

While  $(n - k)$  firms develop their entire final products individually, RJV members jointly develop product components  $[0, \alpha]$ , with  $\alpha \in [0, 1]$ . In our setting,  $\alpha$  represents the degree of cooperation among the RJV members;  $\alpha = 0$  corresponds to the degenerate case in which there is no cooperation within the RJV, whereas  $\alpha = 1$  corresponds to the case in which the RJV members develop the entire final product jointly.

We consider the case where joint-development is limited to given set of product components (i.e., where  $\alpha$  is given),<sup>9</sup> and that it determines the degree of product differentiation among the RJV member firms.

**Degree of product differentiation** The degree of product differentiation between the product of firm  $i$  and the product of firm  $j$  depends on the "distance" between the products of these two firms. Assume that component  $m$  of firm  $i$ 's product takes value  $\alpha_i(m) = i$  if it is developed in-house, and value  $\alpha_i(m) = 0$  if it is developed jointly with the other firms. Also, let  $\xi$  be the measure of the set of jointly developed components,  $\{m \in [0, 1] \mid \alpha_i(m) = 0\}$ . Then, we have  $\alpha = \xi$ , where  $\alpha$  represents the common components between the firms' products. Finally, let  $\Delta = 1 - \xi$ . It can be verified that  $\Delta$  represents a distance between the products of firm  $i$  and firm  $j$ .<sup>10</sup>

The degree of differentiation between the firms' products,  $s$ , is then determined by the common components,  $\alpha$ . We assume that  $s(\alpha)$  is continuous and differentiable, and that the degree of differentiation between members of the RJV, is decreasing with  $\alpha$ . That is,  $ds(\alpha)/d\alpha < 0$ , with  $s \in [s_{\min}, s_{\max}]$ ,  $s_{\min} \in [0, s_{\max})$  and  $s_{\max} < 1$ .<sup>11</sup> Since a non-member firm (i.e., outside the RJV)

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<sup>9</sup>Such limitations may be inherent to product design and/or technology. In Section 4, we discuss the case where the degree of cooperation is endogenous.

<sup>10</sup>This is true as we have (i)  $\Delta \geq 0$ , (ii)  $\Delta = 0$  if and only if the products of firms  $i$  and  $j$  are identical (i.e.,  $\alpha = 1$ ), and (iii)  $\Delta(i, j) + \Delta(j, k) \geq \Delta(i, k)$  is always true.

<sup>11</sup>We assume that  $s_{\min} \geq 0$ , that is, even when all product components are developed jointly by the RJV firms

does not share any common component with the other firms, the distance between its product and any other firm's product is equal to 1, and hence, the degree of differentiation between this firm and any other firm is equal to  $s_{\max}$ .

**Product development costs** The firms have the same cost of developing product components. The marginal development cost of the  $m^{\text{th}}$  component is denoted by  $\eta(m)$ , and  $\eta(\cdot)$  is continuous. Letting  $d(m)$  denote the total cost of developing the first  $m$  product components, we have  $d(m) = \int_0^m \eta(x) dx$ .

RJV members share the development cost of common components,  $d(\alpha)$ , equally. Each firm also carries in-house R&D to develop the remaining product components, for which the firm bears the full cost of development. Let  $D_R(k, \alpha)$  denote the total development cost for a member of the RJV, for a given degree of cooperation of  $\alpha$ . We have  $D_R(k, \alpha) = d(\alpha)/k + (d(1) - d(\alpha))$ , which can be rewritten as

$$D_R(k, \alpha) = d(1) - \frac{k-1}{k}d(\alpha). \quad (1)$$

The first term in (1) represents the cost of developing the entire product alone, and the second term represents the economies from cooperation in product development that are enjoyed by the RJV members. The total development cost decreases with the degree of cooperation,  $\alpha$ , and with the size of the RJV,  $k$ .<sup>12</sup> The development cost for non-member firms is  $D_{-R} = d(1)$ .

Note that, we have  $(d(1) - d(\alpha)) + d(\alpha) = d(1)$ , which implies that there are no economies (or diseconomies) of scope in developing the product components. Therefore, in our setting, the design of the product (whether it is modular or integrated) is not relevant in terms of the cost of developing it.<sup>13</sup>

To sum up, in our setting, the degree of cooperation determines both the degree of product differentiation (due to common product components) and the development cost (due to cost-sharing for the jointly-developed components). We do not consider any spillovers<sup>14</sup> or synergies in product

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( $\alpha = 1$ ), their end-products are not necessarily perceived as perfect substitutes by the consumers. See, for example, Sullivan (1998) and Lado et al. (2004) on how brand names affect the demand for "twin automobiles." Twin automobiles are identical products that are sold under different brand names (e.g., Citroen Evasion, Fiat Ulysses, and Peugeot 806), for which the consumers' willingness to pay is not necessarily the same.

<sup>12</sup>Indeed, we have  $D_R(l, \alpha) \geq D_R(k, \alpha)$  for all  $l \leq k$ .

<sup>13</sup>There may be advantages (or disadvantages) of adopting a modular design instead of an integrated design for product development. By assuming away (dis)economies of scope in developing product components, we adopt a neutral stance, in that we assume neither is superior in terms of product development costs.

<sup>14</sup>Spillovers can be particularly important in the context of process R&D, where firms pursue vertical (quality and/or cost) improvements. For example, in a hybrid-process R&D setting, joint R&D efforts can generate spillovers on in-house R&D, as both R&D efforts have the same direction (quality improvement or cost reduction). In our

development.<sup>15</sup>

**Demand** We generalize the demand system provided by Dixit (1979) and Singh and Vives (1984) for  $n$  asymmetric firms with respect to the degree of differentiation.<sup>16</sup> The utility of the representative consumer is given by

$$U(\mathbf{q}, m) = a \sum_{i=1}^n q_i - \frac{1}{2} \sum_{i=1}^n (q_i)^2 - \sum_{i=1}^{n-1} q_i \sum_{j=i+1}^n (1 - s_{ij}) q_j + m, \quad (2)$$

where  $s_{ij}$  is the degree of differentiation between the product of firm  $i$  and the product of firm  $j$ ,  $\mathbf{q} = (q_1, \dots, q_n)$  is the vector of quantities, and  $m$  is the numeraire good. Then, the inverse demand for an RJV member firm  $i = 1, \dots, k$  is

$$p_i = a - q_i - \sum_{\substack{j=1 \\ j \neq i}}^k (1 - s) q_j - \sum_{j=k+1}^n (1 - s_{\max}) q_j, \quad (3)$$

where  $q_i$  and  $p_i$  denote the quantity and price of firm  $i$  and  $q_j$  the quantity of firm  $j$ . The inverse demand for a non-member firm  $l = k + 1, \dots, n$  is

$$p_l = a - q_l - \sum_{\substack{j=1 \\ j \neq l}}^n (1 - s_{\max}) q_j. \quad (4)$$

