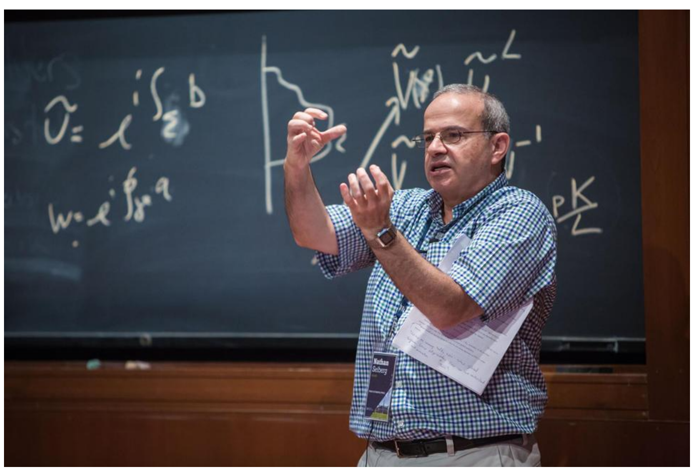
Three roads not (yet) taken



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Natifest

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תודה

Dankie Gracias

شكراً

Спасибо Merci Takk

Köszönjük Terima kasih

Grazie Dziękujemy Dėkojame

Ďakujeme Vielen Dank Paldies

Kiitos Täname teid 谢谢

Thank You

Tak

感謝您 Obrigado Teşekkür Ederiz

감사합니다

Σας Ευχαριστούμ

ขอบคุณ

Bedankt Děkujeme vám

ありがとうございます

Tack

- Question all assumptions, be rigorous wherever possible, do not settle for incomplete understanding
- The more references you have, more people will complain
- Publish what you understand, not what you do not understand

