



SOME COMMENTS ON MASSIVE NEUTRINOS*

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Abstract

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*Talk presented at First Workshop on Grand Unification
New England Center, University of New Hampshire*

April 10-12, 1980

* Research supported in part by the National Science Foundation under Grant No. PHY77-22864.

** Harvard Society of Fellows.

SOME COMMENTS ON MASSIVE NEUTRINOS

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ABSTRACT

In this talk, the concept of massive Majorana neutrinos is explained; one particular scenario is pointed out by which grand unified theories may lead to "large" neutrino masses; and some astrophysical evidence for a neutrino weighing several tens of electron volts is reviewed.

At this conference we have had ample discussion of the possibility that neutrinos may have small, non-zero rest masses, which are likely to be so-called "Majorana masses". I would like to begin by reviewing the subject of what is a "Majorana mass".

First, let us recall why it has been conventionally believed that the neutrinos are massless. While experiment has long provided good upper bounds on neutrino masses, there is also a standard theoretical argument that the neutrino mass should be zero. This argument is based on the two component theory of the neutrino. It is argued that the neutrino has only one helicity state (left-handed), but a massive spin 1/2 fermion would have two helicity states, so the neutrino must be massless. The claim, in other words, is that the neutrino must be massless if the right-handed neutrino ν_R does not exist.

It has long been recognized that these arguments contain in principle a fallacy, although until recently most physicists doubted that nature really makes use of the fallacy. The fallacy is that in the two component theory of the neutrino, we have actually two helicity states, not one. There are *left-handed* neutrinos and *right-handed* antineutrinos. Two helicity states are the right number for a massive spin 1/2 fermion, so why can't we combine the negative helicity ν and positive helicity $\bar{\nu}$ into a massive fermion?

The answer is that lepton number conservation makes it impossible to combine ν and $\bar{\nu}$ into a massive fermion. The neutrino is a lepton, with $L = +1$; the antineutrino is an anti-lepton, with $L = -1$. The two helicity states of a massive fermion must have the same lepton number (*if lepton number is a symmetry!*), because rotations and boosts exchange the two helicity states. So the different lepton numbers of ν and $\bar{\nu}$ (and only that) prevents us from making a massive fermion out of ν and $\bar{\nu}$.

If lepton number is not conserved, we *can* combine them.

To see how this works mathematically, let us recall¹ that the Lie algebra of the Lorentz group $O(3,1)$ can be decomposed as $SU(2) \times SU(2)$:

$$O(3,1) \simeq SU(2) \times SU(2). \quad (1)$$

Since the representations of $SU(2)$ are labeled by an integer or half-integer, the representations of $SU(2) \times SU(2)$ or of $O(3,1)$ are labeled by a pair of numbers (p,q) which are each integers or half-integers.

The usual neutrino field ν_L and its Dirac adjoint $\bar{\nu}_L$ transform as follows:

$$\nu_L \left(\frac{1}{2}, 0 \right) \quad \bar{\nu}_L \left(0, \frac{1}{2} \right) \quad (2)$$

$\bar{\nu}_L$ transforms oppositely to ν_L because the complex conjugation which is involved in going from ν_L to $\bar{\nu}_L$ exchanges the two factors of $SU(2)$ in Eq. (1) and so exchanges p and q . (This in turn is because of some factors of i which must be introduced in relating the $O(3,1)$ Lie algebra to $SU(2) \times SU(2)$.)

