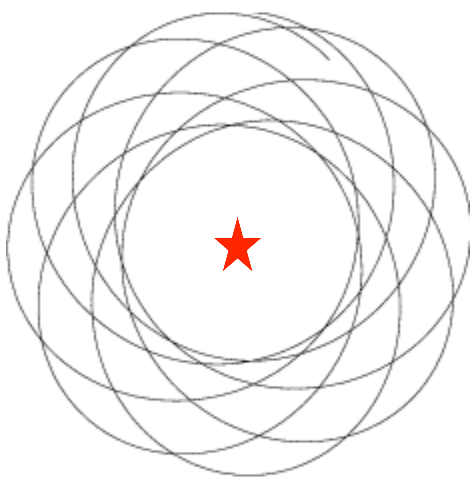
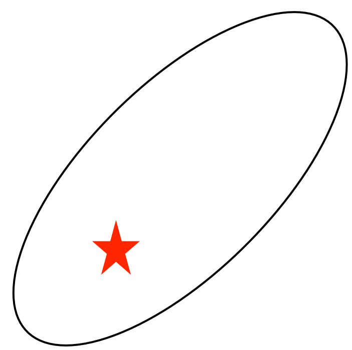
Kozai-Lidov oscillations

- Kozai (1962 - asteroids); Lidov (1962 - artificial satellites)
- arise most simply in restricted three-body problem (two massive bodies on a Kepler orbit + a test particle)
- e.g., wide binary star + planet orbiting one member of the binary
- in Kepler potential $\Phi = -GM/r$, eccentric orbits have a fixed orientation



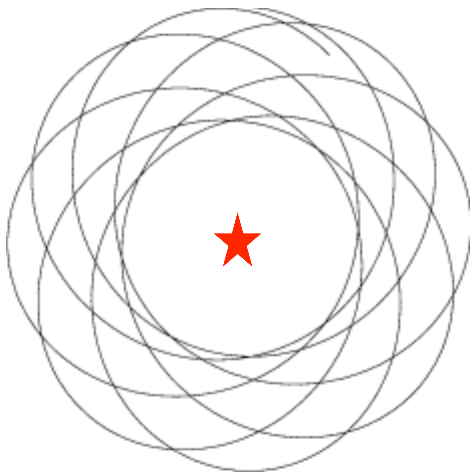
generic axisymmetric potential



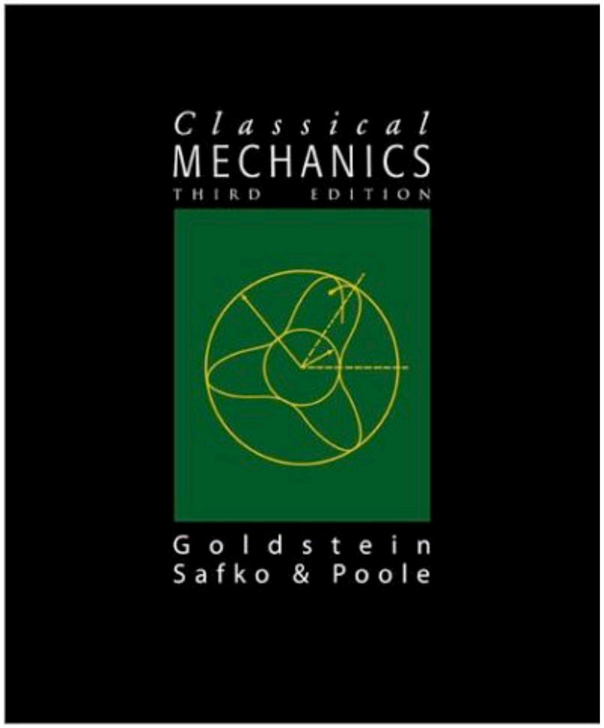
Kepler potential

Kozai-Lidov oscillations

- Kozai (1962 - asteroids); Lidov (1962 - artificial satellites)
- arise most simply in restricted three-body problem (two massive bodies on a Kepler orbit + a test particle)
- e.g., wide binary star + planet orbiting one member of the binary
- in Kepler potential $\Phi = -GM/r$, eccentric orbits have a fixed orientation



generic axisymmetric potential

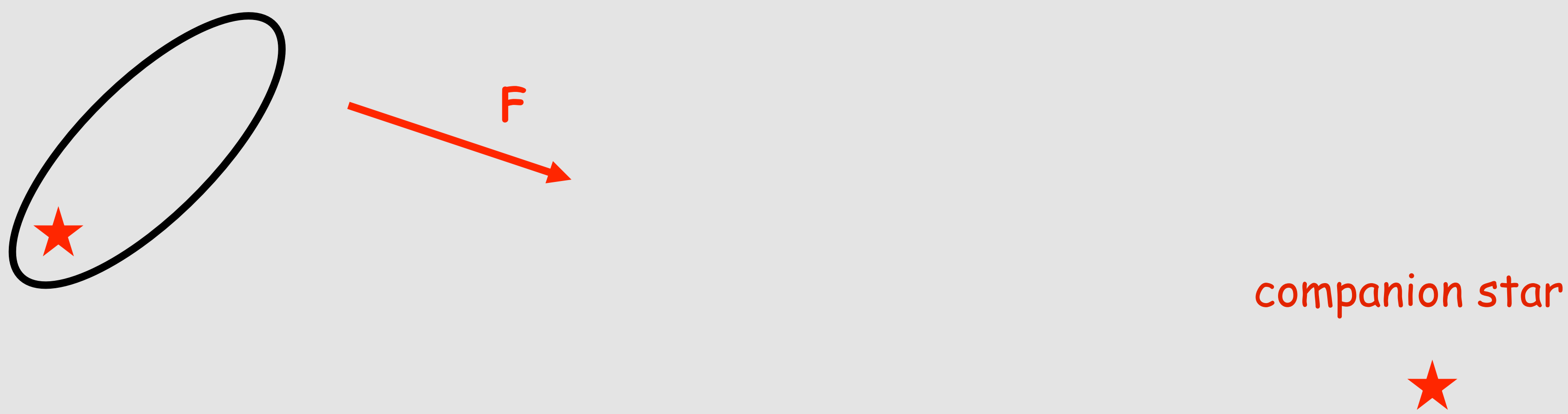


Kozai-Lidov oscillations

- now subject the Kepler orbit to a weak, time-independent external force **F** from the companion star

- because the orbit orientation is fixed even weak external forces act for a long time in a fixed direction relative to the orbit and therefore change the angular momentum or eccentricity

- if $F \sim \varepsilon$ then **timescale** for evolution $\sim 1/\varepsilon$ but **nature** of evolution is independent of ε



Kozai-Lidov oscillations

Consider a planet orbiting one member of a binary star system:

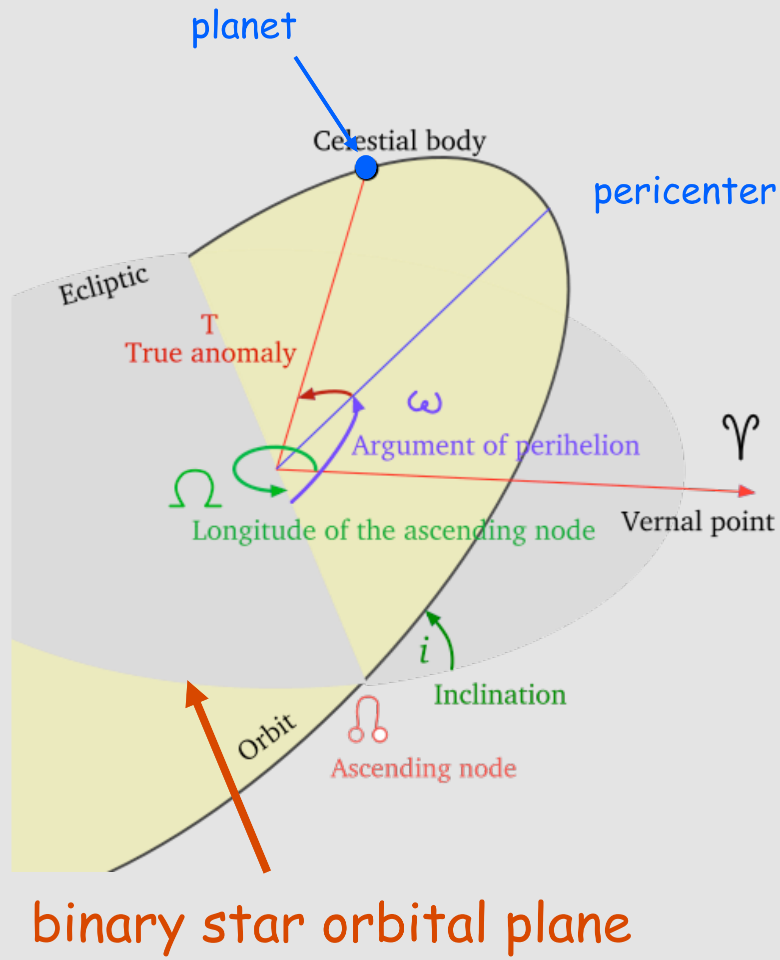
• because the force from the companion star is weak we can average over both planetary and binary star orbits

• keep only the quadrupole term from the companion

• because of averaging the gravitational potential from the companion is fixed, so energy E is conserved ($E=-GM_*/2a$ so semi-major axis a is conserved)

• for circular companion orbit the potential is axisymmetric so J_z is conserved

• accidentally, it turns out that J_z is conserved even if companion orbit is eccentric



Averaged Hamiltonian is

$$H = \epsilon[5e^2 \sin^2 i \sin^2 \omega - (1-e^2)\cos^2 i - 2e^2]$$

where

$$\epsilon \equiv \frac{3GM_c a^2}{8(1-e_c^2)^{3/2} a_c^3}.$$

Action-angle variables are

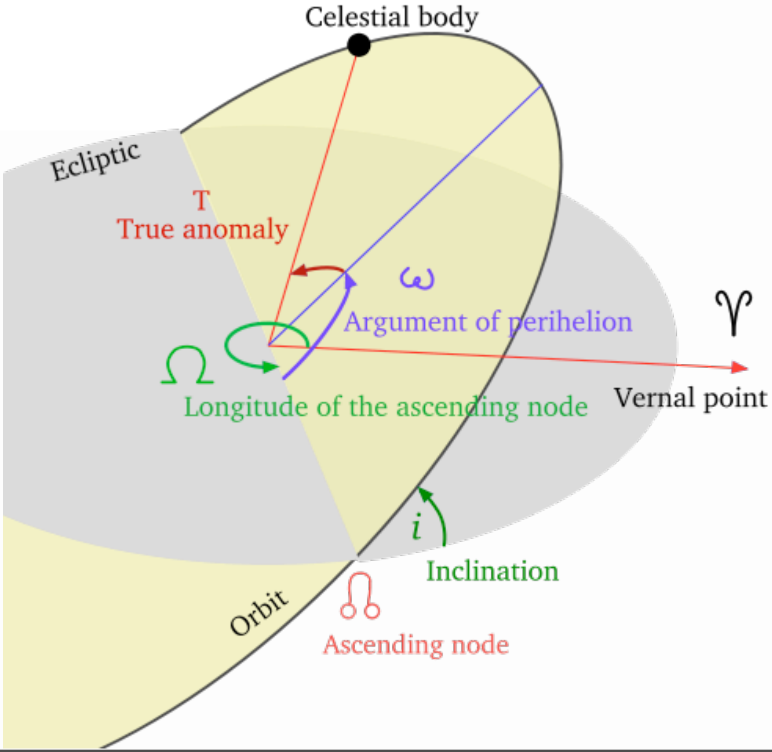
z-angular momentum   longitude of node

$$J_1 = [GM_\star a(1-e^2)]^{1/2}, \quad J_2 = J_1 \cos i, \quad \theta_1 = \omega, \quad \theta_2 = \Omega.$$

angular momentum   argument of pericenter

Hamiltonian is independent of Ω so J_2 is conserved. Remaining motion has one degree of freedom and follows $H =$ constant contours.

$$\frac{dJ_1}{dt} = -\frac{\partial H}{\partial \omega}, \quad \frac{d\theta_1}{dt} = \frac{\partial H}{\partial J_1}.$$



Let $\mathbf{j}$ point in the direction of the angular momentum vector with magnitude $|\mathbf{j}| = (1 - e^2)^{1/2}$. Let $\mathbf{e}$ point towards pericenter with magnitude e. Then

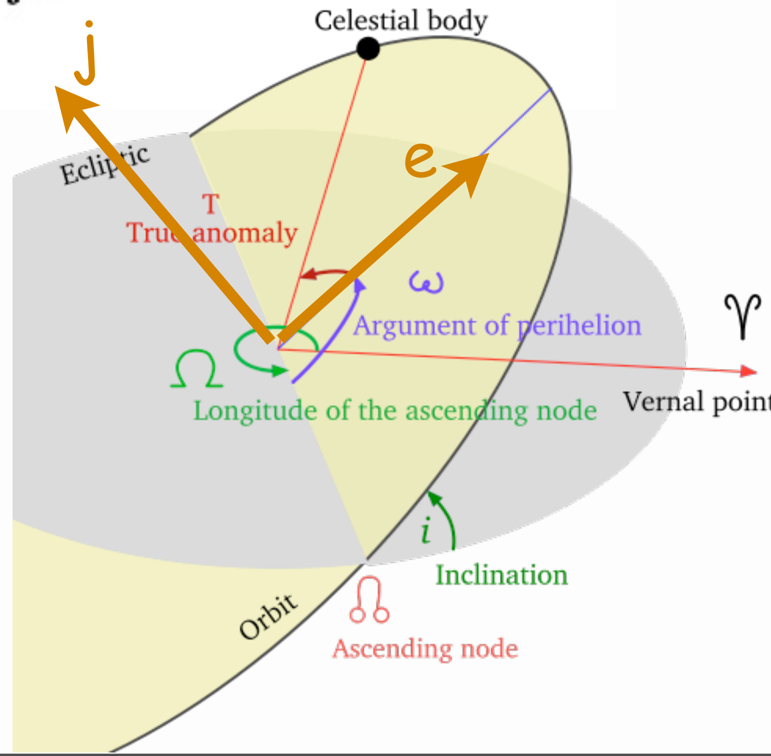
$$H = \epsilon[5(\mathbf{e} \cdot \mathbf{n})^2 - (\mathbf{j} \cdot \mathbf{n})^2 - 2e^2]$$

where $\mathbf{n}$ is the normal to the companion orbit. The equations of motion are

$$\frac{d\mathbf{j}}{d\tau} = \mathbf{e} \times \nabla_{\mathbf{e}} H + \mathbf{j} \times \nabla_{\mathbf{j}} H$$

$$\frac{d\mathbf{e}}{d\tau} = \mathbf{j} \times \nabla_{\mathbf{e}} H + \mathbf{e} \times \nabla_{\mathbf{j}} H$$

where $\tau = t/(GM_\star a)^{1/2}$.