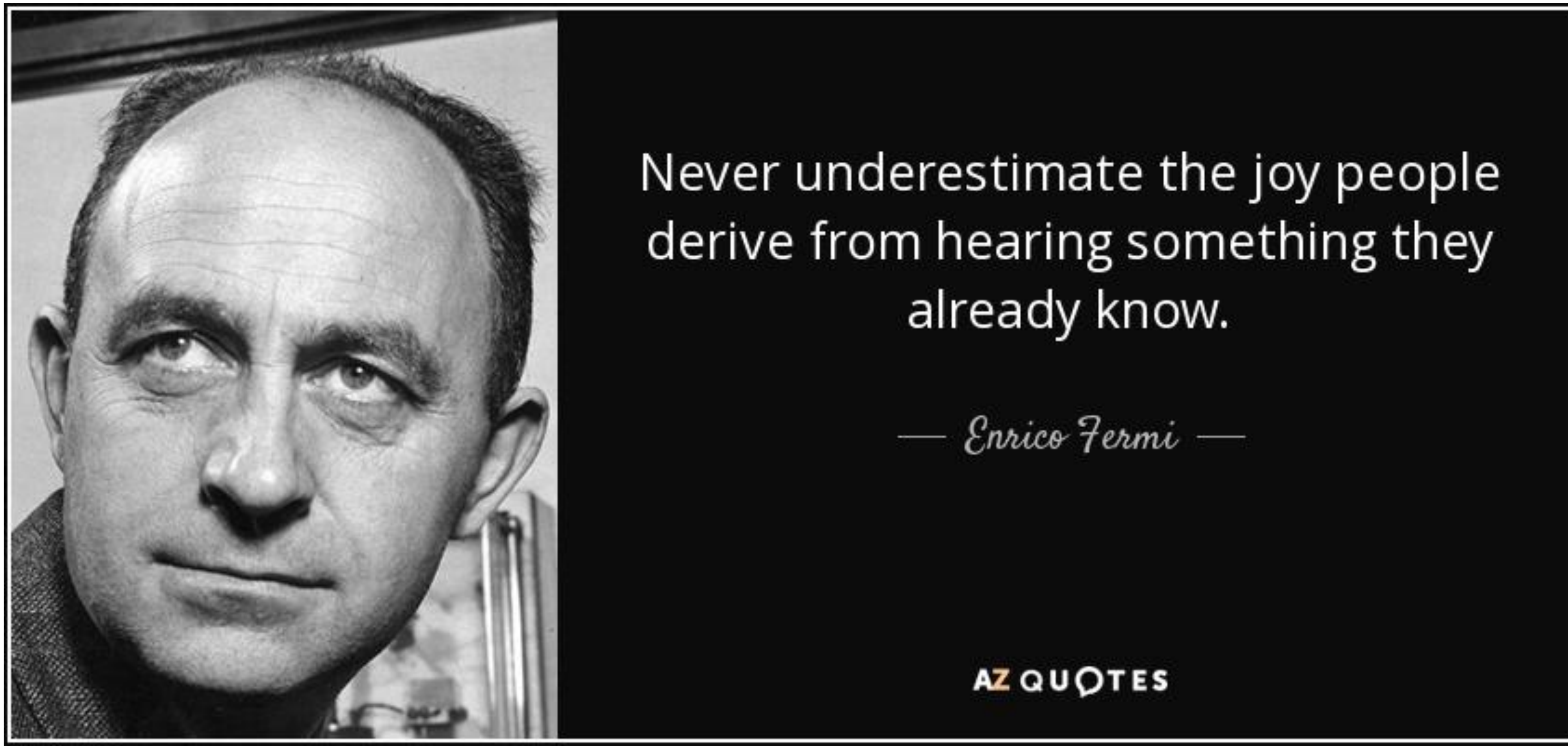
- Direct but not intimidating

- Contributions to QFT and string theory

What to talk about ???

Projects with Nati ?

(Top 10, 2 days – 3+ years)


Never underestimate the joy people derive from hearing something they already know.
— Enrico Fermi —
AZ QUOTES

Three topics that Nati has not (yet) worked on – possibilities for the next 60 years

Topic 1 : Theories with a number of supersymmetries that is not a power of 2

Nothing since 1987 (Schwimmer+Seiberg) ?
Nothing in $d>2$? ($d=3$ $\mathcal{N}=3,5,6$; $d=4$ $\mathcal{N}=3$)

d=4 $\mathcal{N}$=3 superconformal theories

- d=4 $\mathcal{N}$=4 theories are essentially classified
- For $\mathcal{N}$=2 theories – much progress but still seem far from full classification
- What about $\mathcal{N}$=3 ?
- Only free multiplet of $\mathcal{N}$=3 is $\mathcal{N}$=4 vector multiplet. So no free or weakly coupled pure $\mathcal{N}$=3 theories.
- Can have 6r-dimensional Coulomb branch with r free vector multiplets in IR.

d=4 $\mathcal{N}$=3 superconformal theories

- General properties of pure $\mathcal{N}$=3 SCFTs (OA+Evtikhiev, Cordova+Dumitrescu+Intriligator) :

1. No $\mathcal{N}$=3-preserving marginal or relevant deformations ($\mathcal{N}$=2 : relevant, not marginal)

2. No global symmetries (just $SU(3)_R \times U(1)_R$)

3. a=c

Recent examples

(Garcia-Etxebarrio+Regalado)

- N M2-branes in M theory on C^4/Z_k preserve d=3 $\mathcal{N}$=6 SUSY, gives d=3 $\mathcal{N}$=6 SCFTs ($\mathcal{N}$=8 for k=2)
- For k=2,3,4,6 can consider instead $(C^3 x T^2)/Z_k$ (for specific τ when k>2)
- But now can lift to N D3-branes in F-theory on $(C^3 x T^2)/Z_k$ (orientifolds of type IIB for k=2, S-folds for k=3,4,6)
- A family of d=4 $\mathcal{N}$=3 SCFTs !