If the opposite helicity fields ν_R and $\bar{\nu}_R$ existed (they apparently don't, at least not in ordinary particle phenomenology), they would transform as follows:

$$\nu_R \left(0, \frac{1}{2} \right) \quad \bar{\nu}_R \left(\frac{1}{2}, 0 \right). \quad (3)$$

Now, what is a fermion mass term? Fermions (of spin $1/2$) always transform as $(1/2, 0)$ or $(0, 1/2)$, and a mass term always combines two fermi fields of the same type. If one multiplies two fermi fields of the same type, let us say both of type $(1/2, 0)$, the decomposition is

$$\left(\frac{1}{2}, 0 \right) \times \left(\frac{1}{2}, 0 \right) = \left(0, 0 \right) + \left(1, 0 \right), \quad (4)$$

and the $(0,0)$ piece is a Lorentz invariant which can appear in the Lagrangian. (By contrast, combining fermi fields of opposite type gives

$$\left(\frac{1}{2}, 0 \right) \times \left(0, \frac{1}{2} \right) = \left(\frac{1}{2}, \frac{1}{2} \right);$$

there is no Lorentz invariant component which could be a mass term. So a mass term always combines fields of the same type.)

Let us now return to our neutrino fields which transform as indicated in Eqs (2) and (3). If the right-handed neutrino existed, we could take ν_L and $\bar{\nu}_R$, both of them transforming as $(1/2, 0)$, and form the usual Dirac mass term

$$\bar{\nu}_R \nu_L, \quad (5)$$

which combines the two fields as

$$\left(\frac{1}{2}, 0 \right) \times \left(\frac{1}{2}, 0 \right) = \left(0, 0 \right) + \dots$$

This Dirac mass term is obviously invariant under the "lepton number" transformation

$$\begin{aligned} \nu_L &\rightarrow e^{i\alpha} \nu_L \\ \bar{\nu}_R &\rightarrow e^{-i\alpha} \bar{\nu}_R. \end{aligned} \quad (6)$$

Since ν_R does not exist, the only $(1/2, 0)$ field at our disposal is ν_L , and to write a mass term we must write something bilinear in ν_L :

$$\nu_L \nu_L. \quad (7)$$

This is obviously not invariant under the lepton number transformation $\nu_L \rightarrow e^{i\alpha} \nu_L$. It leads to the lepton number violating Lagrangian

$$\mathcal{L} = \int dx \bar{\nu}_L i \not{\partial} \nu_L - \left(\frac{m}{2} \nu_L \nu_L + \text{h.c.} \right), \quad (8)$$

which is the Lagrangian for a massive Majorana neutrino. (The Dirac algebra in Eqs. (7) and (8) will be commented on later.)

The mass term (7) is known as a Majorana mass term; a Majorana mass is simply a mass which violates a lepton or fermion number conservation law.

To clarify the physical content of Lagrangian (8), this Lagrangian describes a massive *neutral* fermion--neutral in the sense that the particle is its own antiparticle. It is obvious that the particle described by (8) must be identical with its own antiparticle, because otherwise we would need four helicity states, two for the particle and two for the antiparticle, but if ν_R does not exist we have only two helicity states at our disposal.

We are quite acquainted in physics with particles which are identical with their own antiparticles--for example, the neutral pi meson π^0 . The only novelty is that we do not usually deal with *fermions* which are their own antiparticles (or which, differently put, do not carry conserved additive quantum numbers).

Roughly speaking, the Majorana neutrino is to the Dirac electron as the neutral π^0 is to the charged π^+ . The Majorana neutrino is described by a two component field and the electron by a four component field. The electron field has twice as many components as the neutrino field, and correspondingly it describes *two* massive spin 1/2 particles, e^+ and e^- , while the Majorana neutrino describes a single neutral massive fermion. Likewise, the neutral π^0 can be described by a real scalar field while the charged π^+ requires a complex scalar field. The complex field has twice as many degrees of freedom as the real field, and describes two states, π^\pm , while the real field describes only one, the π^0 .

If from the beginning of the development of quantum field theory, fermions that do not carry conserved additive quantum numbers had been known, then Majorana fermions would probably be as familiar to us as neutral π^0 's.