**Firms' profits** Once the products are developed, firms produce at a constant marginal cost of  $c$  and compete in the product market with quantities. The net (resp., gross) profit of an RJV member firm is denoted by  $\Pi_R$  (resp.,  $\pi_R$ ), and the net (resp., gross) profit of a non-member firm is denoted by  $\Pi_{-R}$  (resp.,  $\pi_{-R}$ ). The net profits for RJV member firms and non-member firms are  $\Pi_R(k, \alpha) = \pi_R(k, s(\alpha)) - D_R(k, \alpha)$ , and  $\Pi_{-R}(k, \alpha) = \pi_{-R}(k, s(\alpha)) - d(1)$ , respectively. We assume that the market is viable; in equilibrium we have  $\Pi_R(k, \alpha) \geq 0$  and  $\Pi_{-R}(k, \alpha) \geq 0$  for any

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setting, joint R&D is directed at developing a set of product components. Since product components are distinct, joint efforts to develop a set of product components are not likely to generate spillovers on in-house R&D aimed at developing a different set of product components. One could, however, consider spillovers in the development of the same product components. In our setting, this would not only reduce the development costs, but also intensify competition due to the reduced degree of differentiation.

<sup>15</sup>The empirical literature has shown that synergies and complementarities are important drivers for RJV formation (see, for example, Röller et al., 2007). Since R&D is not directed towards vertical improvements in our setting, such synergies could be taken into account with lower costs of joint development. All else equal, considering such synergies would lead to an RJV with a larger number of participants.

<sup>16</sup>See Häckner (2000) for the generalization for  $n$  symmetric firms.

$k$  and  $\alpha$ .

**Rules for RJV formation** We consider the formation of a single RJV in the industry, and define an equilibrium size of the RJV,  $k^*$ , as one that satisfies the following conditions:

- (i)  $\Pi_R(k^*, \alpha) \geq \Pi_R(k^* - 1, \alpha)$  ;
- (ii)  $\Pi_R(k^*, \alpha) \geq \Pi_R(k^* + 1, \alpha)$ ; and
- (iii)  $\Pi_R(k^*, \alpha) \geq \Pi_{-R}(k^* - 1, \alpha)$ .

Our approach corresponds to the exclusive membership rule in the coalition formation literature,<sup>17</sup> where players can join a coalition only with the consent of the existing members of the coalition, but are free to leave it. The first two conditions ensure the optimality of the RJV (none of the firms is willing to accept a new firm to the RJV or to exclude a member from the RJV), whereas the last condition ensures stability (i.e., none of the RJV member firms is better off leaving the RJV unilaterally).

**Timing** Once the RJV is formed, all firms set quantities independently and compete à-la-Cournot.

### 3 Analysis

In this section we study the equilibrium size of the RJV, and investigate how the equilibrium size of the RJV varies with the degree of cooperation in product development. We then compare the equilibrium size of the RJV to the socially optimal size.

For a given size of the RJV,  $k$ , the firms' profits at the second stage of the game can be found in Appendix A. With the following Lemma, we characterize how the firms' profits, gross of developments costs, vary with the size of the RJV.

**Lemma 1** *The equilibrium gross profits of the member firms are decreasing with the size of the RJV for  $k < n/2 + 1/(1 - s_{\max}) \equiv \tilde{k}$  (and are increasing with it otherwise), whereas the equilibrium gross profits of the non-member firms are increasing with the size of the RJV for all  $k$ .*

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<sup>17</sup>See Bloch (2002) for a survey on coalitions and networks. Our paper does not contribute to this strand of literature as we adopt a relatively simple (existing) rule for RJV formation, and do not consider multiple RJV's within an industry.

**Proof.** See Appendix B. ■

The effect of an additional firm joining the RJV (i.e., size of the RJV increasing from  $k$  to  $k+1$ ) on a firm's gross profit is different when the firm is a member and when it is a non-member. This is because when the  $(k+1)^{th}$  firm becomes a member, the differentiation between this new member and the  $k$  initial member firms decreases from  $s_{\max}$  to  $s(\alpha) < s_{\max}$ , but the differentiation between the new member firm and the non-member firms remains unchanged.<sup>18</sup>

We start by explaining the effect of this change on the member firms' gross profits.<sup>19</sup> First, the inclusion of an additional firm to the RJV generates a negative direct effect. This is because the reduced differentiation between the  $(k+1)^{th}$  firm and the initial  $k$  firms leads to an inward shift of their demand. The magnitude of this negative direct effect is different for the new member and the existing member firms. For the new member firm, the magnitude of this effect depends on the initial size of the RJV,  $k$ . In particular, a larger initial RJV implies a larger negative effect, as the new member suffers from reduced differentiation with respect to a larger number of firms. For the initial members, however, the magnitude of this negative effect is the same, regardless of the number of existing member firms (as they suffer from reduced differentiation with respect to a single firm, i.e., the new member).

Furthermore, the inclusion of the  $(k+1)^{th}$  firm to the RJV generates indirect effects on the quantity produced by the  $k$  initial member firms. First, as the demand for the existing member firms' products shifts inwards, each member firm contracts its output (hence, a negative indirect effect). At the same time, as the  $(k+1)^{th}$  firm reduces its quantity, each initial member firm reacts by increasing its output (hence, a positive indirect effect). When  $k$  is large, the latter effect dominates the former (as the reduction in the  $(k+1)^{th}$  firm's quantity is relatively large). Therefore, in this case the net indirect effect is positive.