Recent examples

- Naively labeled by N,k
- Extra "discrete torsion" parameters (OA+Tachikawa) :
- $k=2$: 4 well-known orientifolds
- $k=3$: $L=1,3$
- $k=4$: $L=1,4$
- $k=6$: $L=1$
- Dimensions of N "Coulomb branch generators" : $(k, 2k, 3k, \ldots, (N-1)k, NL)$
- Dual to F-theory on $AdS_5 x(S^5 x T^2)/Z_k$

Special examples

- Minimal pure $\mathcal{N}=3$ theory is N=1, k=L=3 with a single generator of dimension 3
- Theories with N=2 and L=1 happen to give $\mathcal{N}=4$ SYM theories:
- k=3 : SU(3)
- k=4 : SO(5)
- k=6 : G_2 (brane construction !)

Some questions

- Can we find more pure $\mathcal{N}=3$ theories ?
  - Deformations of $\mathcal{N}=1$ theories ?
  - Bootstrap ? (More constrained than $\mathcal{N}=2$)

- Can we classify all pure $\mathcal{N}=3$ theories ?

Topic 2 : Conformal bootstrap

Nothing (at least since d=2 RCFTs) ?

(Based on work very much in progress with Alday, Bissi, Perlmutter)

Conformal bootstrap

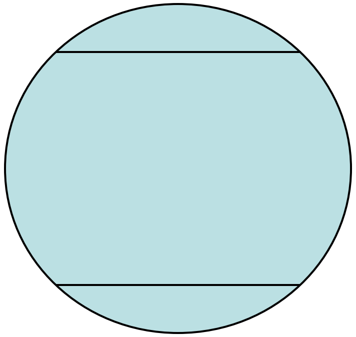
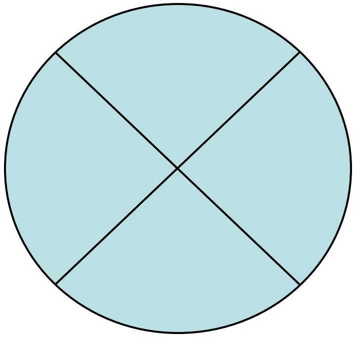
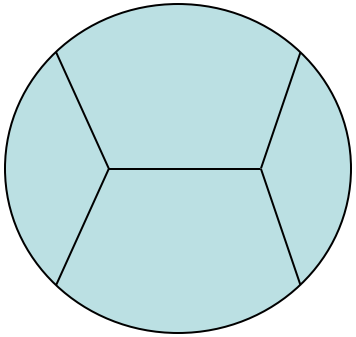
- CFT : primary operators $O_i(x)$ and OPE coefficients c_{ijk}. Consistency of the OPE in $\langle O_1(x_1)O_2(x_2)O_3(x_3)O_4(x_4)\rangle$ requires that

$$\sum_n c_{12n}c_{34n}\, G_n(u,v) \approx \sum_n c_{14n}c_{23n}\, G_n(v,u)$$

  for conformal cross-ratios u,v and "(super)conformal blocks" G_n depending on operator dimensions.
- "Crossing equation" necessary but not sufficient for consistent CFT.

Holographic bootstrap

- QG on AdS_{d+1} is a CFT_d → automatically obeys crossing. When weakly coupled can expand correlators in "Witten diagrams":

<OOOO>=  +  +  +

- Dual to a "large N CFT" with "single-trace ops" O_i, $\Delta_i \sim m_i$, "double-trace" $[O_iO_j]$, etc. c_{OiOjOk} = 3-point coupling = $O(1/N)$, $c_{OiOj[OiOj]}$=$O(1)$, $\Delta_{[OiOj]}= \Delta_i+\Delta_j+O(1/N^2)$, …

Holographic bootstrap

- A field theory like Φ^4 or Φ^3 on AdS_{d+1} can also give a solution to crossing, though not full CFT_d (can be decoupled sector).
- Any bulk theory gives a solution to crossing perturbatively in $1/N$. In <OOOO> :