(Some further points about the Majorana mass term (7) should be clarified. To form (0,0) from (1/2,0) x (1/2,0), one combines the two fields *antisymmetrically* since only the *antisymmetric* combination of spin 1/2 and spin 1/2 makes spin 0. Therefore the two fields ν_L in (7) or (8) are combined antisymmetrically with respect to their spinor indices. But fermi fields anticommute and should be combined antisymmetrically, so the Majorana mass term is in fact consistent with Fermi statistics. By " $\nu_L \nu_L$ " in Eq. (7) is meant the following. The field ν_L is a two component spinor field ν_L^α , $\alpha = 1, 2$. By " $\nu_L \nu_L$ " we mean the antisymmetric combination of the two fields, which transforms as (0,0). Introducing the two index antisymmetric tensor $\epsilon_{\alpha\beta}$, $\epsilon_{12} = +1$, the antisymmetric combination can be written more explicitly as

$$v_L^\alpha v_L^\beta = 1/2 \epsilon_{\alpha\beta} v_L^\alpha v_L^\beta .)$$

The conclusion is that, despite the two component nature of the neutrino, the neutrino may have a mass--if lepton number is not conserved.

As is well known, in grand unified theories², lepton number is generally not conserved, for the same reason that baryon number is generally not conserved. Grand unified theories generally combine quarks and antiquarks, leptons and antileptons, into the same representation of a gauge group. The bosons of the unified group mediate transitions among the various states, and thus mediate violations of the various quantum numbers. Because of the lepton number violation that is introduced by grand unification, unified theories will generically have non-zero neutrino masses. (The major exception is the minimal SU(5) theory, in which neutrino masses are prevented by B-L conservation.) Moreover, this subject is lent some importance by the fact that neutrino masses are by far the most sensitive way to search for lepton number violation of the sort that unified theories suggest.

On the scale of grand unification, SU(2) x U(1) is a very good symmetry^{3,4}, and this leads to a simple estimate of the scale of neutrino masses that should be expected. (Much of the discussion below follows Ref. (4).) The simple Majorana mass term $v_L v_L$ that we have discussed above is not SU(2) x U(1) invariant, because the neutrino field v_L is an SU(2) x U(1) nonsinglet. To form a gauge invariant expression, one must introduce the Weinberg-Salam doublet

$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

and replace the neutrino field v_L by the gauge invariant form $(\phi^0 v_L - \phi^+ e_L^-)$. This is a gauge invariant version of v_L because, after symmetry breaking, $(\phi^0 v_L - \phi^+ e_L^-) = \langle \phi \rangle v_L + \dots$. The gauge invariant version of $v_L v_L$ is then⁴

$$(\phi^0 v_L - \phi^+ e_L^-)(\phi^0 v_L - \phi^+ e_L^-) = \langle \phi^0 \rangle^2 v_L v_L + \dots \quad (9)$$

This operator has dimension five, so it is a nonrenormalizable interaction and will not be present in the fundamental Lagrangian. However, it may be induced as an effective interaction by the exchange of very massive particles with lepton-number violating couplings. ((9) will in fact then describe the dominant lepton number violation at low energies, because it is the lowest dimension lepton number nonconserving operator that can be formed from the usual particle fields.)

If the operator (9) does appear in the effective Lagrangian, then, on dimensional grounds, it will appear with a coupling constant that has dimensions of inverse mass. The mass in question will presumably be related to the mass scale of grand unification since we expect that the effective interaction (9) will be induced by diagrams with exchange of superheavy particles. So let us parametrize the coefficient of the operator (9) as a

dimensionless constant f , which will depend on the model, divided by the grand unified mass scale M :

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= \frac{f}{M} (\phi^0_{\nu_L} - \phi^+_{e_L}) (\phi^0_{\nu_L} - \phi^+_{e_L}) \\ &= \frac{f}{M} \langle \phi^0 \rangle^2 \nu_L \nu_L + \dots \end{aligned} \quad (10)$$