Averaged Hamiltonian is

$$H = \epsilon[5e^2 \sin^2 i \sin^2 \omega - (1 - e^2) \cos^2 i - 2e^2]$$

where

$$\epsilon \equiv \frac{3GM_c a^2}{8(1 - e_c^2)^{3/2} a_c^3}.$$

Action-angle variables are

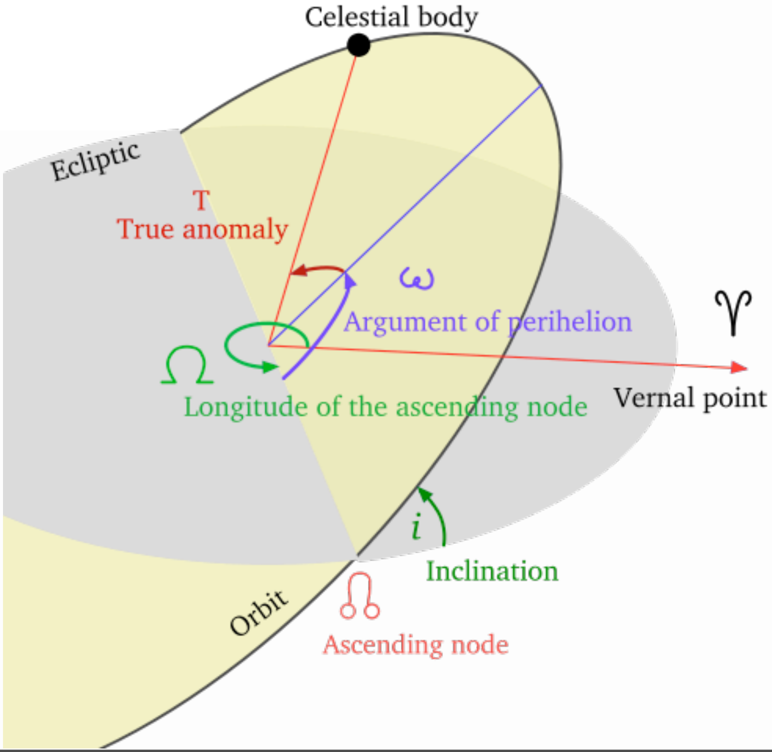
z-angular momentum longitude of node

$$J_1 = [GM_\star a(1 - e^2)]^{1/2}, \quad J_2 = J_1 \cos i, \quad \theta_1 = \omega, \quad \theta_2 = \Omega.$$

angular momentum argument of pericenter

Hamiltonian is independent of Ω so J_2 is conserved. Remaining motion has one degree of freedom and follows $H =$ constant contours.

$$\frac{dJ_1}{dt} = -\frac{\partial H}{\partial \omega}, \quad \frac{d\theta_1}{dt} = \frac{\partial H}{\partial J_1}.$$



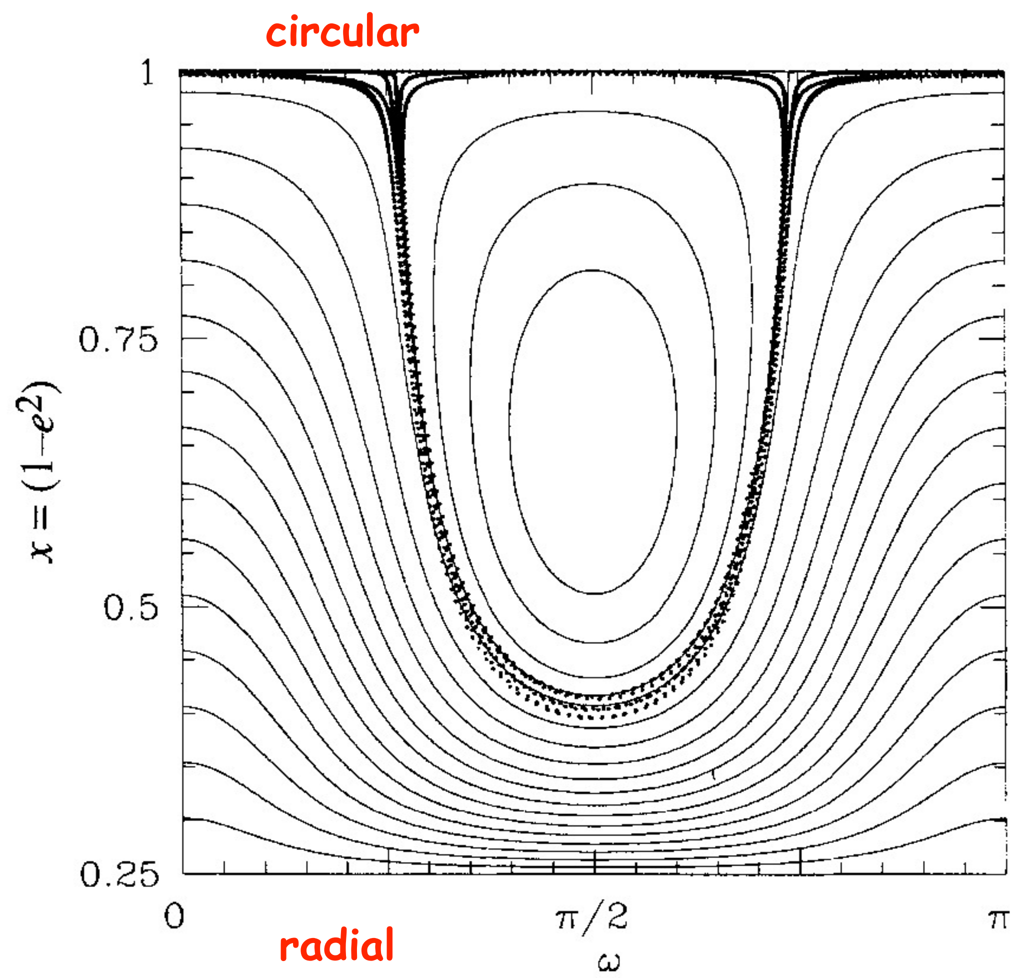
Kozai-Lidov oscillations

• initially circular orbits remain circular if and only if the initial inclination is < $39^o = \cos^{-1}(3/5)^{1/2}$

• for larger initial inclinations the phase plane contains a separatrix

• circular orbits cannot remain circular, and are excited to high inclination and eccentricity -- not a rigid hoop (surprise # 1)

•circular orbits are chaotic (surprise # 2)



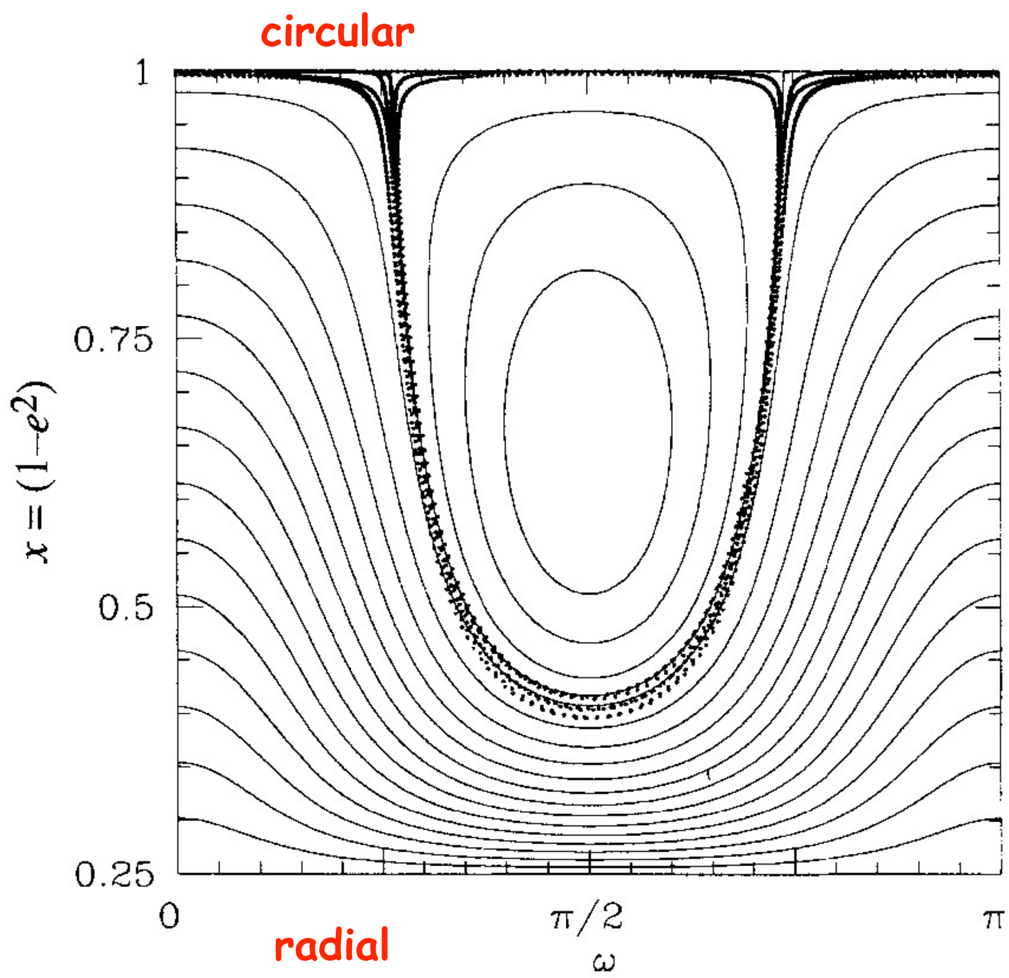
Kozai-Lidov oscillations

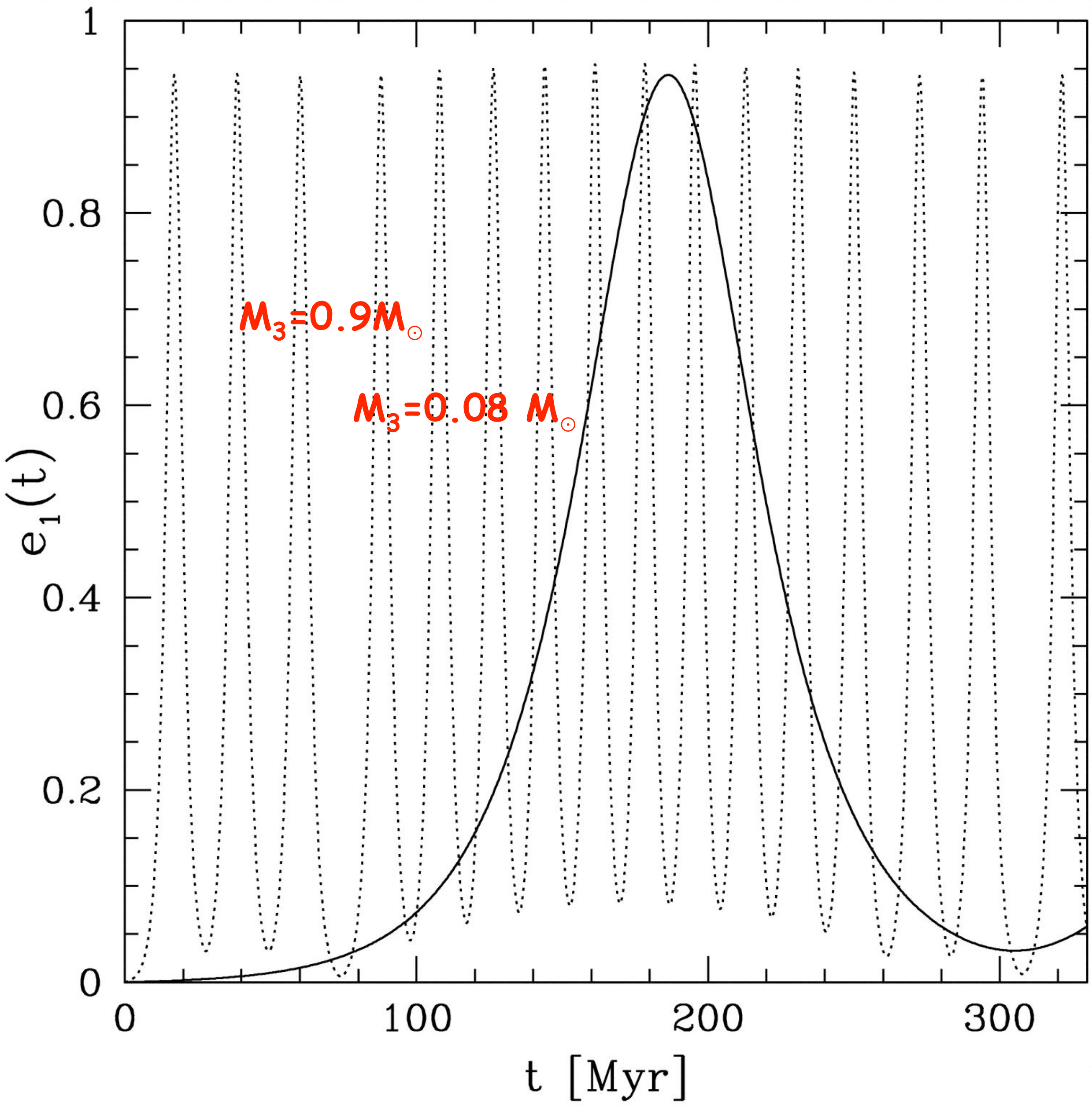
• circular orbits cannot remain circular, and are excited to high inclination and eccentricity (surprise # 1)