In sum, the direct effect, together with the indirect effects, make that the gross profits of the member firms are increasing with the size of RJV if  $k$  is sufficiently large, and are decreasing otherwise.

In contrast with the member firms, increasing the size of the RJV has only a positive indirect effect on the non-member firms' gross profits. Any direct effect is absent, as the degree of differentiation between non-member firms is the same as the one between a non-member firm and a

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<sup>18</sup>This is due to our assumption that the degree of differentiation between any given two non-member firms is the same as the degree of differentiation between a given member firm and a non-member firm, which is  $s_{\max}$ .

<sup>19</sup>In Appendix C, we detail how the best response of the existing member firms, the new member firm, and the non-member firms shift when an additional firm joins the RJV.

member firm. As for the indirect effect, we find that the output for all initial members and the  $(k + 1)^{th}$  firm decreases as the  $(k + 1)^{th}$  firm joins the RJV. Non-member firms react to this change by expanding their output, hence the sign of the indirect effect for non-member firms is positive.

The table below summarizes the sign of these various effects on member and non-member firms' gross profits.

<b>Table:</b> The effect of a marginal increase in the size of the RJV on gross profits		
	member firms	non-member firms
Direct Effect	(-)	$\emptyset$
Indirect Effects	(+) if $k$ high (-) if $k$ low	(+)
Net Effect	(+) if $k > \tilde{k}$ (-) if $k \leq \tilde{k}$	(+)

We now characterize the equilibrium size of the RJV. At the first stage of RJV formation, the profit of an RJV member firm is given by

$$\Pi_R^*(k, \alpha) = \pi_R^*(k, s(\alpha)) - \left[ d(1) - \frac{k-1}{k}d(\alpha) \right]. \quad (5)$$

**Proposition 1** *Assume that an equilibrium exists. If  $\tilde{k} < n$ , then the equilibrium size of the RJV is either (i)  $n$ , or (ii) smaller than  $\tilde{k}$ .*

**Proof.** Let us assume for the moment that  $k$  is a real number, and not an integer. We have

$$\frac{\partial \Pi_R^*(k, \alpha)}{\partial k} = \frac{\partial \pi_R^*(k, s(\alpha))}{\partial k} + \frac{d(\alpha)}{k^2}.$$

From Lemma 1, the first term is positive if  $k > \tilde{k}$ , and negative otherwise, and the second term is always positive. Therefore, if there is an equilibrium, we either have a corner solution, i.e.,  $k^* = n$ , or  $k^* < \tilde{k}$ . ■

This result implies that if  $n$  is sufficiently large so that  $\tilde{k} \simeq n/2$ , then the equilibrium RJV involves either all firms ( $k^* = n$ ) or less than half of the firms in the industry ( $k^* < n/2$ ). Another consequence is that we can have multiple equilibria,<sup>20</sup> one of which involves an industry-wide

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<sup>20</sup>Here, we mean in terms of the size of the RJV. Naturally, given an equilibrium size of the RJV,  $k^*$ , and the number of firms in the industry,  $n$ , there are  $\binom{n}{k^*}$  equilibria.

RJV, and another, which involves an RJV with (approximately) less than half of the firms in the industry.<sup>21</sup>

We now study the sign of  $\partial^2 \Pi_R^* / \partial \alpha \partial k$ , which helps us determine how the equilibrium size of RJV varies with the degree of cooperation.<sup>22</sup> For the purpose of this analysis, assume that  $k$  is a real number. We have  $\partial^2 \Pi_R^* / \partial \alpha \partial k \geq 0$  if and only if

$$\underbrace{\frac{\partial^2 \pi_R^*(k, s)}{\partial s \partial k}}_{\text{direct and indirect effects}} \quad \underbrace{\frac{ds(\alpha)}{d\alpha}}_{\text{differentiation effect}} \quad + \quad \underbrace{\frac{d'(\alpha)}{k^2}}_{\text{development cost effect}} \quad \geq 0. \quad (6)$$

(-) or (+)                      (-)                      (+)

The first term can be either positive or negative. Since we always have  $ds(\alpha)/d\alpha < 0$ , this means that the effect of  $\alpha$  through the direct and the indirect effects tends either to increase or to decrease the size of the RJV. Finally, since  $d'(\alpha) > 0$ , the last term is positive, which tends to increase the size of the RJV. Overall, the sign of  $\partial^2 \Pi_R^* / \partial \alpha \partial k$  is indeterminate unless  $ds(\alpha)/d\alpha$  is very small.<sup>23</sup>

The following remarks that characterize the sign of  $\partial^2 \pi_R^* / \partial s \partial k$  for the two extreme cases, no RJV and industry-wide RJV, prove useful in decomposing the different effects that are at play.

**Remark 1** *If there is no RJV (i.e.,  $k = 1$ ), we have  $\partial^2 \pi_R^* / \partial s \partial k > 0$ .*

**Proof.** See Appendix D. ■

This result implies that for relatively small RJVs,  $\partial^2 \Pi_R^* / \partial \alpha \partial k$  can be negative if the marginal development cost is sufficiently low. The development cost, in general, operates as a source of "complementarity" between the size of the RJV and degree of cooperation. When the degree of cooperation increases, the benefits in terms of cost-sharing from increasing the size of the RJV are larger (and vice versa).

**Remark 2** *Assuming that the industry is sufficiently large (i.e.,  $n > 2/(1 - s_{\max})$ ), if there is an industry-wide RJV (i.e.,  $k = n$ ) we have  $\partial^2 \pi_R^* / \partial s \partial k < 0$  if  $s(\alpha)$  is sufficiently large.*

**Proof.** See Appendix D. ■

For relatively large RJVs,  $\partial^2 \Pi_R^* / \partial \alpha \partial k$  can thus be positive if  $s(\alpha)$  is sufficiently large. The direct effect of the degree of differentiation on profits is a source of "substitutability" between the

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<sup>21</sup>Note that, depending on the parameter values, there may be no equilibria as well.

<sup>22</sup>This will also prove useful in Section 4, when we extend our analysis to endogenize  $\alpha$ .