With this definition the neutrino mass is

$$m_\nu = \frac{f}{M} \langle \phi^0 \rangle^2. \quad (11)$$

With $\langle \phi^0 \rangle = 300$ GeV and M equal to the usual unification scale of 10^{15} GeV, this is approximately

$$m_\nu = (.1 \text{ eV}) f. \quad (12)$$

The value of f is extremely model dependent and depends on the nature of diagrams which are assumed to generate the effective action (10). A particularly simple possibility is that this effective interaction may be generated by a tree diagram with an exchange of a superheavy fermion (figure (1)). This possibility was considered by Gell-Mann, Ramond, and Slansky in work that stimulated much of the current interest in neutrino masses.⁵ (A similar mechanism in an $SU(2) \times SU(2) \times U(1)$ model has been given by Mohapatra and Senjanovic.)⁶ If no suitable heavy fermion exists or if the Yukawa couplings in figure (1) are extremely small, one might consider instead a loop diagram, such as the diagram of figure (2) (suggested by Weinberg).

Although f is extremely model dependent, in many models f will be much less than one, perhaps of order 10^{-3} or 10^{-4} . For example, from figure (1), we would get $f = \lambda^2$, the square of the Yukawa coupling constant. Yukawa couplings, of course, seem to be rather small. From figure (2), we would get f of order α^2 . If f is as small as one might guess from looking at figures (1) and (2), then neutrino masses are $\lesssim 10^{-4}$ eV and are too small to be detected except in mixing experiments in which the path length is the radius of the earth (as in an experiment described at this conference by Lo Secco) or the earth-sun separation (as in the Davis solar neutrino experiment, which, of course, may have already provided evidence for neutrino mixing). Also--to anticipate ourselves a bit--if f is equal to or less than one, then neutrino masses are much the small to play a role in cosmology.

Many scenarios might lead to neutrino masses larger than the above pessimistic estimates. I will here just point out one simple possibility.

If the heavy fermion which is exchanged in figure (1) is much lighter than the other superheavy particles, then the neutrino masses will be enhanced, since the neutrino mass from figure (1) is $\lambda^2 \langle \phi \rangle^2$ divided by the mass of the heavy fermion. In models in which the heavy fermion mass is a free parameter, we can make the neutrino masses as large as we wish by choosing the heavy fermion light enough. However, the procedure is not very

natural and there is no predictive power.

It may happen, however, that the heavy fermion of figure (1) is naturally massless at the tree level and receives its mass from a loop correction, proportional to the other superheavy masses. In this case the heavy fermion of figure (1) will be naturally much lighter than the other superheavy particles, by a calculable factor. The neutrino masses will then be automatically "large".

This actually happens⁷ in the minimal form of the O(10) model. In that model, the heavy fermion is massless at the tree level but gets mass from a two loop diagram, which is shown in figure (3). The mass is therefore (roughly) of order $\alpha^2 M$, M being the typical grand unified mass (for a more accurate discussion see reference (7)). The neutrino masses are then naturally enhanced by a factor $1/\alpha^2$.

After a certain amount of analysis, in which one must use relations among Yukawa couplings provided by O(10), one finds in this model the following formula for neutrino masses:

$$m_\nu = m_Q \frac{M_W}{\alpha^2 M}. \quad (13)$$

Here M is the grand unified mass, M_W is the W boson mass, and m_Q is the mass of the up quark in the same generation as whatever neutrino we are considering. With $M_W \approx 100$ GeV and $M \approx 10^{15}$ GeV (but actually the right M to use here is quite uncertain) this becomes

$$m_\nu = 10^{-9} m_Q. \quad (14)$$

Quantitatively, this means

$$\begin{aligned} m_{\nu_e} &= 10^{-9} m_u \approx 5 \times 10^{-3} \text{ eV} \\ m_{\nu_\mu} &= 10^{-9} m_c \approx 1 \text{ eV} \\ m_{\nu_\tau} &= 10^{-9} m_t \approx 30 \text{ eV} \end{aligned} \quad (15)$$

(if, for instance, $m_t = 30$ GeV).

Also, it should be noted that the proportionality in equation (14) between neutrino masses and up quark masses is a proportionality not just between masses but between mass *matrices*. This means that, in this model, the neutrino mixing angles are equal to the Cabibbo-Kobayashi-Maskawa angles.