•circular orbits are chaotic (surprise # 2)

• as the initial inclination approaches 90˚, the maximum eccentricity achieved in a Kozai oscillation approaches unity ⇒ tidal dissipation or collision (surprise # 3)

• mass and separation of companion affect period of Kozai oscillations, but not the amplitude (surprise # 4)





eccentricity oscillations of a planet in a binary star system

- a_{planet} = 2.5 AU
- companion has inclination 75°, semi-major axis 750 AU, mass 0.08 $M_\odot$ (solid) or 0.9 $M_\odot$ (dotted)

(Takeda & Rasio 2005)

Kozai-Lidov oscillations

- circular orbits are excited to high inclination and eccentricity (surprise # 1)

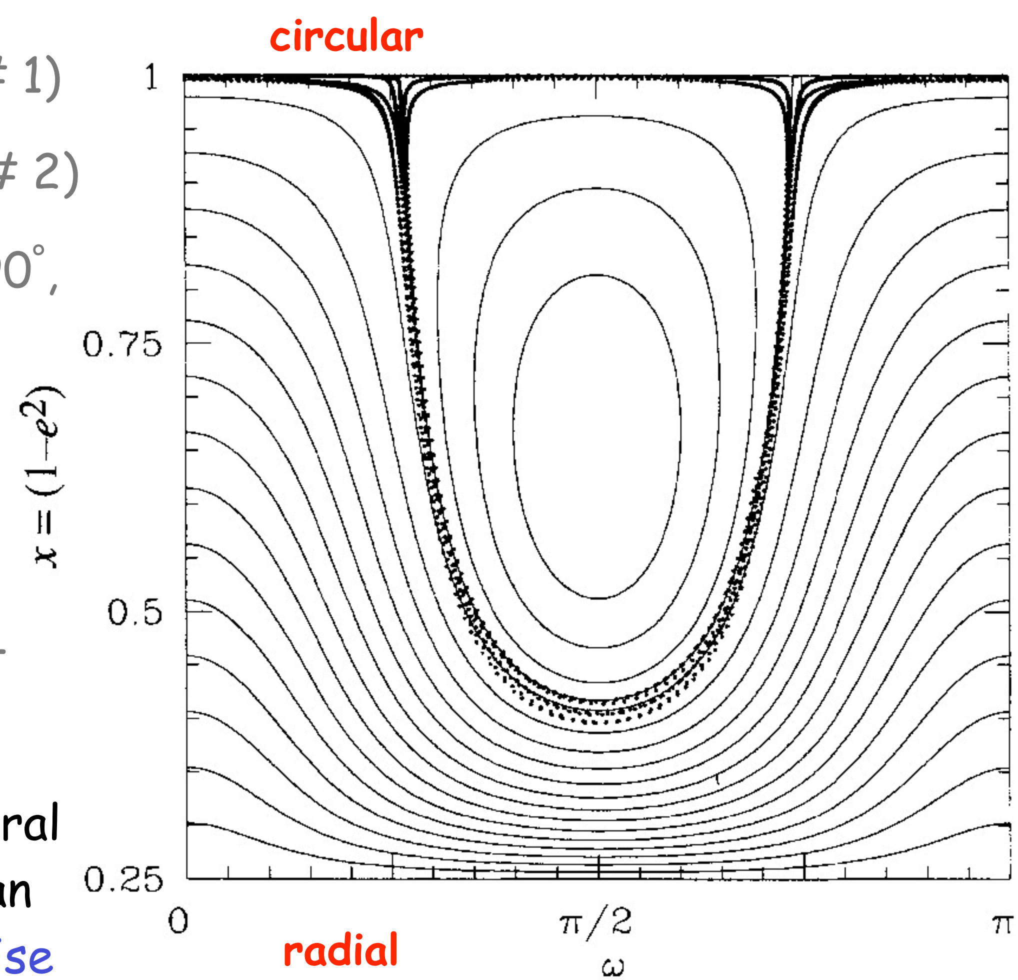
- circular orbits are chaotic (surprise # 2)

- as the initial inclination approaches 90°, the maximum eccentricity approaches unity ⇒ tidal dissipation or collision

(surprise # 3)

- mass and separation of companion affect period of Kozai oscillations, but not the amplitude (surprise # 4)

- small additional effects such as general relativity or octupole tidal potential can strongly affect the oscillations (surprise # 5)



1. Irregular satellites of the giant planets

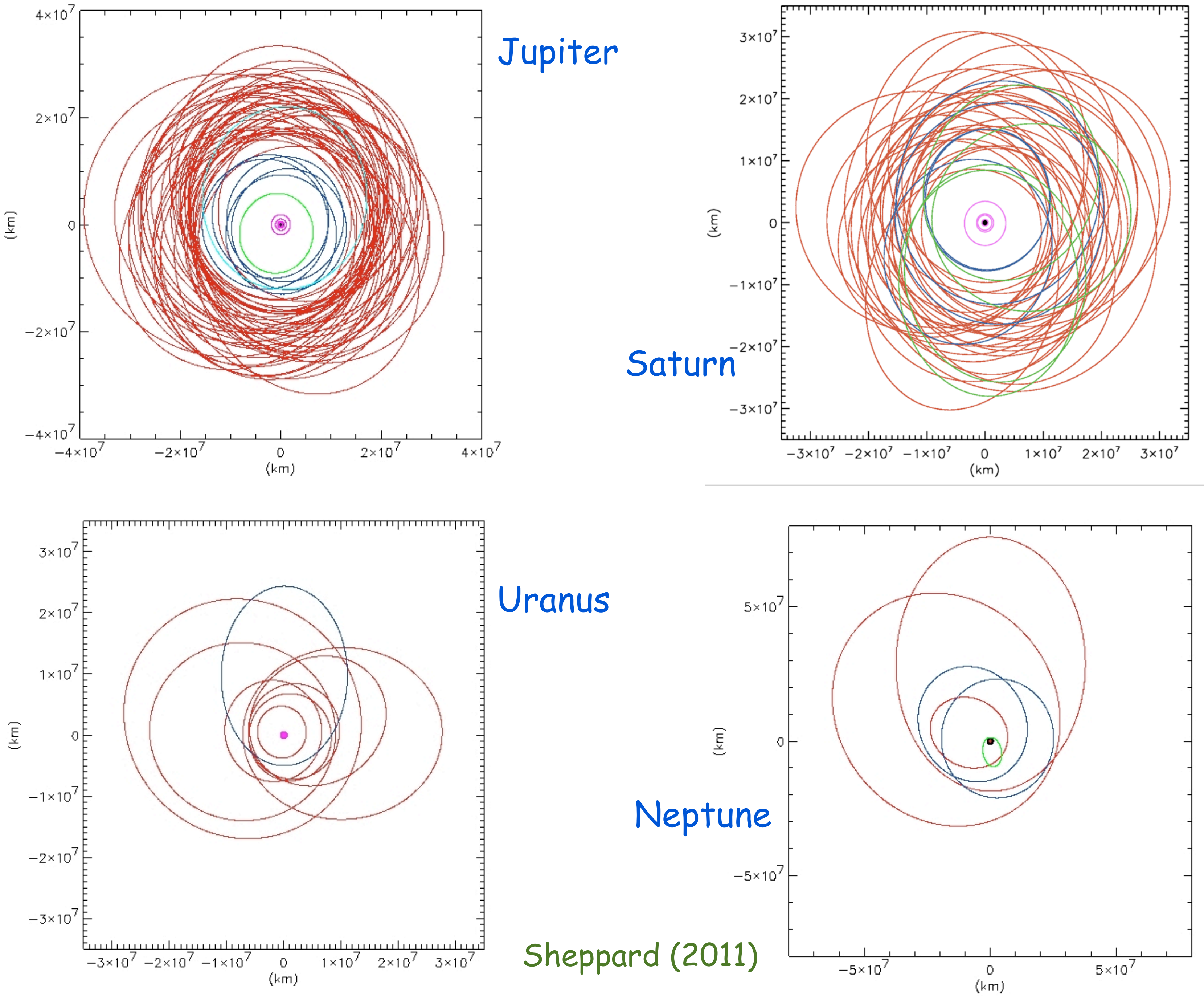
Hill (or tidal, or Roche) radius

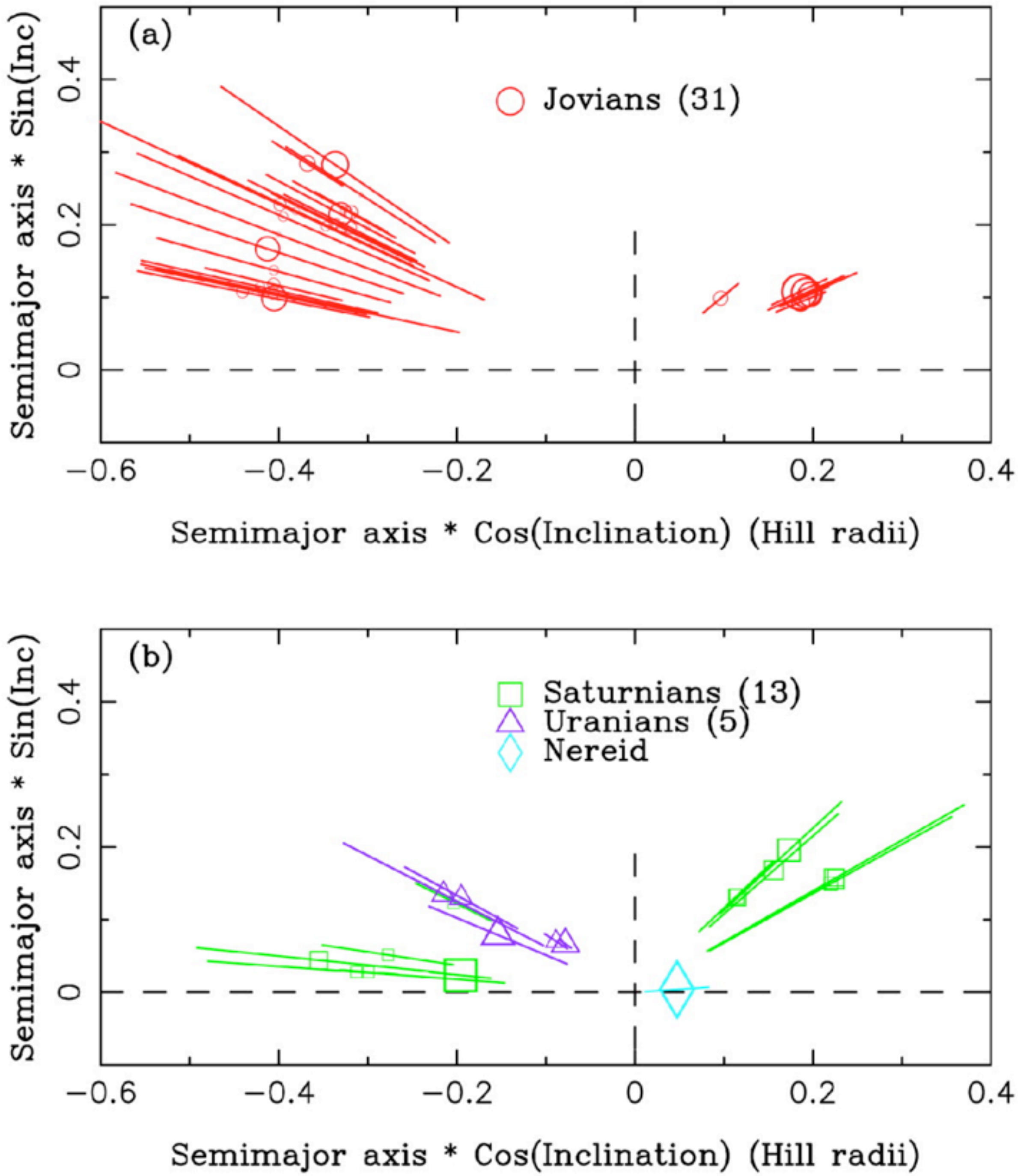
$$r_H = a_p \, (m/3M_\odot)^{1/3}$$

represents approximately the maximum radius at which an orbit stays bound to the planet