<sup>23</sup>That is,  $\Pi_R^*(\alpha, k)$  is neither supermodular nor submodular on the entire range for all parameter constellations.

size of the RJV and degree of cooperation. When the degree of cooperation is larger, the degree of product differentiation between RJV-member firms,  $s(\alpha)$ , decreases, which raises the direct cost (i.e., the magnitude of the direct effect) of cooperation.

The indirect effects can be either a source of substitutability or complementarity. When the size of the RJV is small, members contract output whenever a new member firm joins the RJV, and this effect is exacerbated when  $s(\alpha)$  is smaller (i.e., when  $\alpha$  is larger), which then implies that the size of the RJV and the degree of cooperation are substitutes. On the other hand, when the size of the RJV is large, members expand output when a new member firm joins the RJV, and this effect is exacerbated when  $s(\alpha)$  is smaller (i.e., when  $\alpha$  is larger), which then implies complementarity between the size of the RJV and the degree of cooperation.

The following Proposition characterizes the variation of the size of the RJV with respect to the degree of cooperation.

**Proposition 2** *If the degree of product differentiation is not sensitive to the degree of cooperation (i.e.,  $\partial s/\partial\alpha$  is very small in absolute terms), the size of the RJV increases with the degree of cooperation. Otherwise, the size of the RJV can either increase or decrease with the degree of cooperation.*

**Proof.** Let us assume that  $k$  is a real number, and not an integer. Since the equilibrium size of the RJV maximizes the joint profits of the members, the equilibrium size solves the first order condition,

$$\frac{\partial\pi_R^*(k, s(\alpha))}{\partial k} + \frac{d(\alpha)}{k^2} = 0. \quad (7)$$

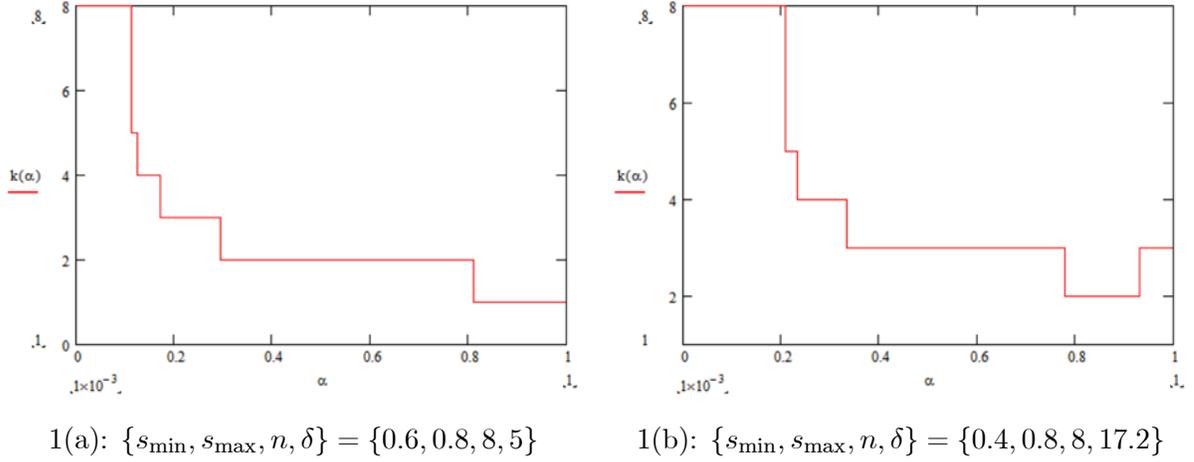
This first order condition defines implicitly a function  $k^*(\alpha)$ . Given that the second order condition holds, from the implicit function theorem, the variations of  $k^*(\alpha)$  with respect to  $\alpha$  are given by the sign of  $\partial^2\Pi_R^*/\partial\alpha\partial k$ . From (6), the sign of  $\partial^2\Pi_R^*/\partial\alpha\partial k$  is indeterminate in general. It is positive if  $ds(\alpha)/d\alpha \approx 0$ , in which case  $k^*$  increases with  $\alpha$ . It is negative if  $\partial^2\pi_R^*/\partial\alpha\partial k < 0$  and  $d'(\alpha)$  is sufficiently low, in which case  $k^*$  decreases with  $\alpha$ . ■

When the degree of product differentiation is sensitive to the degree of cooperation, the overall effect of a higher  $\alpha$  on the equilibrium size of the RJV,  $k^*$ , depends ultimately on  $s(\cdot)$  and  $d(\cdot)$ . As we show with a numerical example, the relationship between the equilibrium size of the RJV and the degree of cooperation can be non-monotonic.

Let  $d(\alpha) = \delta\alpha/2$ , and  $s(\alpha) = s_{\max} - (s_{\max} - s_{\min})\alpha^2$ . Figure 1(a) below is depicted for  $\{s_{\min}, s_{\max}, n, \delta\} = \{0.6, 0.8, 8, 5\}$ , whereas Figure 1(b) is depicted for  $\{s_{\min}, s_{\max}, n, \delta\} = \{0.4, 0.8, 8, 17.2\}$ ,

that is, for a lower value for  $s_{\min}$  and a higher value for the development cost parameter  $\delta$ .

**Figure 1:** Variation of the equilibrium size of RJV with the degree of cooperation



As can be seen from the figures, there is an industry-wide RJV (i.e.,  $k^* = n$ ) when the degree of cooperation is very low. This is because a low  $\alpha$  implies a low negative direct effect in absolute terms (reduced differentiation) between the cooperating firms, which makes cooperation less costly. The benefits through reduced development costs then offset the negative direct effect.<sup>24</sup>

In Figure 1(a) the equilibrium size of the RJV decreases with the degree of cooperation (and eventually no RJV is formed, i.e.,  $k^* = 1$ ). One can think about a higher  $\alpha$  as representing a higher cost of cooperation, and therefore a higher  $\alpha$  leads to fewer firms cooperating. However, in our setting, the degree of cooperation also interacts with the profits of the firms through the indirect effects, which can be positive. As Figure 1(b) shows, the equilibrium size of the RJV can also increase with the degree of cooperation. In this example, the equilibrium size of the RJV first decreases and then increases with  $\alpha$ .