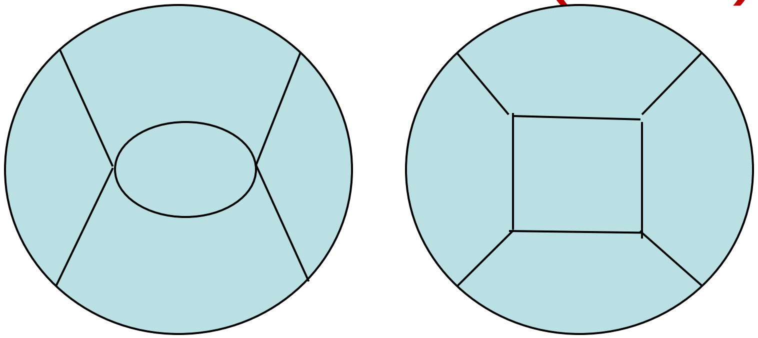
O(1) : Just disconnected diagram, [OO]

$O(1/N^2)$: only O' and correction to [OO]

- At order $1/N^2$ reverse also seems true ! (Heemskerk,Penedones,Polchinski,Sully; Alday,Bissi,Lukowski;…) (when gap)

One-loop or $O(1/N^4)$

- In a QFT (or effective field theory like SUGRA), tree-level action determines also loop amplitudes, up to a finite number of coupling constants ("renormalization conditions"). Should be true also in AdS.
- One-loop bulk diagrams not yet computed, except in ϕ^4 (Penedones, Fitzpatrick+Kaplan). Contribute to <OOOO> at $O(1/N^4)$:



One-loop or $O(1/N^4)$

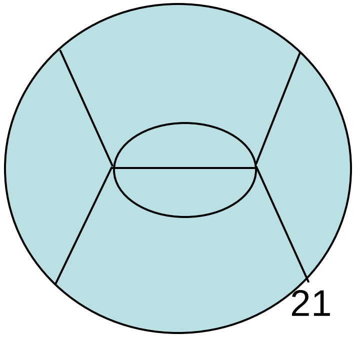
- So can we take a solution to crossing at $O(1/N^2)$ = a tree-level bulk theory, and use only crossing to compute a solution at $O(1/N^4)$ = one-loop diagrams ?
- Preliminary results : yes ! (Up to inevitable freedom in changing bulk couplings)
- More precisely, can do it if only [OO] appears at $O(1/N^4)$. If also [O’O’] appear, may need input from additional 4-point functions at order $O(1/N^2)$.

One-loop or $O(1/N^4)$

- So can use crossing to compute one-loop diagrams in AdS in theories like Φ^4 or Φ^3 or 5d $\mathcal{N}=8$ SUGRA on AdS_5, just from tree-level <OOOO>. Position/Mellin space.
- In theories like $\mathcal{N}=4$ SYM, even at very strong coupling (=10d SUGRA), need more input from $\langle O_m O_m O_n O_n \rangle$, but then should be able to compute $\langle O_2 O_2 O_2 O_2 \rangle$ at $O(1/N^4)$ from leading large N answers. (Up to undeterminable local bulk couplings.)

Higher-loops

- In principle can extend to higher loops = higher orders in 1/N, but new bulk couplings appear, and, related to this, also higher-trace operators appear in OPE, so need extra tree-level information (like 5-point functions) to get full answer.



Topic 3 : Disordered field theories

Nothing ?

(Based on OA, Komargodski, Yankielowicz +
work in progress with Narovlansky)

Disorder

- Random inhomogeneities common in condensed matter
- Model as QFTs with random coupling constants ("quenched disorder"). Analyze typical behavior in ensemble
- My motivation : How does renormalization group work in the presence of disorder ? Are there new types of fixed points (phases) ?