- at $r < 0.05r_H$, satellites of the giant planets tend to be on nearly circular, prograde orbits near the planetary equator ("regular" satellites). Probably formed from a protoplanetary disk
- at $r > 0.05r_H$ the satellites have large eccentricities and inclinations, including retrograde orbits (irregular" satellites). Probably captured from heliocentric orbits
- irregular satellites are much smaller than regular ones but there are a lot more of them (97). Total satellite count:

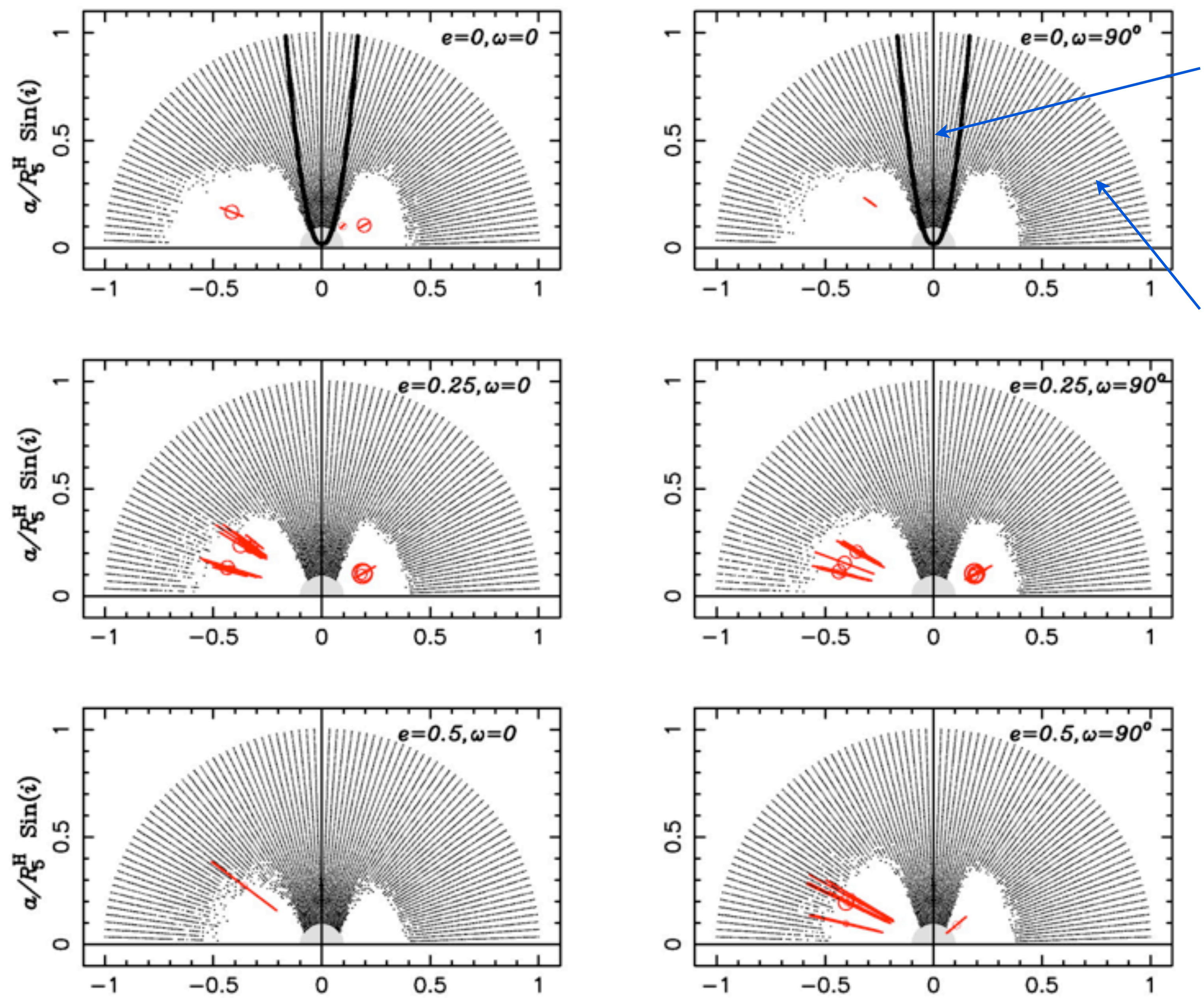
Jupiter: 65 Saturn: 62 Uranus: 27 Neptune: 13


Jupiter
Saturn
Uranus
Neptune
(km)
Sheppard (2011)



there are no irregular satellites with inclinations between 40° and 140°

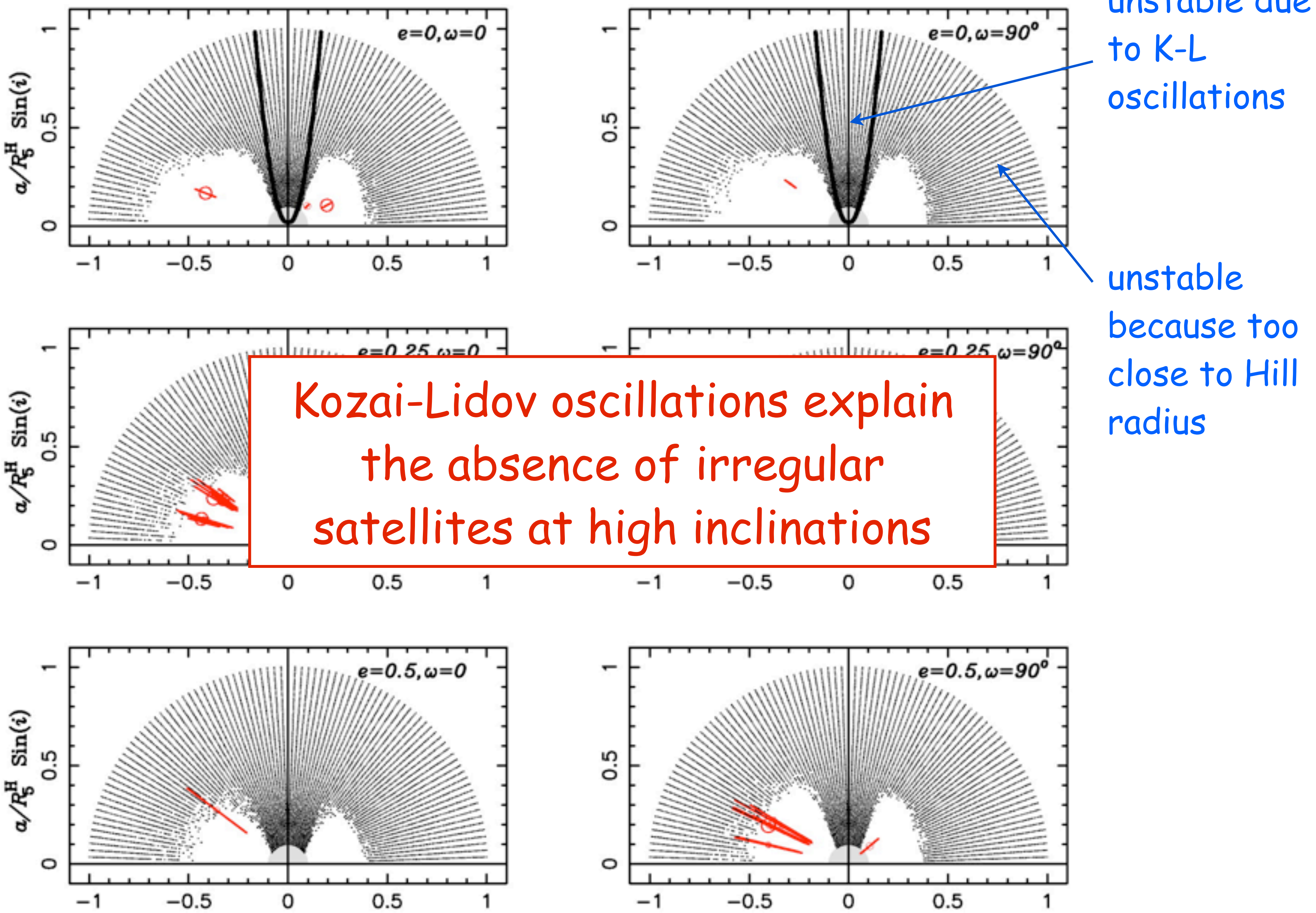
Nesvorny et al. (2003)



unstable due to K-L oscillations

unstable because too close to Hill radius

Nesvorny et al. (2003)



Kozai-Lidov oscillations explain the absence of irregular satellites at high inclinations

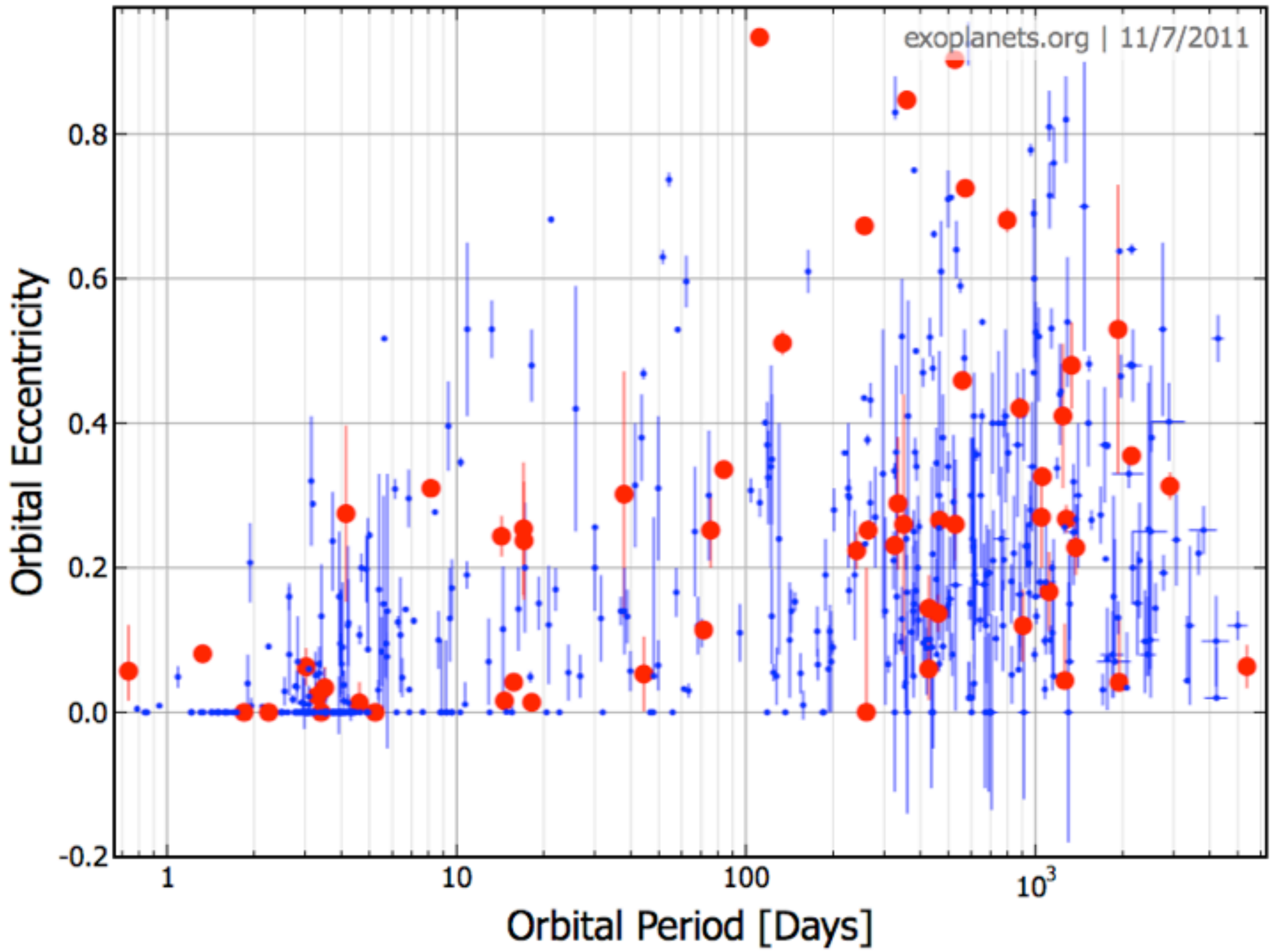
Nesvorny et al. (2003)

2. Exoplanet eccentricities

Kozai-Lidov oscillations may excite eccentricities of planets in **some** binary star systems, but probably not all planet eccentricities:

- not all have stellar companion stars (so far as we know)
- suppressed by additional planets
- suppressed by general relativity (!)