**Social Welfare** Social welfare is defined as the sum of industry profit and consumer surplus. Hence, for a given  $k$  and a given  $\alpha$ , we have

$$W = k\Pi_R^*(k, \alpha) + (n - k)\Pi_{-R}^*(k, \alpha) + CS, \quad (8)$$

where  $\Pi_R^*(k, \alpha)$ ,  $\Pi_{-R}^*(k, \alpha)$ , and  $CS$  can be found in Appendix E. Let  $k^w$  denote the socially optimal size of the RJV.

<sup>24</sup>Note that we have not considered any other cost of forming an RJV. Naturally, the presence of such costs may prevent firms to form an industry-wide RJV when the degree of cooperation is low.

**Proposition 3** *The equilibrium size of the RJV is too small from the industry’s perspective (unless there is an industry-wide RJV), whereas it is too large from consumers’ point of view.*

**Proof.** In order to determine how  $k^*$  compares with  $k^w$ , we assume that  $k$  is a real number, and compute the following:

$$\left. \frac{dW}{dk} \right|_{k=k^*} = (n - k) \left. \frac{\partial \Pi_{-R}^*(k, \alpha)}{\partial k} \right|_{k=k^*} + \left. \frac{\partial CS}{\partial k} \right|_{k=k^*}.$$

From Lemma 1, we know that  $\partial \Pi_{-R}^*(k, \alpha) / \partial k > 0$ . Since  $k^*$  maximizes the RJV member firms’ profits, the equilibrium size of the RJV is therefore too small from the industry’s perspective. Finally, we have  $\partial CS / \partial k < 0$ ; the proof consists of proving that this inequality holds under all parameter constellations, and is done under Mathematica (available upon request). ■

The first part of Proposition 3 is due to the externality generated by the RJV member firms’ output decisions. When a non-member firm joins the RJV, it—together with the other member firms—contracts output (due to the reduced degree of differentiation), which benefits the remaining non-member firms in the industry. That is, from the non-member RJV firms’ point of view, the equilibrium size of the RJV is too small. This also implies that the size of the RJV that maximizes the industry profits is necessarily larger than the equilibrium size, as  $k^*$  maximizes the RJV member firms’ profits (unless all firms are included in the RJV).

The consumers, on the other hand, would be better off with a smaller size of RJV than the one generated in equilibrium. This is because, in particular, consumers value variety, and the degree of differentiation among the RJV members is less than the degree of differentiation among non-member firms (which is the same as the degree of differentiation between any member firm and any non-member firm). Furthermore, the size of the RJV affects consumer surplus through conflicting competition effects. We find that when an additional firm joins the RJV, the equilibrium price of non-member firms increases, whereas the equilibrium price of member firms decreases if  $k < \tilde{k}$ , and increases otherwise.

Since the size of the RJV has two opposing effects on social welfare, it is not possible to conclude how  $k^*$  and  $k^w$  compare in general. For the functional forms and parameter values used for Figures 1(a) and 1(b), we find that the equilibrium size of RJV either coincides with the social optimal (in

particular for low or high values of  $\alpha$ )<sup>25</sup> or it is too low.<sup>26</sup>

## 4 Discussion

In this section, we consider the case when the degree of cooperation is also endogenous, and we also discuss how our results change when firms compete in prices instead of in quantities.

### 4.1 Endogenous degree of cooperation

In our setting, it is not possible to derive analytical results when both the size of the RJV and the degree of cooperation are endogenous. In order to provide some insights about what might happen in this case, we use simulations.

Let  $\tilde{\alpha}^*(k) = \arg \max_{\alpha} \{\Pi_R(k, \alpha)\}$ , where  $\Pi_R(k, \alpha)$  is given by (5). We adopt a similar approach for RJV formation as in our main model, and consider that the equilibrium size of the RJV and degree of cooperation,  $\{k^*, \alpha^*\}$ , satisfy the following conditions:

- (i)  $\Pi_R(k^*, \alpha^*) \geq \Pi_R(k^* - 1, \tilde{\alpha}^*(k^* - 1))$ ;
- (ii)  $\Pi_R(k^*, \alpha^*) \geq \Pi_R(k^* + 1, \tilde{\alpha}^*(k^* + 1))$ ;
- (iii)  $\Pi_R(k^*, \alpha^*) \geq \Pi_{-R}(k^* - 1, \tilde{\alpha}^*(k^* - 1))$ ; and
- (iv)  $\alpha^* = \tilde{\alpha}^*(k^*)$ .

Conditions (i) and (ii) mean that the size of the RJV is optimal for members, while condition (iii) implies stability. For these conditions, we assume that member firms adjust the degree of cooperation when the size of the RJV changes. Finally, condition (iv) states that the degree of cooperation maximizes member firms' profits.

The timing is as follows: once the RJV is formed and the degree of cooperation is set, all firms set quantities independently and compete à-la-Cournot. We study the variations of  $k^*$  and  $\alpha^*$  with respect to the number of firms,  $n$ .

First, consider the case where the effect of the degree of cooperation on product differentiation is negligible.

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<sup>25</sup>Note that in our analysis (in particular, for Proposition 3), we considered  $k$  as a real number. Since we restrict  $k$  to be an integer number in our simulations,  $k^*$  and  $k^w$  coincide in some instances.

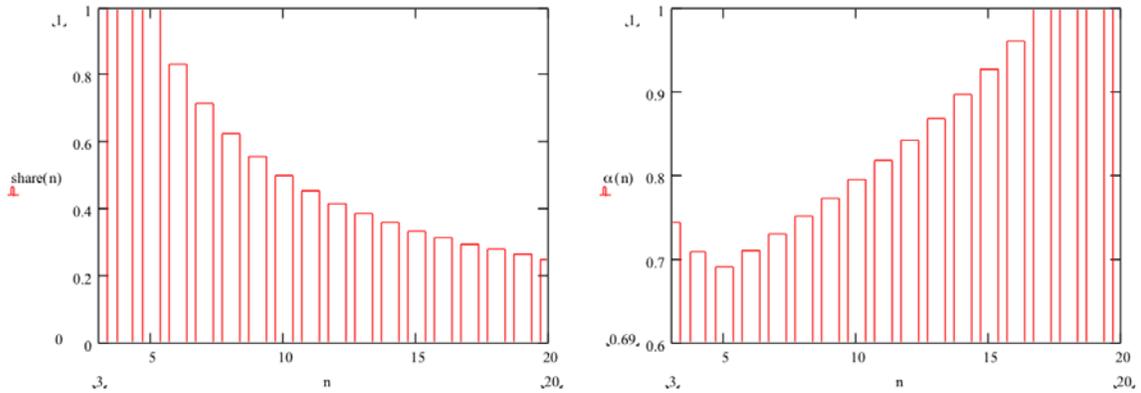
<sup>26</sup>However, simulations for other parameter values and/or other functional forms for  $s(\alpha)$  and  $d(\alpha)$  confirm that the equilibrium size of RJV can be either too low or too high compared to the social optimum.