These results should not be taken too literally, for two reasons. First, although it is true that in this model the neutrino masses are proportional to the up quark masses, the constant of proportionality, quoted in equation (14) as 10^{-9} , is actually uncertain in a quite wide range because the superheavy masses that enter the diagram of figure (3) are not really known. Second, the model in question also predicts $m_d = m_e$, and so definitely requires modification. However, it may be that the mechanism considered

here could have applications in other models or in a modified form of this model.

As the final subject in this talk, I would like to turn attention to astrophysics and point out that there actually are two interesting astrophysical arguments that suggest the existence of a neutrino with a mass of several tens of electron volts. These arguments are not new, but do not seem to be well known among particle physicists. One argument involves the mean mass density of the universe; the second concerns galactic halos, a subject about which my knowledge comes mainly from conversations with M. Davis and M. Lecar.

Considering first the average mass density of the universe, we know that according to general relativity, if the average density ρ of the universe is less than a certain critical density, ρ_c , the universe will expand forever, but if it is greater than ρ_c , the universe will recollapse. Because it depends on the uncertain Hubble constant, ρ_c isn't known accurately, but it is roughly 10^{-29} gm/cm³.

Moreover, if ρ/ρ_c is less than 1, it goes to zero in time; if ρ/ρ_c is greater than 1, it diverges in time (as the universe recollapses to a singularity).

Experimentally, ρ/ρ_c isn't known reliably. The baryon density of the universe (which is estimated by measuring the total starlight from galaxies, and taking into account the average number of baryons per star and the average amount of starlight per star) seems to correspond to ρ/ρ_c of a few percent. However, indirect measurements (such as the observation of galactic halos, discussed below) suggest that ρ/ρ_c might be a few tenths. Thus, experiment suggests that ρ/ρ_c is less than one, but by a fairly modest factor, not by many orders of magnitude.

From the point of view of particle physics, there is something surprising about this. According to general relativity, if ρ/ρ_c is less than one, it goes to zero as a function of time for large time (in fact $(\rho/\rho_c) \sim 1/t$ for large t). But our universe, with an age of order 10^{10} years, is extremely old by elementary particle physics standards. The age of the universe is about 10^{40} in units of $1/\text{GeV}$, or about 10^{60} in units of $1/(\text{Planck mass})$.

If ρ/ρ_c is really going to zero in time, why, such a long time after the big bang, does ρ/ρ_c still differ from one only by one order of magnitude or so? Why is ρ/ρ_c not equal to, say, 10^{-20} or 10^{-40} ?

The fact that in such an old universe ρ/ρ_c is fairly close to one suggests that ρ/ρ_c is not diverging in time either to zero or to infinity (as occurs if $\rho/\rho_c > 1$), but that ρ/ρ_c is exactly equal to one.

That ρ/ρ_c might equal exactly one is an old speculation, which goes back at least to Dirac. Until recently it was just a speculation, in the sense that no rational reason was ever given for $\rho/\rho_c = 1$. Recently, for the first time, a possible reason has been given. Guth⁸ has described a class of

theories in which it is possible to show, for dynamical reasons, that ρ/ρ_c must equal one.

Although there are many unanswered questions about Guth's theory, this theory is an important development because it is the first theory that has put the value of ρ/ρ_c on a scientific basis, rather than leaving it as a matter for speculation. Whether or not Guth's theory proves to be correct, it should encourage us to believe that the value of ρ/ρ_c is capable of being rationally understood.

In any event, if in our present universe the mass density ρ is equal to ρ_c , where is this mass density to be found? Since, as mentioned above, the baryon density seems to give a ρ that is only a few percent of ρ_c , perhaps most of the mass is in some form other than baryons. This reasoning led Cowsik and Mc Lelland⁹ and Lee and Weinberg¹⁰ to consider the possibility that the neutrino might have a mass and that most of the mass of the universe might consist of massive neutrinos.