red = binary

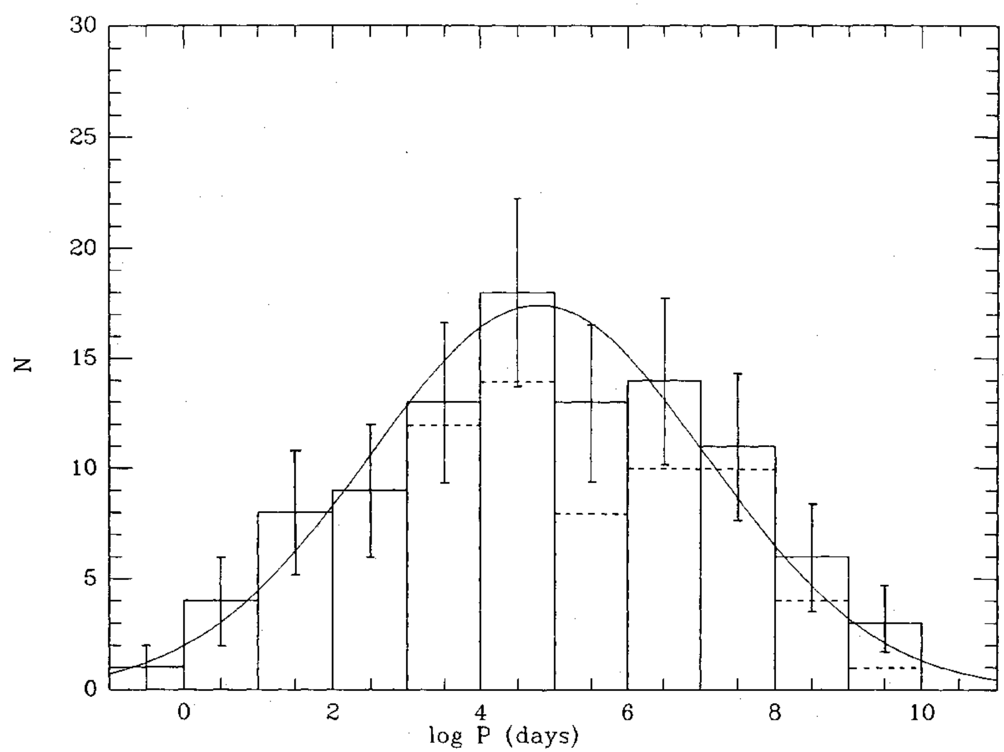


3. Formation of close binary stars

Binary stars are common: roughly 2/3 of nearby stars are in binaries, with a wide distribution of periods:

$$dn \propto \exp\left[-\frac{(\log P/P_0)^2}{2\sigma_P^2}\right] \qquad P_0 = 170\,\mathrm{yr},\ \sigma_P = 2.3$$

(Duquennoy & Mayor 1991)



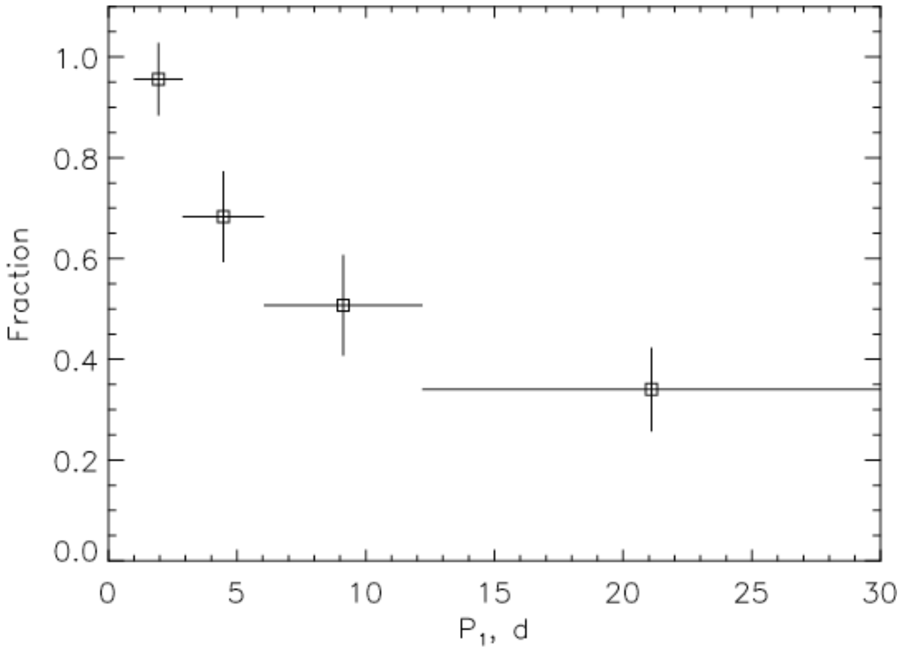
2. Formation of close binary stars

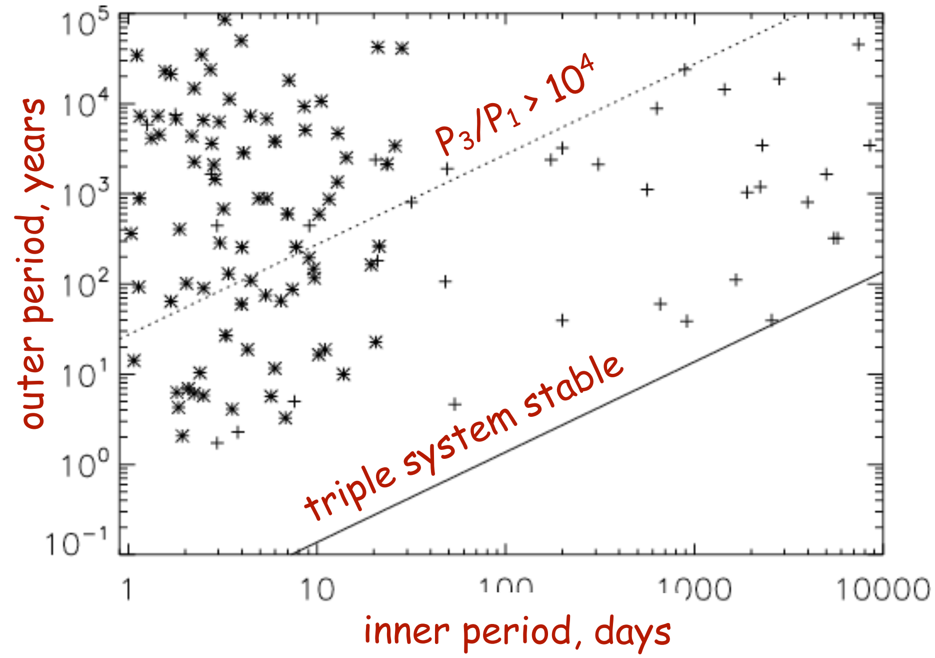
Binary stars are common: roughly 2/3 of nearby stars are in binaries, with a wide distribution of periods:

$$dn \propto \exp\left[-\frac{(\log P/P_0)^2}{2\sigma_P^2}\right] \quad P_0 = 170\,\mathrm{yr},\ \sigma_P = 2.3$$

(Duquennoy & Mayor 1991)

If formation of inner and outer binary in a hierarchical triple star is independent we expect (1) about (2/3)X(2/3)~0.5 of all systems to be triple and (2) characteristics of inner and outer binary to be independent





If formation of inner and outer binary in a hierarchical triple star is independent we expect (1) about (2/3)X(2/3)~0.5 of all systems to be triple and (2) characteristics of inner and outer binary to be independent

This is not true: 96% of binaries with P < 3 d are in triples, but only 34% of binaries with P > 12 d are in triples (Tokovinin et al. 2006)

How can a tertiary companion that is 1000 X further away affect the formation of a binary star?

How do you form a binary with a separation of a few stellar radii when stars shrink by orders of magnitude during their formation?

Formation of close binary stars

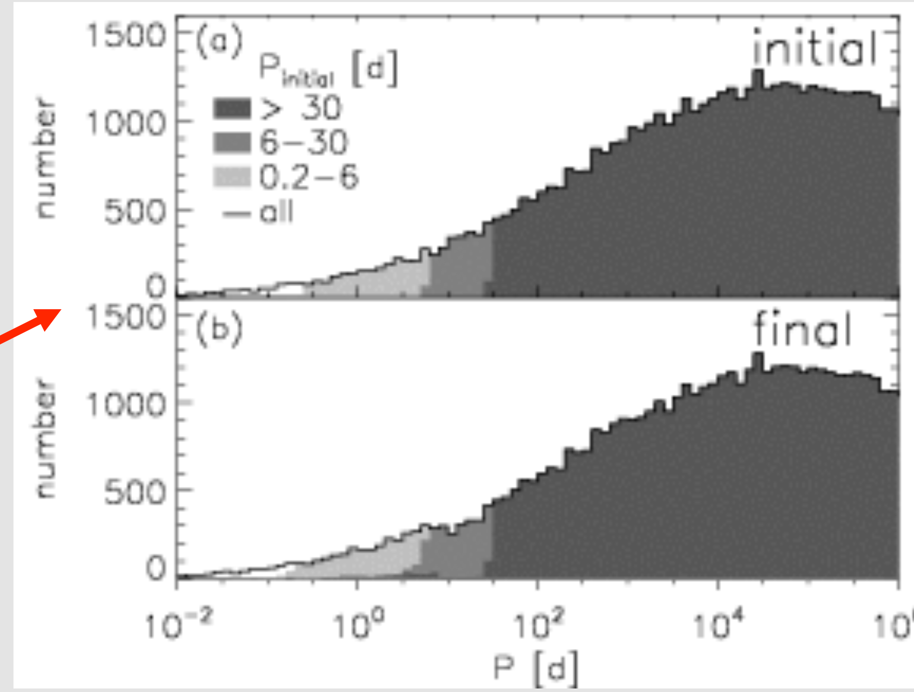
follow orbit evolution of binary or triple star systems, including:

- secular evolution of orbit due to quadrupole tidal field from a tertiary
- apsidal precession due to rotational distortion of stars in the inner binary
- apsidal precession due to mutual tidal distortion of stars in the inner binary
- stellar spins
- tidal friction (Eggleton & Kiseleva-Eggleton 2001)
- relativistic precession

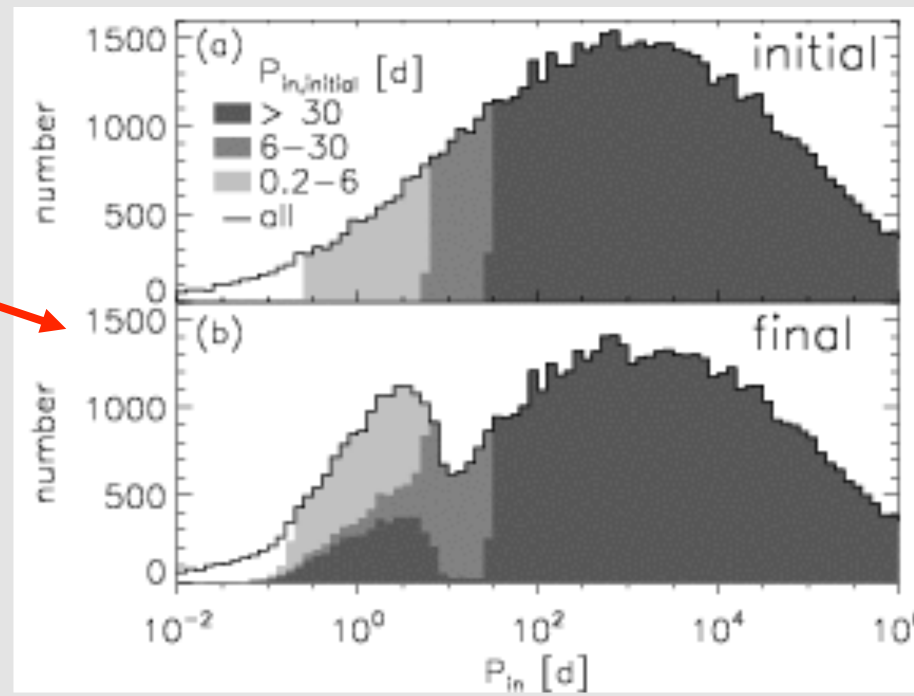
Fabrycky & Tremaine (2007)

Formation of close binary stars

- choose binary stars at random from the Duquennoy & Mayor (1991) distribution, then evolve under tidal friction