**Remark 3** *If the degree of cooperation has a negligible effect on the degree of differentiation (i.e.,  $ds(\alpha)/d\alpha \approx 0$ ), in equilibrium there is an industry-wide RJV ( $k^* = n$ ), and full cooperation ( $\alpha^* = 1$ ).*

If there is no cost (or a very low cost) to cooperation, a higher degree of cooperation means larger benefits to the member firms (through cost-sharing), and hence, RJV members set the highest degree of cooperation in product development (i.e.,  $\alpha^* = 1$ ). Furthermore, adding an additional member to the RJV has an insignificant negative effect, while profits further increase due to the additional member's contribution to the development cost. Therefore, in equilibrium, all firms join the RJV (i.e.,  $k^* = n$ ).

Second, consider the case where the degree of cooperation affects the degree of differentiation in a non-negligible way. Since we cannot characterize the equilibria analytically for this case, we revert to numerical simulations. We assume that  $d(\alpha) = \delta\alpha/2$ ,  $s(\alpha) = s_{\max} - (s_{\max} - s_{\min})\alpha^4$ , and that  $\{s_{\min}, s_{\max}, \delta\} = \{0.8, 0.9, 17\}$ . Figures 2(a) and 2(b) show the variation of the equilibrium (relative) size of RJV<sup>27</sup> and degree of cooperation, respectively, with the size of the industry.

**Figure 2:** Variation of the equilibrium size of RJV and degree of cooperation with the size of the industry



2(a): Equilibrium size of RJV ( $k^*/n$ )

2(b): Equilibrium degree of cooperation ( $\alpha^*$ )

In Figure 2, the RJV is industry-wide while the degree of cooperation is relatively small for industries with a small number of firms. By contrast, for industries with a large number of firms, the relative size of the RJV is small while there is full cooperation (i.e.,  $\alpha^* = 1$ ). We can therefore state the following remark.

**Remark 4** *If the degree of cooperation has a non-negligible effect on the degree of differentiation,*

<sup>27</sup>We use the relative size of the RJV,  $k^*/n$ , instead of its absolute size, as it seems to be a more relevant measure of the size of the RJV when the number of firms in the industry varies.

*RJV member firms can maximize cooperation in one dimension (i.e., the size of the RJV or the degree of cooperation) by minimizing cooperation in the other dimension.*

Another way to see how endogenizing the degree of cooperation affects the size of the RJV is to compare the equilibrium outcome in Figure 2 with the equilibrium outcome when the degree of cooperation is exogenous. For example, if we assume that when firms cannot set the degree of cooperation, cooperation is full (i.e.,  $\alpha = 1$ ), we find that the size of the RJV is lower than when the degree of cooperation is endogenous. This shows that when firms can decide both on the size of the RJV and the degree of cooperation, they can trade off between these two dimensions of cooperation.

## **4.2 Bertrand competition**

In our setting, we have assumed that firms compete à-la-Cournot. We now discuss how our analysis would change if we assumed instead that firms compete à-la-Bertrand. The insights we provide in this section are obtained by solving the Bertrand model for a given number of firms (three firms).

Lemma 1, where we characterize how gross profits vary with the number of RJV members, is central to our results under Cournot competition. When firms compete in quantities, we found that non-member firms always benefit from an increase in the size of the RJV, and that member firms benefit too if the size of the RJV is sufficiently high. This no longer holds under Bertrand competition. When an additional firm joins the RJV, due to the reduced differentiation, the prices of the existing member firms decrease, which hurts both the member firms and the non-member firms.

At the RJV formation stage, the members then face the following trade-off: on the one hand, a larger RJV intensifies competition; on the other, it increases the economies from joint development. Note this trade-off is similar under Cournot competition, except that the competition effect can in some cases be positive. Therefore, we can conjecture that the equilibrium size of the RJV to be smaller under Bertrand competition than under Cournot competition. Finally, in terms of comparative statics, the relation between  $k^*$  and  $\alpha$  is also indeterminate, since there are both a source of substitutability due to the negative competition effect and a source of complementarity due to the positive development cost effect.

## 5 Conclusion

In this paper, we have provided an oligopoly model of RJV in product development to study the effect of the degree of cooperation (jointly developed product components) on the size of the RJV (number of participating firms). We show that the equilibrium size of the RJV can vary non-monotonically with the degree of cooperation. While savings from development costs operate as a source of "complementarity" between the size of the RJV and the degree of cooperation, the effect of the reduced degree of differentiation on firms' profits operates as a source of "substitutability" between the two. We also find that the equilibrium size of the RJV is too small from the industry's perspective (unless there is an industry-wide RJV), whereas it is too large from consumers' point of view. Since RJV's involve a competitive cost which is borne only by the member firms, and since this cost (the effect of reduced differentiation) is more significant under price competition than under quantity competition, we conjecture that the former yields a smaller size RJV than the latter.

In order to study the effects on the size of RJV of the degree of cooperation, we adopted the simplest possible model of oligopoly. In particular, we considered the formation of a single RJV within the industry. Considering multiple RJV's would perhaps provide additional insights, but it would also require shifting the paper's focus from considerations of cooperation to the analysis of coalition formation.