If this is true, it is easy to determine^{9,10} what neutrino mass is required. The neutrino *number* density is easy to calculate from the standard big bang theory because at a high temperature (of order 1 MeV) the neutrinos were in thermal equilibrium, and since then the neutrino number has been conserved. In fact, the neutrino number density is expected to be approximately $100/\text{cm}^3$ for each species, ν_e , ν_μ , or ν_τ . (This number density is not affected by the neutrino mass, which, for the range of masses of interest, was a negligible perturbation when the neutrinos were last in thermal equilibrium. Likewise, the neutrino mass was a negligible perturbation with respect to nucleosynthesis.)

With a neutrino number density of $100/\text{cm}^3$, the mass density is simply $100/\text{cm}^3$ times the mass, summed over species:

$$\rho(\text{neutrinos}) = (100/\text{cm}^3) \sum_i m_i. \quad (16)$$

The condition that this neutrino mass density equals the critical density turns out to be

$$\sum_i m_i = (50 \text{ eV}) (h/75)^2, \quad (17)$$

where h is the Hubble constant in units of $\text{km}/\text{sec}/M_{\text{pc}}$. ($h = 75$ is currently favored). For $50 < h < 100$, the sum of the neutrino masses required to give $\rho = \rho_c$ ranges from 25 eV to 100 eV.

It is very interesting that there is also a second astrophysical argument which suggests a neutrino mass in roughly the same range. This argument, which is due originally to Gunn and Tremaine¹¹, involves the existence of dark galactic halos.

Galaxies are observed to be surrounded by dark matter. The dark matter is detected by its gravitational field; the gravitational fields of galaxies, including ours, are stronger than expected on the basis of the stars making

up the galaxy.

The parameters characterizing the dark matter are observed to be roughly as follows. The radius and mass of the dark matter are about five or ten times those of the visible part of the galaxy (figure (4)). These quantities are measured by observing the orbits of particles (either neutral hydrogen atoms, which are detected by the radiation they emit, or large objects such as globular clusters) which are in orbit around the galaxy. From the velocity and orbital radius of an orbiting particle one can, using Newton's laws, determine the galactic mass. The galactic masses determined in this way exceed by a factor of five or ten the masses expected based on the stars contained in galaxies. Even more convincingly, the orbital velocity of a particle in orbit at radius R is determined, according to Newton's laws, by the total mass contained within the orbit. By studying the "rotation curve" of orbital velocity as a function of R one can determine the mass distribution of the galaxy as a function of R (for a review, see ref. (12)). In this way, it is determined that eighty or ninety percent of the mass of a galaxy lies outside the visible region.

The density of the dark matter is not accurately known, but the maximum observed density of dark matter seems to be about 10^{-24} gm/cm³.

What does the dark matter consist of? Could it be a cloud of massive neutrinos, gravitationally bound to the galaxy? If so, then, as Gunn and Tremaine pointed out¹¹, there is an interesting *lower* bound for the neutrino mass.

The velocity of a neutrino which is gravitationally bound to a galaxy cannot exceed the escape velocity from the galaxy, V_{escape} , which is generally roughly 300 km/sec. For these nonrelativistic neutrinos, the momentum is simply $p = mV$. So the momentum of a neutrino which is gravitationally bound to the galaxy cannot exceed a maximum momentum

$$p_{\text{max}} = m V_{\text{escape}}$$

For neutrinos of momentum less than p_{max} , fermi statistics do not permit a number density greater than

$$2 \int_0^{p_{\text{max}}} \frac{d^3 p}{(2\pi\hbar)^3} = \frac{p_{\text{max}}^3}{3\pi^2 \hbar^3} \quad (18)$$

Actually, although it will not significantly affect the conclusion, we should note that, as Gunn and Tremaine showed, the maximum *plausible* neutrino number density based on big bang cosmology is one half of the maximum allowed by fermi statistics, or $p_{\text{max}}^3 / 6\pi^2 \hbar^3$.