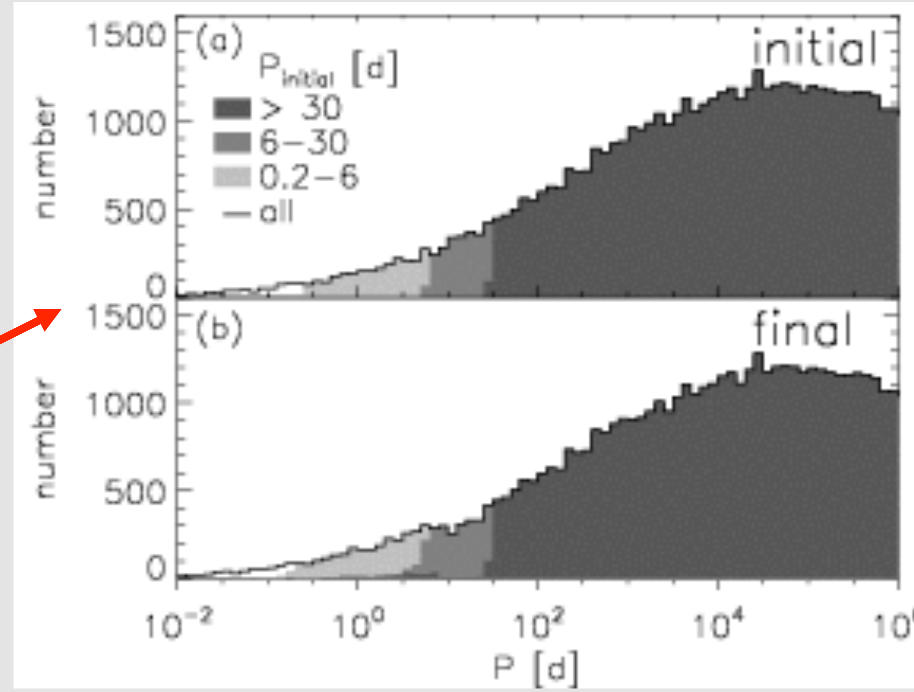


- choose triple stars by sampling twice from the binary-star distribution and discard if unstable, then evolve under tidal friction

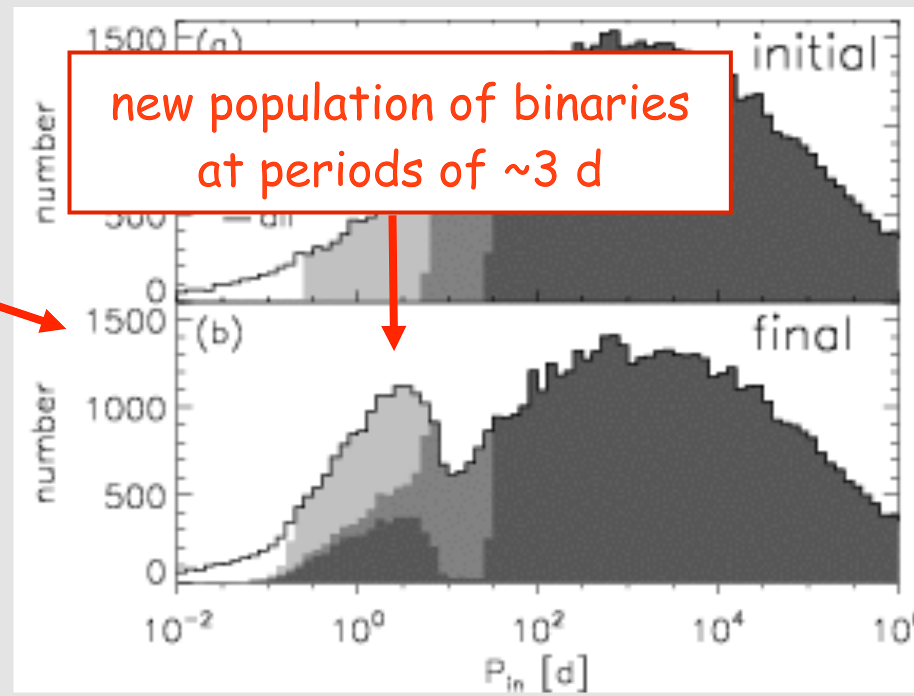


Formation of close binary stars

• choose binary stars at random from the Duquennoy & Mayor (1991) distribution, then evolve under tidal friction

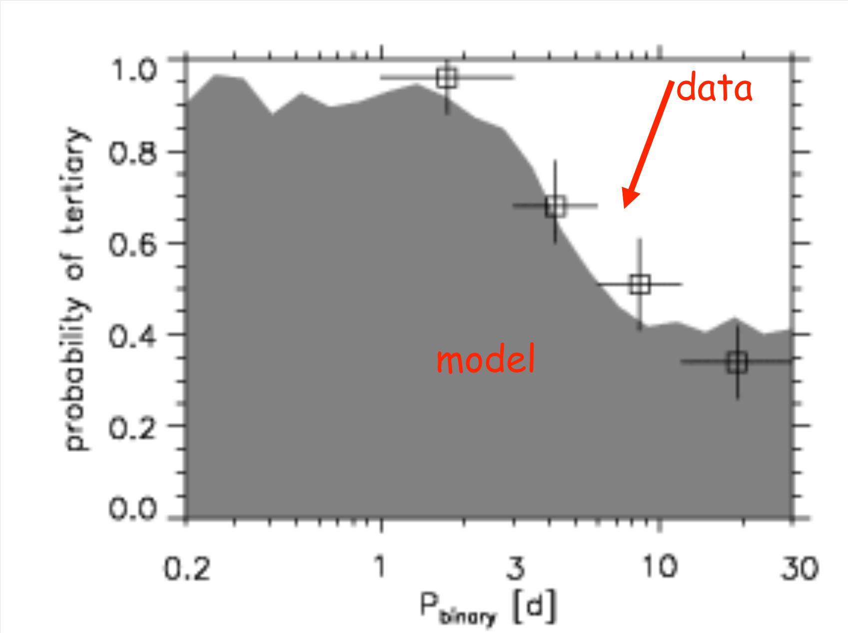


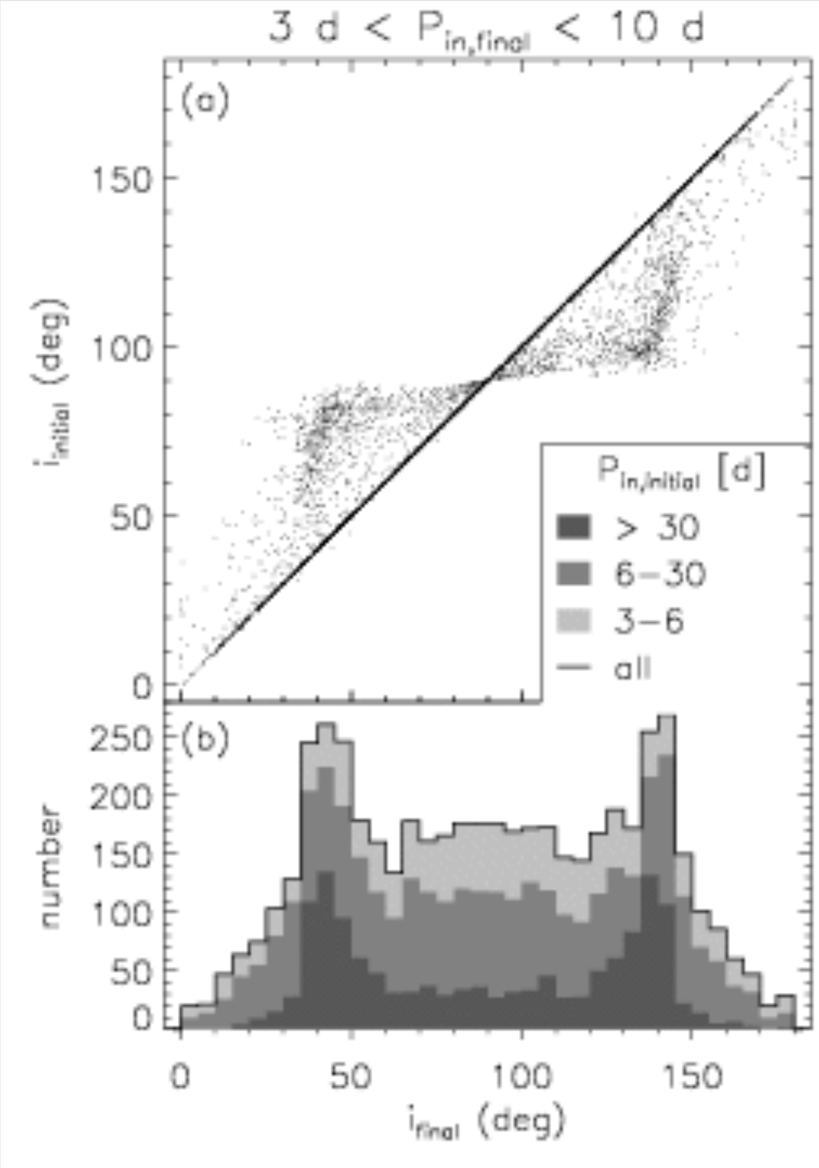
• choose triple stars by sampling twice from the binary-star distribution and discard if unstable, then evolve under tidal friction



• combine the distributions assuming (a) 25% of systems are triple; (b) period distribution is cut off at 6 d (radius of dynamically stable protostars)

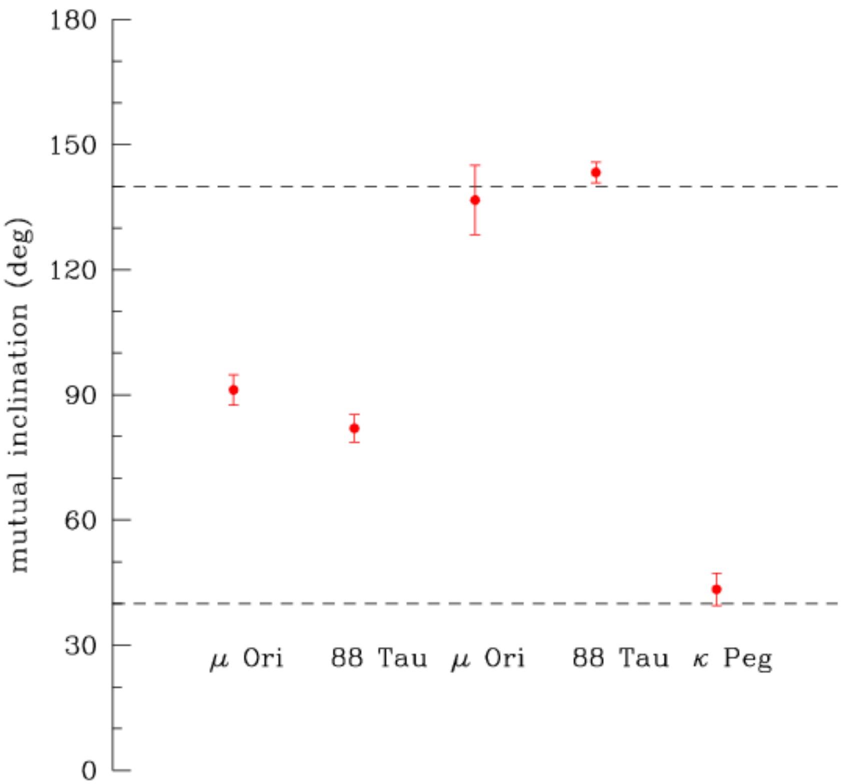
• Kozai-Lidov cycles may be responsible for almost **all** close binary stars

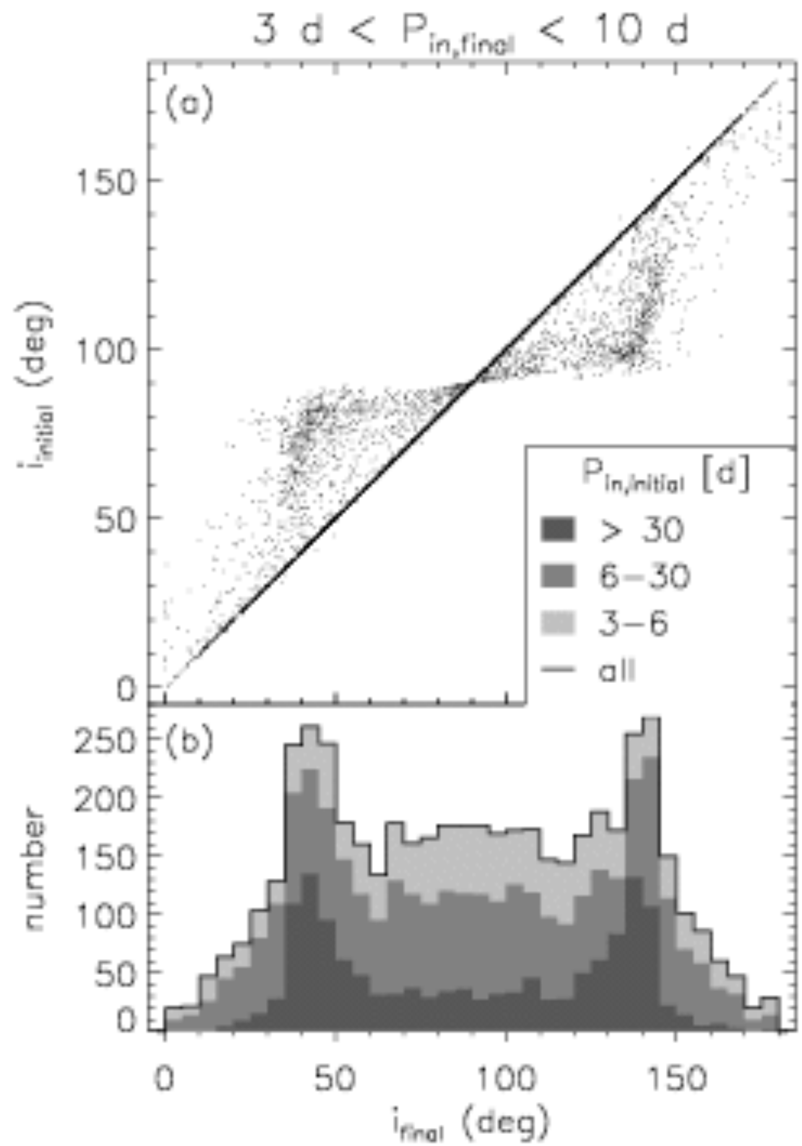




• in this simple model, there is a strong peak near 40 and 140 degrees in the mutual inclinations of systems with 3 d < P_{in} < 10 d