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## Appendix

### A. Equilibrium profits

The profit of firm  $i = 1, \dots, n$  is equal to  $\pi_i = (p_i - c) q_i$ . Each firm chooses its quantity,  $q_i$ , so as to maximize its profit. The first-order condition for profit maximization is

$$p_i - c + q_i \frac{\partial p_i}{\partial q_i} = p_i - c - q_i = 0,$$

that is,  $p_i - c = q_i$ . We search for a symmetric equilibrium, such that in equilibrium  $q_1 = \dots = q_k$  for member firms and  $q_{k+1} = \dots = q_n$  for non-member firms. Using the first-order conditions and the symmetry conditions, we find:

$$\pi_R^* = (q_R^*)^2 = \frac{(1 + s_{\max})^2 (a - c)^2}{((1 + s) ((n - 1) (1 - s_{\max}) + 2) + k (s_{\max} - s) ((n - k) (1 - s_{\max}) + 2))^2}$$

for the RJV member firms; and

$$\pi_{-R}^* = (q_{-R}^*)^2 = \frac{(1 + s + k (s_{\max} - s))^2 (a - c)^2}{((1 + s) ((n - 1) (1 - s_{\max}) + 2) + k (s_{\max} - s) ((n - k) (1 - s_{\max}) + 2))^2}$$

for the non-member firms.

### B. Proof of Lemma 1

For the first part of the Lemma, notice that the numerator of  $\pi_R^*$  does not depend on  $k$ , and that the denominator is increasing with  $k$  if  $2 + (n - 2k) (1 - s_{\max}) > 0$ , that is, if  $k < \tilde{k}$ . For the second part of the Lemma, note that we have  $\pi_{-R}^*$  increasing with  $k$  as

$$\frac{\partial \pi_{-R}^*}{\partial k} = \frac{2 (s_{\max} - s) (1 - s_{\max}) (a - c)^2 (1 + s + (s_{\max} - s) k) [k^2 (s_{\max} - s) + (1 + s) (2k - 1)]}{((1 + s) ((n - 1) (1 - s_{\max}) + 2) + k (s_{\max} - s) ((n - k) (1 - s_{\max}) + 2))^3} > 0,$$

since both the both numerator and denominator are positive. ■

### C. Effects at work in Lemma 1

As shown in Appendix A, the firms' equilibrium gross profits are equal to the square of equilibrium quantities. Therefore, to study the variations of gross profits with respect to the number of member firms, we study how quantities vary when a new firm joins the RJV. To understand the various

effects at work, we analyze how this change affects the best-response of the firms.

**Non-members firms** When the  $(k + 1)^{th}$  firm joins an RJV initially of size  $k$ , the change in the best-response of non-member firm  $i$  is given by

$$q_i^{NR}(k + 1) - q_i^{NR}(k) = -\frac{1}{2} \sum_{\substack{j=1 \\ j \neq i}}^n (1 - s_{\max}) (q_j(k + 1) - q_j(k)),$$

where  $q_i^{NR}(k)$  and  $q_j(k)$  denote the best-response of firm  $i$  and firm  $j \neq i$ , respectively, for an RJV of size  $k$ . If the  $(k + 1)^{th}$  firm becomes a member of the RJV, it does not affect the degree of differentiation  $s_{ij}$  between firm  $i$  and firm  $j \neq i$  (no direct effect). The only change in the best response of firm  $i$  comes from a change in the quantities of the other firms. If they contract their output, firm  $i$  expands its output (and vice versa).

**New member firm** The change in the best-response of the new member, firm  $k + 1$ , is given by

$$\begin{aligned} q_{k+1}^R(k + 1) - q_{k+1}^{NR}(k) &= -\frac{1}{2} \sum_{j=1}^k [(1 - s) q_j(k + 1) - (1 - s_{\max}) q_j(k)] \\ &\quad - \frac{1}{2} \sum_{j=k+2}^n (1 - s_{\max}) (q_j(k + 1) - q_j(k)). \end{aligned}$$

The best-response of the new member firm changes via two different channels: (i) a direct effect: its differentiation with the  $k$  existing members is reduced, from  $s_{\max}$  to  $s$ ; (ii) indirect effects: the other firms adjust their quantities.

**Existing member firms** The change in the best response of an existing member, firm  $i = 1, \dots, k$ , is given by

$$\begin{aligned} q_i^R(k + 1) - q_i^R(k) &= -\frac{1}{2} \sum_{j=1, j \neq i}^k (1 - s) [q_j(k + 1) - q_j(k)] \\ &\quad - \frac{1}{2} [(1 - s) q_{k+1}(k + 1) - (1 - s_{\max}) q_{k+1}(k)] \\ &\quad - \frac{1}{2} \sum_{j=k+2}^n (1 - s_{\max}) (q_j(k + 1) - q_j(k)). \end{aligned}$$

The best-response of an existing member firm changes via two different channels: (i) a direct effect: its differentiation with the firm that joins the RJV goes from  $s_{\max}$  to  $s$ ; (ii) indirect effects: the other firms adjust their quantities.

## D. Proof of Remarks 1 and 2

We find that

$$\frac{\partial^2 \pi_R^*}{\partial s \partial k} = \frac{2(a-c)^2 (1+s_{\max})^2 [(1-s_{\max})(n-2k)+2] T(k, n, s, s_{\max})}{((1+s)((n-1)(1-s_{\max})+2) + k(s_{\max}-s)((n-k)(1-s_{\max})+2))^4}. \quad (9)$$

where  $s = s(\alpha)$ ,

$$T = 2(1-s_{\max})(s_{\max}-s)k^2 - 2(2+(1-s_{\max})n)(s_{\max}-s)k + \chi,$$

and

$$\chi = (1+3s_{\max}-2s)(1+s_{\max}+n(1-s_{\max})) > 0.$$

From (9),  $\partial^2 \pi_R^* / \partial s \partial k$  has the sign of  $[(1-s_{\max})(n-2k)+2] T(k, n, s, s_{\max})$ . The first term,  $[(1-s_{\max})(n-2k)+2]$ , is positive if  $k < \tilde{k}$ , and negative otherwise.  $T$  is a second-degree polynomial in  $k$ . Assume that  $T = 0$  has two solutions,  $k_1$  and  $k_2$ . We know that  $k_1 k_2$  has the sign of  $2(1-s_{\max})(s_{\max}-s)\chi$ , hence it is positive. Furthermore, since  $k_1 + k_2 = (2+(1-s_{\max})n)/(1-s_{\max}) > 0$ ,  $k_1$  and  $k_2$  are both positive. It follows that  $T$  is either always positive if it has no root, or first positive, then negative, then positive. If  $T$  is positive, (9) has the sign of  $(1-s_{\max})(n-2k)+2$ . Whereas, if  $T$  is negative, then (9) has the opposite sign.