The neutrino mass density is now simply the mass times the number density, so the maximum possible neutrino mass density is

$$\rho_{\text{max}} = \frac{m p_{\text{max}}^3}{6\pi^2 \hbar^3} = \frac{m^4 (300 \text{ km/sec})^3}{6\pi^2 \hbar^3} \quad (19)$$

The condition that the observed density of dark matter of about 10^{-24} gm/cm³ should be less than ρ_{max} now gives a lower bound on the neutrino mass. With $m^4 (300 \text{ km/sec})^3 / 6\pi^2 h^2 \geq 10^{-24}$ gm/cm³, we find

$$m \geq 20 \text{ eV}, \quad (20)$$

which is the bound first derived by Gunn and Tremaine.

If there are several species of massive neutrinos, then ρ_{max} involves a sum over neutrino species. Since ρ_{max} is proportional to the fourth power of the neutrino mass, the inequality (20) becomes

$$\sum_i m_i^4 \geq (20 \text{ eV})^4. \quad (21)$$

(It should be noted that, on the basis of additional assumptions, Gunn and Tremaine derived additional inequalities that were inconsistent with (20), and concluded that a neutrino in this mass range could not exist. It is probably for this reason that their paper was not widely noticed among particle physicists. The additional inequalities of Gunn and Tremaine involved assumptions about the process of galaxy formation, and in my opinion are not nearly as reliable as (20).)

It is very interesting that two separate arguments lead to estimates of neutrino masses, (17) and (20), which, within the uncertainties of astrophysical quantities, substantially coincide. Both estimates indicate the existence of a neutrino weighing several tons of electron volts. Given that the two estimates, of such different nature coincide, it is natural to suspect that a neutrino in this mass range really exists.

Acknowledgement

I would like to thank M. Davis, M. Lecar, and G. Steigman for discussions.

References

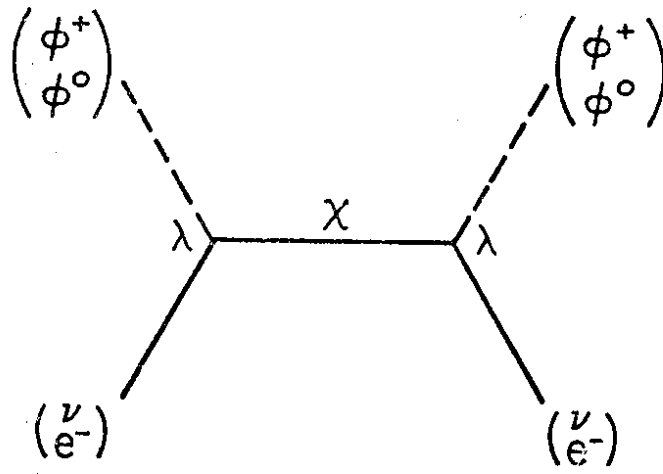
- (1) For example, see S. Weinberg, in *Lectures on Particles and Field Theory* (Prentice-Hall, Inc. Englewood Cliff, N.J., 1964), Vol. II, p. 439.
- (2) H. Georgi and S. L. Glashow, *Phys. Rev. Lett.* 32 (1974) 438; J. C. Pati and A. Salam, *Phys. Rev. D* 8 (1973) 1240.
- (3) F. Wilczek and A. Zee, *Phys. Rev. Lett.* 43 (1979) 1571.
- (4) S. Weinberg, *Phys. Rev. Lett.* 43 (1979) 1566.
- (5) M. Gell-Mann, P. Ramond, and R. Slansky, unpublished.
- (6) R. N. Mohapatra and G. Senjanovic, *Phys. Rev. Lett.* 44 (1980) 912.
- (7) E. Witten, *Phys. Lett.* 91B (1980) 81.
- (8) A. Guth, to appear.

