

# Quantum Theory: Exact or Approximate?

Stephen L. Adler\*

*Institute for Advanced Study, Einstein Drive, Princeton, NJ 08540, USA*

Angelo Bassi†

*Department of Theoretical Physics, University of Trieste,*

*Strada Costiera 11, 34014 Trieste, Italy and*

*Istituto Nazionale di Fisica Nucleare, Trieste Section, Via Valerio 2, 34127 Trieste, Italy*

Quantum mechanics has enjoyed a multitude of successes since its formulation in the early twentieth century. It has explained the structure and interactions of atoms, nuclei, and subnuclear particles, and has given rise to revolutionary new technologies. At the same time, it has generated puzzles that persist to this day.

These puzzles are largely connected with the role that measurements play in quantum mechanics [1]. According to the standard quantum postulates, given the Hamiltonian, the wave function of quantum system evolves by Schrödinger's equation in a predictable, deterministic way. However, when a physical quantity, say z-axis spin, is “measured”, the outcome is not predictable in advance. If the wave function contains a superposition of components, such as spin up and spin down, which each have a definite spin value, weighted by coefficients  $c_{\text{up}}$  and  $c_{\text{down}}$ , then a probabilistic distribution of outcomes is found in repeated experimental runs. Each repetition gives a definite outcome, *either* spin up or spin down, with the outcome probabilities given by the absolute value squared of the corresponding coefficient in the initial wave function. This recipe is the famous Born rule. The puzzles posed by quantum theory are how to reconcile this probabilistic distribution of outcomes with the deterministic form of Schrödinger's equation, and to understand precisely what constitutes a “measurement”. At what point do superpositions break down, and definite outcomes appear? Is there a quantitative criterion, such as size of the measuring apparatus, governing the transition from coherent superpositions to definite outcomes?

These puzzles have inspired a large literature in physics and philosophy. There are two

---

\*Electronic address: [adler@ias.edu](mailto:adler@ias.edu)

†Electronic address: [bassi@ts.infn.it](mailto:bassi@ts.infn.it)

distinct approaches. One is to assume that quantum theory is exact, but that the interpretive postulates need modification, to eliminate apparent contradictions. Many worlds, decoherent histories, Bohmian mechanics, and quantum theory as information, all fall in this category. Although their underlying mathematical formulations differ, empirically they are indistinguishable, since they predict the same experimental results as does standard quantum theory.

The second approach is to assume that quantum mechanics is not exact, but instead is a very accurate approximation to a deeper level theory, which reconciles the deterministic and probabilistic aspects. This may seem radical, even heretical, but looking back in the history of physics, there are precedents. Newtonian mechanics was considered to be exact for several centuries, before being supplanted by relativity and quantum theory, to which classical physics is an approximation. But apart from this history, there is another important motivation for considering modifications of quantum theory. This is to give a quantitative meaning to experiments testing quantum theory, by having an alternative theory, making predictions that differ from those of standard quantum theory, to which these experiments can be compared.

Although a modification of quantum theory may ultimately require a new dynamics, we focus here on phenomenological approaches, that look for modifications of the Schrödinger equation that describe what happens in measurements. A successful phenomenology must accomplish many things: (1) It must explain why repetitions of the same measurement lead to definite, but differing, outcomes. (2) It must explain why the probability distribution of outcomes is given by the Born rule. (3) It must permit quantum coherence to be maintained for atomic and mesoscopic systems, while predicting definite outcomes for measurements with realistic apparatus sizes in realistic measurement times. (4) It should conserve overall probability, so that particles do not spontaneously disappear. (5) It should not allow superluminal transmission of signals, while incorporating quantum nonlocality.

It is not obvious that a phenomenology should exist that satisfies these requirements, but remarkably, through work over the last two decades, one does. One ingredient is the observation that rare modifications, or “hits”, localizing the wave function, will not alter atomic-level coherences, but when accumulated over a macroscopic apparatus can lead to definite outcomes which differ from run to run [2]. A second ingredient is the observation that the classic “gambler’s ruin” problem in probability theory gives a mechanism that can

explain the Born rule governing outcome probabilities [3], as follows. Suppose that Alice and Bob each have a stock of pennies, and flip a fair coin. If the coin shows heads, Alice gives Bob a penny, while if the coin shows tails, Bob gives Alice a penny. The game ends when one player has all the pennies and the other has none. Mathematical analysis shows that the probability of each player winning is proportional to their initial stake of pennies. Map the initial stake into the modulus squared of the initial spin component coefficient, and one has a mechanism for obtaining the Born rule.

The combination of these two ideas leads to a definite model, called the Continuous Spontaneous Localization (CSL) model [4], in which a Brownian motion noise term coupled to the local mass density is added to the Schrödinger equation, with a nonlinear noise squared term included to preserve wave function normalization. The standard form of this model has a linear evolution equation for the noise averaged density matrix, forbidding superluminal communication. Other versions of the model exist, as reviewed in [5, 6], and a pre-quantum dynamics has been proposed for which this model would be a natural phenomenology [7].