First, to prove Remark 1, we compute  $\partial^2 \pi_R^* / \partial s \partial k$  for  $k = 1$ . We find that

$$\left. \frac{\partial^2 \pi_R^*}{\partial s \partial k} \right|_{k=1} = \frac{2(a-c)^2 [n(1-s_{\max})+2s_{\max}]}{(1+s_{\max})[1+s_{\max}+(1-s_{\max})n]^3}$$

is always positive. Second, to prove Remark 2, we compute  $\partial^2 \pi_R^* / \partial s \partial k$  for  $k = 1$ :

$$\left. \frac{\partial^2 \pi_R^*}{\partial s \partial k} \right|_{k=n} = \frac{2(a-c)^2 [2-n(1-s_{\max})][1+3s_{\max}-2s+n(1-3s_{\max}+2s)]}{(1+s_{\max})[1+s_{\max}+(1-s_{\max})n]^3}.$$

If  $n(1-s_{\max}) > 2$ , this cross derivative at  $k = n$  has the opposite sign of  $[1+3s_{\max}-2s+n(1-3s_{\max}+2s)]$ . When  $s$  is close to  $s_{\max}$  (i.e.,  $\alpha$  is close to 0), this expression is positive, and therefore  $\partial^2 \pi_R^* / \partial s \partial k < 0$  at  $k = n$ . ■

## E. Social Welfare

We begin by the derivation of the consumer surplus. The utility of the representative consumer is given by

$$U(\mathbf{q}, m) = a \sum_{i=1}^n q_i - \frac{1}{2} \sum_{i=1}^n (q_i)^2 - \sum_{i=1}^{n-1} q_i \sum_{j=i+1}^n (1 - s_{ij}) q_j + m,$$

where  $s_{ij}$  is the degree of differentiation between the product of firm  $i$  and the product of firm  $j$ .

Let

$$A \equiv \sum_{i=1}^{n-1} q_i \sum_{j=i+1}^n (1 - s_{ij}) q_j.$$

Since the first  $k$  firms are RJV members and the next  $n - k$  firms are non members, one can develop  $A$  as follows:

$$\begin{aligned} A &= \sum_{i=1}^k q_i \left( \sum_{j=i+1}^k (1 - s_{ij}) q_j + \sum_{j=k+1}^n (1 - s_{ij}) q_j \right) + \sum_{i=k+1}^{n-1} q_i \sum_{j=i+1}^n (1 - s_{ij}) q_j, \\ &= \sum_{i=1}^k q_i \sum_{j=i+1}^k (1 - s) q_j + \sum_{i=1}^k q_i \sum_{j=k+1}^n (1 - s_{\max}) q_j + \sum_{i=k+1}^{n-1} q_i \sum_{j=i+1}^n (1 - s_{\max}) q_j. \end{aligned}$$

The first term of  $A$  can be rewritten as

$$\sum_{i=1}^k q_i \sum_{j=i+1}^k (1 - s) q_j = (1 - s) (q_R)^2 \frac{k(k-1)}{2},$$

the second term as

$$\sum_{i=1}^k q_i \sum_{j=k+1}^n (1 - s_{\max}) q_j = (1 - s_{\max}) q_R q_{-R} (n - k) k,$$

and finally the third term as

$$\sum_{i=k+1}^{n-1} q_i \sum_{j=i+1}^n (1 - s_{\max}) q_j = (1 - s_{\max}) (q_{-R})^2 \frac{(n-k)(n-k-1)}{2}.$$

Summing up,  $A$  can be rewritten as follows:

$$A = (1 - s) (q_R)^2 \frac{k(k-1)}{2} + (1 - s_{\max}) q_R q_{-R} (n - k) k + (1 - s_{\max}) (q_{-R})^2 \frac{(n-k)(n-k-1)}{2}.$$

Now, we compute the total price paid by the representative consumer,

$$\begin{aligned}\sum_{i=1}^n p_i q_i &= \sum_{i=1}^n q_i \left( a - q_i - \sum_{i \neq j} (1 - s_{ij}) q_j \right) \\ &= a \sum_{i=1}^n q_i - \sum_{i=1}^n (q_i)^2 - \sum_{i=1}^n q_i \sum_{i \neq j} (1 - s_{ij}) q_j.\end{aligned}$$

Notice that, by symmetry, we have

$$\sum_{i=1}^n q_i \sum_{i \neq j} (1 - s_{ij}) q_j = 2 \sum_{i=1}^{n-1} q_i \sum_{j=i+1}^n (1 - s_{ij}) q_j = 2A.$$

Therefore, the consumer surplus can be written as

$$CS = U(\mathbf{q}, m) - \sum_{i=1}^n p_i q_i = \frac{1}{2} \sum_{i=1}^n (q_i)^2 + A + m. \quad (10)$$

Replacing the expression of  $A$  in (10), we find that

$$CS = \frac{1}{2} \left[ k(k + s - ks) q_R^2 + [n - k + (1 - s_{\max})(n - k)(n - k - 1)] q_{-R}^2 + 2k(n - k)(1 - s_{\max}) q_{-R} q_R \right] + m.$$

The net profits of member and non-member firms are

$$\Pi_R^*(k, \alpha) = \pi_R^*(k, s(\alpha)) - \left[ d(1) - \frac{k-1}{k} d(\alpha) \right],$$

$$\Pi_{-R}^*(\alpha) = \pi_{-R}^*(s(\alpha)) - d(1),$$

respectively, where  $\pi_R^*$  and  $\pi_{-R}^*$  are defined in Appendix A. Therefore, the industry profits are

$$PS = k\pi_R^*(k, s(\alpha)) - k \left[ d(1) - \frac{k-1}{k} d(\alpha) \right] + (n - k) (\pi_{-R}^*(s(\alpha)) - d(1)).$$