Muterspaugh et al (2007) list five triple systems in this period range



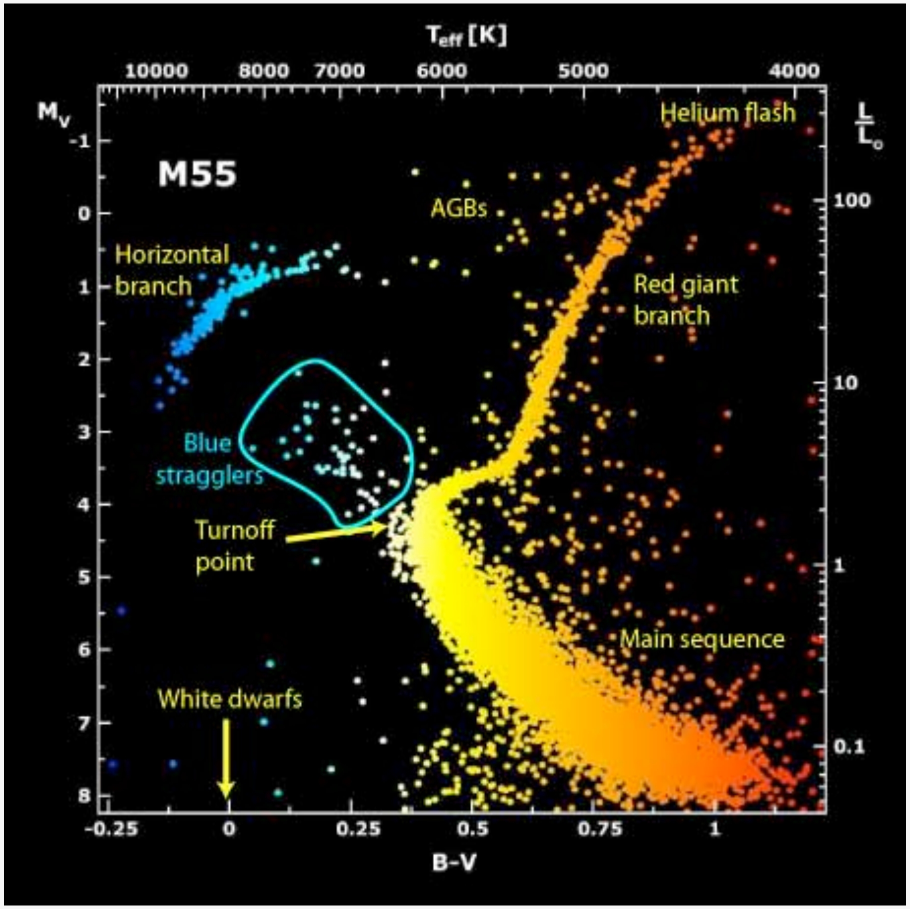


4. Blue stragglers

Blue stragglers are stars in globular clusters that appear to be anomalously young

Possible origins:

- stellar collision and merger
- mass transfer or coalescence in a primordial binary system



4. Blue stragglers

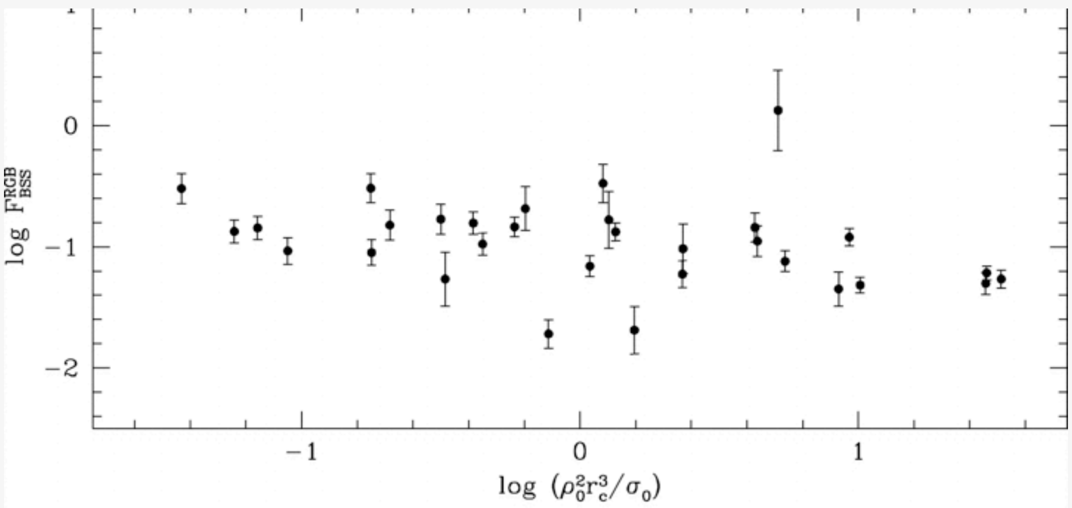
Possible origins:

- stellar collision and merger
- mass transfer or coalescence in a primordial binary system

Problems:

- frequency is not correlated with expected collision rate

(or any other cluster properties)



Leigh et al. (2007)

4. Blue stragglers

Possible origins:

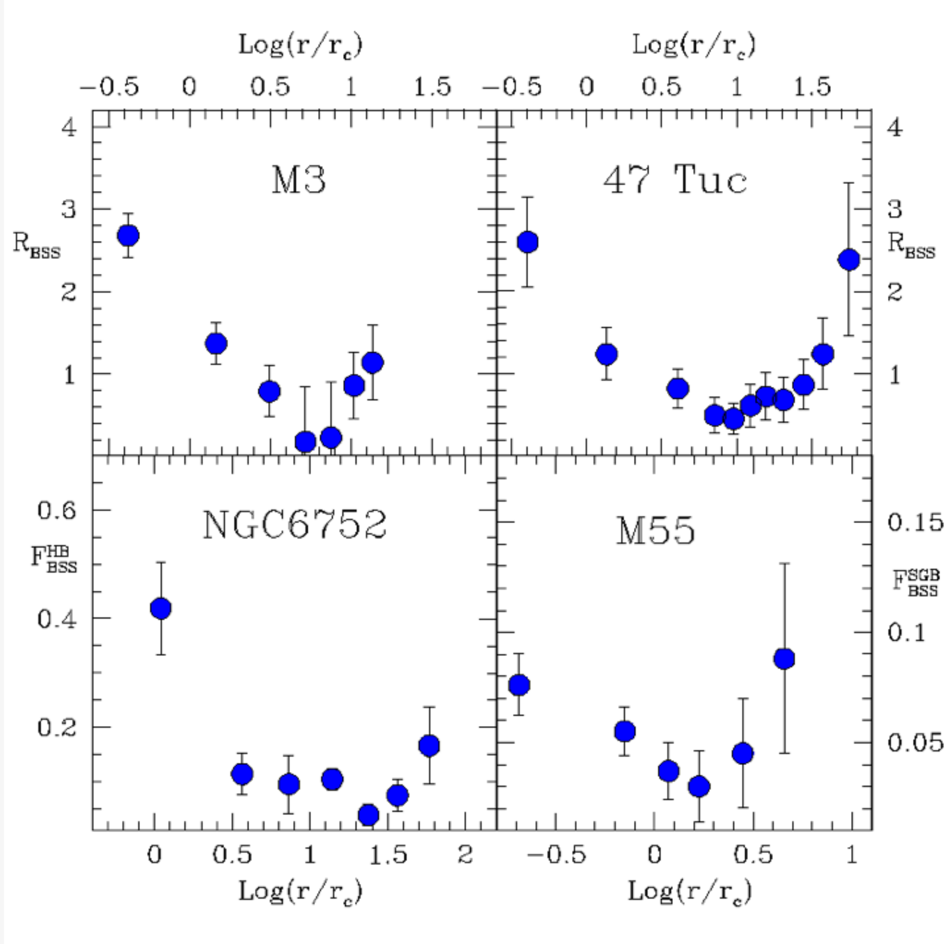
• stellar collision and merger

• mass transfer or coalescence in a primordial binary system

Problems:

• frequency is not correlated with expected collision rate (or any other cluster properties)

• radial distribution is difficult to interpret (maybe both mechanisms operate?)



Ferraro (2005)

4. Blue stragglers

Possible origins:

• stellar collision and merger

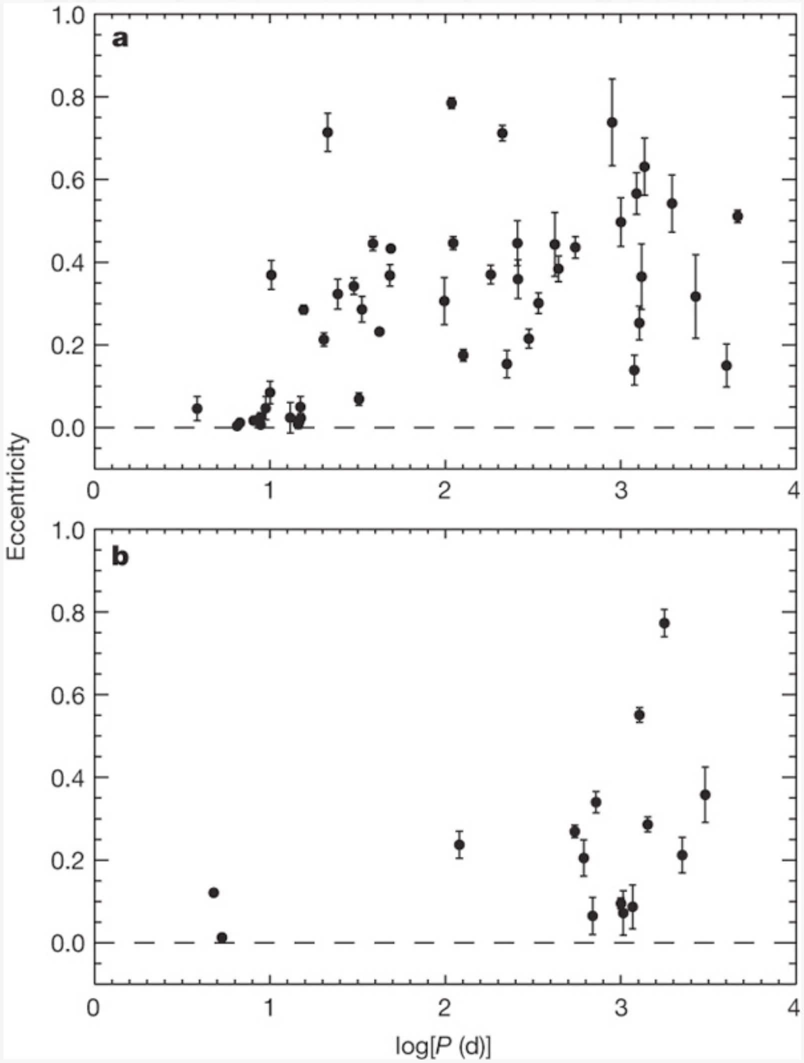
• mass transfer or coalescence in a primordial binary system

Problems:

• frequency is not correlated with expected collision rate

• radial distribution is difficult to interpret

• binary fraction of blue stragglers in NGC 188 is three times that in solar neighborhood

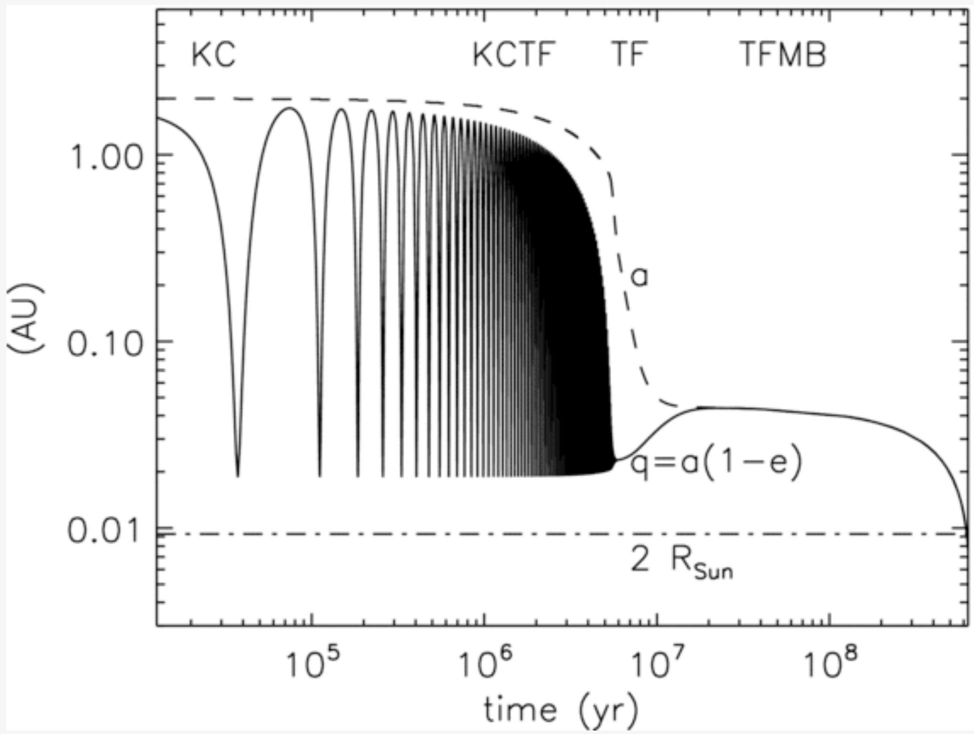


Mathieu & Geller (2009)