The CSL model has two intrinsic parameters. One is a rate parameter  $\lambda$ , with dimensions of inverse time, governing the noise strength. The other is a length  $r_C$ , which can be interpreted as the spatial correlation length of the noise field. Conventionally,  $r_C$  is taken as  $10^{-5}\text{cm}$ , but any length within an order of magnitude of this would do. Demanding that a pointer composed of  $\sim 10^{15}$  nucleons should settle to a definite outcome in  $\sim 10^{-7}$  seconds or less, with the conventional  $r_C$ , requires that  $\lambda$  should be greater than  $\sim 10^{-17}\text{s}^{-1}$ . That is, requiring that measurements happen in reasonable times with a minimal apparatus places a *lower bound* on  $\lambda$ . If one requires that latent image formation in photography, rather than subsequent development, constitutes a measurement, the fact that few atoms move significant distances in latent image formation requires an enhanced lower bound for  $\lambda$  a factor of  $\sim 10^8$  larger [8]. Note that the Hubble constant is  $\simeq 2 \times 10^{-18}\text{s}^{-1}$ , so the conventional value of  $\lambda$  could be compatible with a cosmological origin of the noise field, which seems unlikely if  $\lambda$  were much enhanced.

An *upper bound* on  $\lambda$  is placed by the requirement that apparent violations of energy conservation, taking the form of spontaneous heating produced by the noise, should not be too large; the best bound comes from heating of the intergalactic medium [8]. Spontaneous radiation from atoms places another stringent bound [9], which can however be evaded if the noise is non-white, with a frequency cutoff [10, 11]. Laboratory and cosmological bounds on

$\lambda$  (for  $r_C = 10^{-5}$ cm) are summarized in the Table.

Accurate tests of quantum mechanics that have been performed or proposed include diffraction of large molecules in fine mesh gratings [12], and a cantilever mirror incorporated into an interferometer [13]. The Table shows the current limit on  $\lambda$  that has been obtained to date in fullerene diffraction, and the limit that would be obtained if the proposed cantilever experiment attains full sensitivity [14]. To confront the conventional (enhanced) value of  $\lambda$ , one would have to diffract molecules a factor of  $10^6$  ( $10^2$ ) larger than fullerenes.

In terms of distinguishing between conventional quantum theory, and modified quantum theory as given by the CSL model, experiments do not yet tell us whether quantum theory is exact, or approximate. Future lines of research include refining the sensitivity of current experiments, to reach the capability of making this decision, and achieving a deeper understanding of the origin of the CSL noise field.

- 
- [1] A. J. Leggett, *Science* **307**, 871 (2005).
  - [2] G.C. Ghirardi, A. Rimini and T. Weber, *Phys. Rev. D* **34**, 470 (1986).
  - [3] P. Pearle, *Phys. Rev. D* **13**, 857 (1976).
  - [4] G.C. Ghirardi, P. Pearle and A. Rimini, *Phys. Rev. A* **42**, 78 (1990).
  - [5] A. Bassi and G.C. Ghirardi, *Phys. Rep.* **379**, 257 (2003).
  - [6] P. Pearle, in *Open Systems and Measurements in Relativistic Quantum Field Theory* (Lecture Notes in Physics vol 526) ed H-P Breuer and F Petruccione (Berlin: Springer, 1999).
  - [7] S. L. Adler, *Quantum Theory as an Emergent Phenomenon*, Cambridge University Press (2004).
  - [8] S. L. Adler, *J. Phys. A* **40**, 2935 (2007).
  - [9] Q. Fu, *Phys. Rev. A* **56**, 1806 (1997).
  - [10] S. L. Adler and F. M. Ramazanoğlu, *J. Phys. A* **40**, 13395 (2007).
  - [11] A. Bassi and S. L. Adler, *J. Phys. A* **40**, 15083 (2007); **41**, 395308 (2008).
  - [12] M. Arndt, O. Nairz, J. Vos-Andreae, C. Keller, G. van der Zouw, and A. Zeilinger, *Nature* **401**, 680 (1999).
  - [13] W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester, *Phys. Rev. Lett.* **91**, 130401 (2003).
  - [14] A. Bassi, E. Ippoliti, and S. L. Adler, *Phys. Rev. Lett.* **94** 030401 (2005).

---

**Upper bounds on the parameter  $\lambda$  of the CSL model  
(with noise correlation length  $r_C \sim 10^{-5}$  cm)**

| <b>Laboratory Experiments</b>                | <b>Distance (in orders of magnitude) from the conventional value<br/><math>\lambda \sim 10^{-17}\text{s}^{-1}</math></b> | <b>Cosmological Data</b>              | <b>Distance (in orders of magnitude) from the conventional value<br/><math>\lambda \sim 10^{-17}\text{s}^{-1}</math></b> |
|--|--|---------------------------------------|--|
| Fullerene diffraction experiments            | 13   | Dissociation of cosmic hydrogen       | 17   |
| Decay of supercurrents                       | 14   | Heating of intergalactic medium (IGM) | 8  |
| Spontaneous X-ray emission from Ge           | 6  | Heating of interstellar dust grains   | 15   |
| Proton decay                                 | 18   |                                       |  |
| Mirror cantilever interferometric experiment | 9  |                                       |  |

TABLE I: The table gives upper bounds on  $\lambda$  from laboratory experiments and cosmological data, compared with the conventional CSL model value  $\lambda \sim 10^{-17}\text{s}^{-1}$ . Reducing the numbers by 8 gives the distance of each bound from the enhanced value  $\lambda \sim 10^{-9}\text{s}^{-1}$  obtained if one assumes that latent image formation constitutes measurement. The X-ray emission bound excludes an enhanced  $\lambda$  for white noise, but this constraint is relaxed if the noise spectrum is cut off below  $10^{18}\text{s}^{-1}$ . Large molecule diffraction would confront the CSL value of  $\lambda$  for molecules heavier than  $\sim 10^9$  Daltons, and would confront the enhanced  $\lambda$  for molecular weights greater than  $\sim 10^5$  Daltons. (The molecular diffraction bound on  $\lambda$  decreases as the inverse square of the molecular weight, provided the molecular radius is less than  $r_C$ .)