Simplifying assumptions

- Disorder couples to a single scalar operator,

$$S[h] = S_0 + \int d^d x \, h(x) O(x)$$

- Coupling h varies independently and randomly (Gaussian) at every point,

$$\overline{h(x)} = 0, \quad \overline{h(x) h(y)} = c^2 \, \delta(x-y)$$

"Background field" h(x) becomes dynamical

- Euclidean QFT (2nd order phase transitions); can generalize to couplings constant in time (more complicated)

Precise setup

- Disorder-averaged correlation functions

$$\overline{\langle O_1(x_1) \dots O_n(x_n)\rangle} = \int [Dh] e^{-\frac{1}{2c^2}\int d^d x\, h^2(x)} \frac{\int [D\Phi] O_1(x_1) \dots O_n(x_n) e^{-S[h]}}{\int [D\Phi] e^{-S[h]}}$$

- <u>Not</u> $\frac{1}{Z}\int [Dh] e^{-\frac{1}{2c^2}\int d^d x\, h^2(x)} \int [D\Phi] O_1(x_1) \dots O_n(x_n) e^{-S[h]}$
- Connected disordered correlators such as

$$\overline{\langle O_1(x_1) O_2(x_2)\rangle}^{conn} = \overline{\langle O_1(x_1) O_2(x_2)\rangle} - \overline{\langle O_1(x_1)\rangle \langle O_2(x_2)\rangle}$$

generated by disordered free energy

$$W_D = \int [Dh] \log(Z[h]) e^{-\frac{1}{2c^2}\int d^d x\, h^2(x)}$$

- <u>Independent</u> of general disordered correlators

Replica trick

- A general method is replica trick :

$$W_D = \int [Dh] \log(Z[h])\, e^{-\frac{1}{2c^2}\int d^d x\, h^2(x)} =$$

$$= \frac{d}{dn}|_{n=0} \int [Dh] Z^n[h]\, e^{-\frac{1}{2c^2}\int d^d x\, h^2(x)}$$

$$Z^n[h] = \int \prod_{A=1}^{n} [D\Phi_A]\, e^{-\sum_{A=1}^{n} S_A[h(x)]}$$

So a limit of standard QFTs.

- Same for correlators. Different origin for O(x) in general disordered correlators ($O_1(x)$) and in connected correlators ($\frac{1}{n}\sum_{A=1}^{n} O_A(x)$).

RG flow

- Limit of standard QFTs → couplings flow as usual + extra coupling c^2
- So can have standard ($c=0$) and disordered scale-invariant fixed points
- In connected correlators standard RG equation, renormalization of local operators
- In general correlators, have extra "anomalous dimension" and mixings, e.g.

$$\left(M\frac{\partial}{\partial M}+\beta_i\frac{\partial}{\partial\lambda_i}+\beta_{c^2}\frac{\partial}{\partial c^2}+2\gamma\right)\overline{\langle O_1(x_1)O_2(x_2)\rangle}+\tilde{\gamma}\,\overline{\langle O_1(x_1)O_2(x_2)\rangle}^{conn}=0.$$

RG flow

- At fixed point we get "logarithmic CFTs" (Cardy), e.g. : $\overline{\langle O(0)O(x)\rangle}^{conn} \propto 1/|x|^{2\Delta}$
  $\overline{\langle O(0)O(x)\rangle} \propto \log(x)/|x|^{2\Delta}$
  Related to degeneracy in replica theory.
- New types of critical exponents.
- Can perform exact computations at large N (field theory / holography). (Subtleties)
- Time-independent disorder – what can be said? Replica theory non-local in time; what does RG equation look like ? Fixed points ? Lifshitz scaling (Hartnoll+Santos) ?

Summary

- These were some suggestions for the next 60 years (120 עד)
- But Nati has often preferred to take the road less travelled by, and that has made all the difference…

Strings 2017 מיתרים תשע״ז

June 26-30, 2017, Tel Aviv, Israel