4. Blue stragglers

Possible origin:

- stellar collision and merger
- mass transfer or coalescence in a primordial binary system
- Kozai-Lidov oscillations in a triple system leading to merger (Perets & Fabrycky 2009)



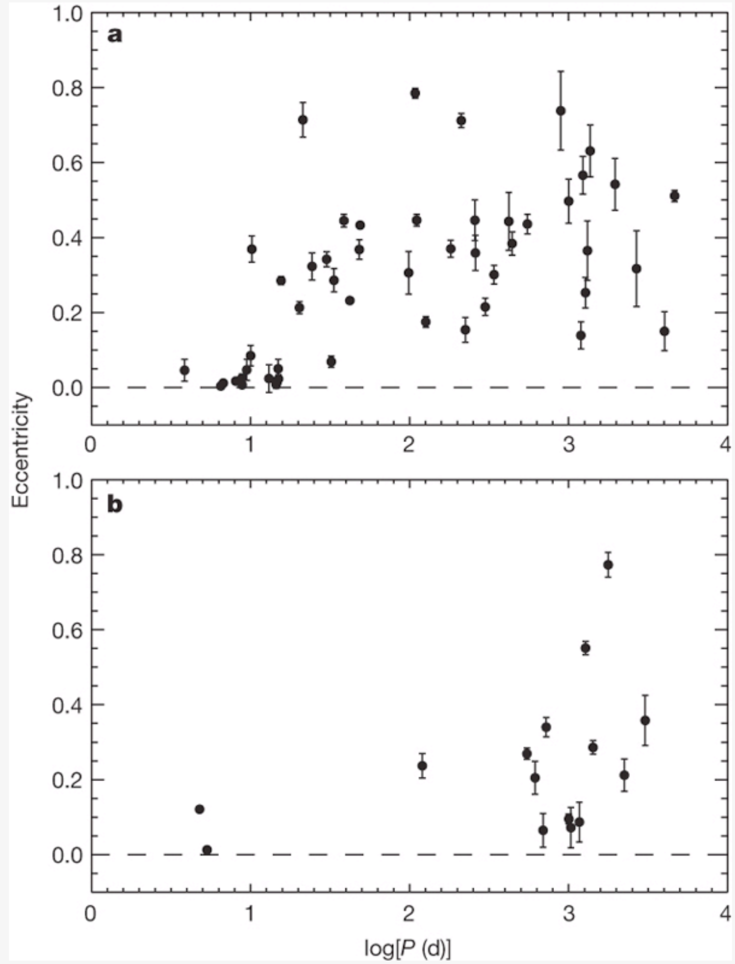
4. Blue stragglers

Possible origin:

• stellar collision and merger

• mass transfer or coalescence in a primordial binary system

• Kozai-Lidov oscillations in a triple system leading to merger (Perets & Fabrycky 2009)



Problems:

• frequency is not correlated with expected collision rate

• radial distribution is difficult to interpret

• binary fraction of blue stragglers in NGC 188 is three times that in solar neighborhood

5. Type Ia supernovae

These arise from white dwarfs that exceed the Chandrasekhar limit, either through

- mass accretion from a main-sequence companion star
- mergers of white dwarf-white dwarf binaries

- if most close binaries are in triples then most SN Ia progenitors are in triples so Kozai-Lidov oscillations will strongly affect rate (Thompson 2011)
- may explain "prompt" Ia supernovae
- predicts periodic gravitational pulses (Gould 2011)
- why have we not found nearby WD-WD binaries? Possible color contamination by main-sequence third body

6. Planetary migration

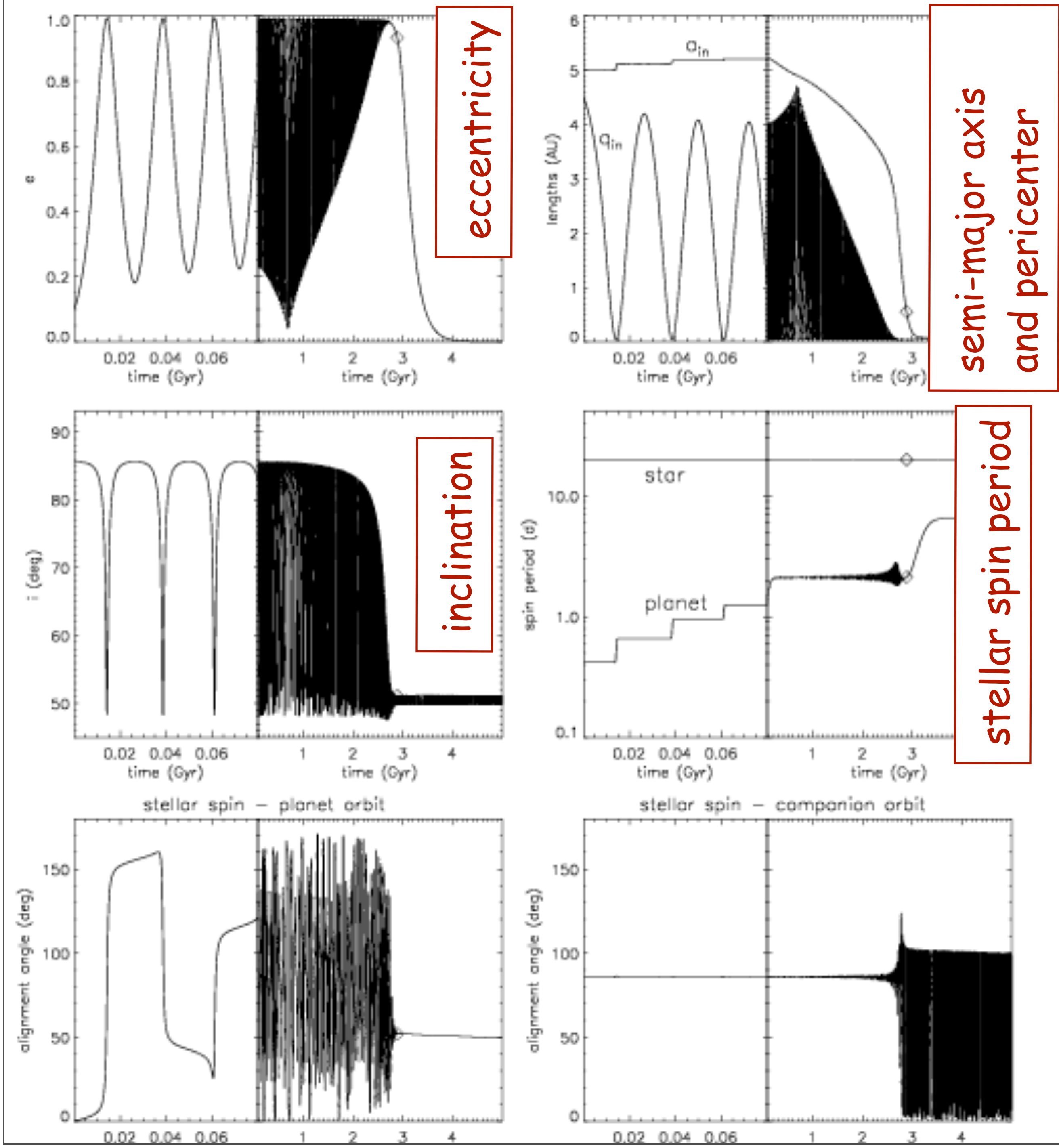
Planet-planet scattering + tidal friction may form hot Jupiters

• suppose scattering leads to an isotropic distribution of velocities

• tidal friction is only important for pericenter $q < 0.02$ AU, so must scatter onto nearly radial orbit. Probability $\sim q/a$

• if Kozai-Lidov oscillations are present angular momentum oscillates but L_z is conserved. Probability of $q < 0.02$ AU at some point in the cycle is $\sim (q/a)^{1/2}$

• Kozai-Lidov oscillations due to outer planets are a critical part of all high-eccentricity migration scenarios

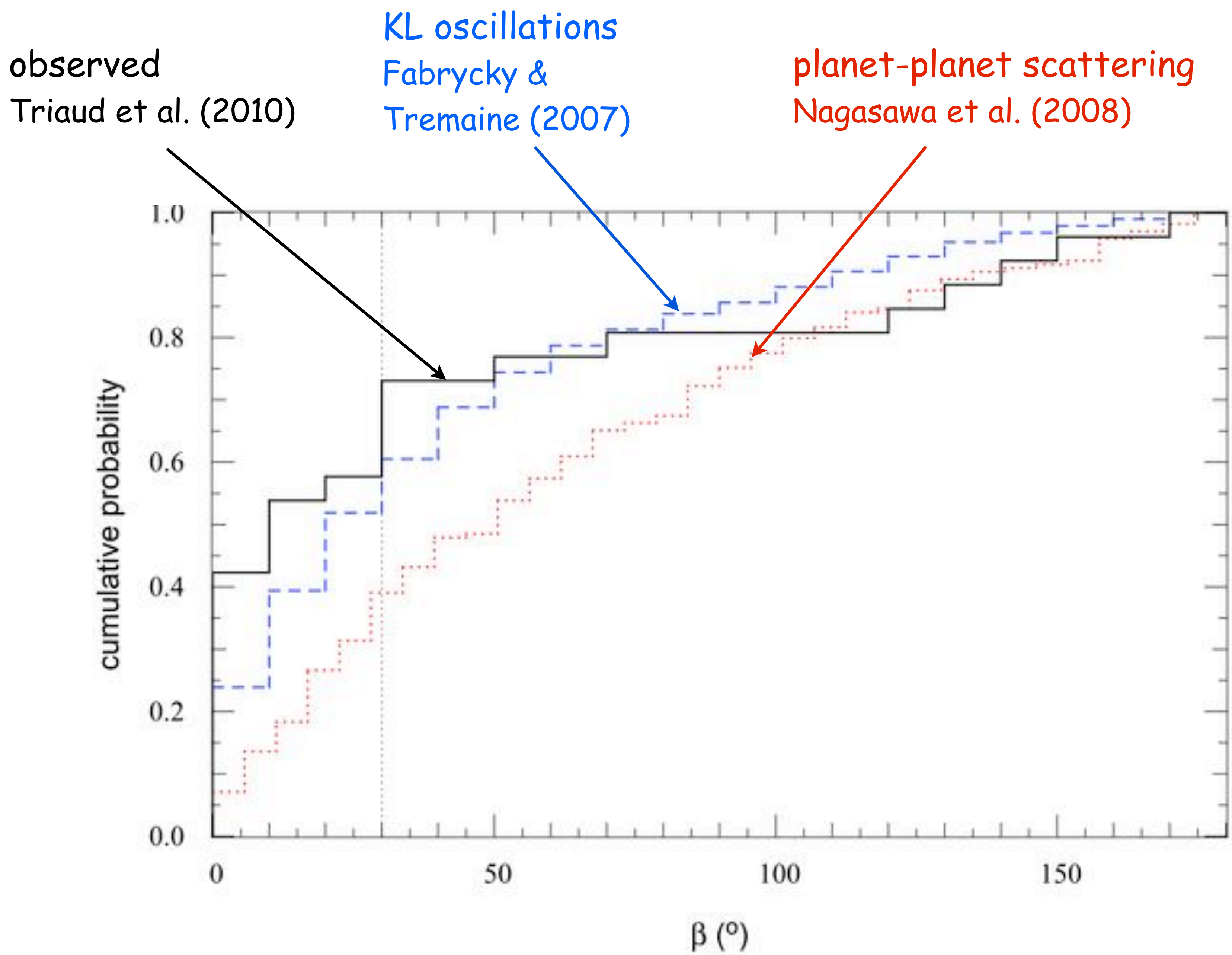


Kozai oscillations with tidal friction in a model of HD 80606b

initial conditions:

- a = 5 AU
- i = 86 ±
- a_{out} = 1000 AU

(Wu & Murray 2003)



distribution of projected obliquities

7. Black-hole mergers

Kozai-Lidov oscillations may accelerate the merger of binary black holes (the "final parsec problem") where external field may come from triaxial galaxy potential or a third black hole (Blaes et al. 2002, Yu 2002, Tanikawa & Umemura 2011)

8. Comets

Kozai-Lidov oscillations induced by the Galactic tidal field drive comets onto orbits that intersect the planetary system

Kozai-Lidov oscillations

- distant satellites of the giant planets have inclinations near 0 or 180^o but not near 90^o
- may excite eccentricities of planets in binary star systems, but probably not all planet eccentricities
- may enhance merger rate of binary black holes in the centers of galaxies
- source of long-period comets
- formation of close binary stars
- formation of blue stragglers
- formation of hot Jupiters
- obliquities of host stars of transiting exoplanets
- Type Ia supernovae, gamma-ray bursts, gravitational wave sources
- **homework:** why do Earth satellites stay up?