- (9) R. Cowsik, J. McLelland, Phys. Rev. Lett. 29 (1972) 669.
- (10) B. W. Lee and S. Weinberg, Phys. Rev. Lett. 39 (1977) 165.
- (11) S. Tremaine and J. E. Gunn, Phys. Rev. Lett. 42 (1979) 407. For discussions of additional astrophysical aspects of massive neutrinos, see J. E. Gunn, B. W. Lee, I. Lerche, D. N. Schramm, and G. Steigman, Ap. J. 223 (1978) 1015; G. Steigman, C. L. Sarazin, H. Quintana, and J. Faulkner, Astron. J. 83 (1978) 1050.
- (12) S. M. Faber and J. S. Gallagher, in *Annual Reviews of Astronomy and Astrophysics*, ed. G. Burbidge, D. Layzer, and J. G. Philips, 1979.

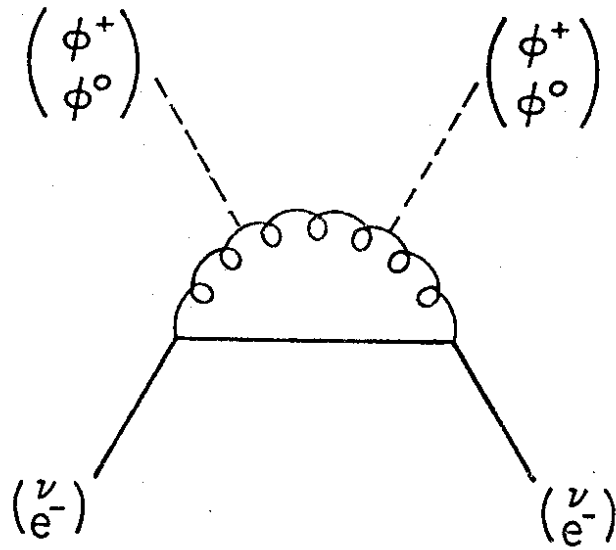
FIGURE CAPTIONS

- Figure 1: A diagram in the $O(10)$ model which leads to neutrino masses. A lepton number violating effective interaction is generated by exchange of the heavy fermion χ .
- Figure 2: A hypothetical one loop diagram which might generate a lepton number violating interaction. Superheavy particles are circulating in the loop.
- Figure 3: A two loop diagram which, in the simplest $O(10)$ model, gives mass to the χ . Circulating in the loop are ordinary quarks and leptons and superheavy fermions.
- Figure 4: Galaxies are believed to consist of a visible region (stars) surrounded by a much larger and more massive halo of dark matter.

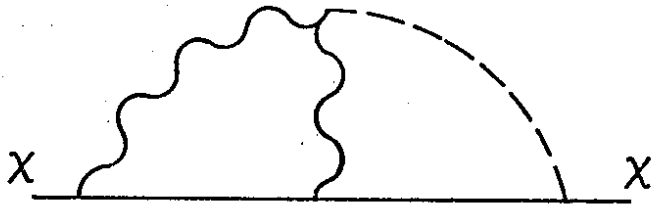
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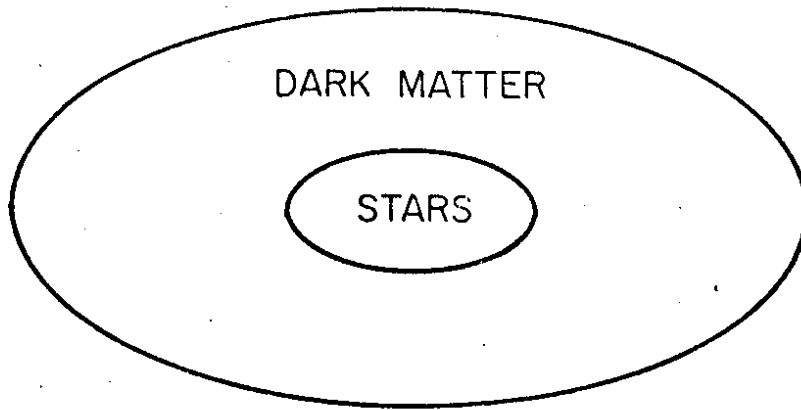
(2)



(3)



(4)



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