IMPLICATIONS OF A FRAME-DEPENDENT DARK ENERGY ACTION

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CONVENTION FOR METRIC (1,-1,-1,-1)

PAPERS: S.L.A. arXiv: 1306.0482

S.L.A. + F.R. RAMAZANOGLU: arXiv: 1308.1448

S.L.A. arXiv: 1605.05217

S.L.A. arXiv: 1704.00388

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I **USUAL COSMOLOGICAL CONSTANT ACTION** $\quad S_{cosm} = -\frac{\Lambda}{8\pi G}\int d^4x\,({}^{(4)}g)^{1/2}$

II **FRAME-DEPENDENT ACTION** $\quad S_{eff} = -\frac{\Lambda}{8\pi G}\int d^4x\,({}^{(4)}g)^{1/2}\,(g_{00})^{-2}$

FOR R-W METRIC $\quad ds^2 = (dt)^2 - a^2(t)\,(d\vec{x})^2$

$g_{00} = 1$, I AND II THE SAME

I FOUR SPACE GENERAL COORDINATE INVARIANT

II THREE SPACE " " " " " "

INVARIANT UNDER WEYL SCALING $\quad g_{\mu\nu} \to \lambda^2 g_{\mu\nu}$

IS FRAME DEPENDENCE ALLOWED?

- BEFORE CMB: NO: LAWS OF PHYSICS CANNOT DEPEND ON STATE OF UNIFORM MOTION

- AFTER CMB: YES: DIPOLE GIVES OUR VELOCITY RELATIVE TO CMB REST FRAME

$$V = 369 \text{ km/s}$$

IN PRINCIPLE, OTHER PHYSICS COULD BE TIED TO THE CMB REST FRAME

MOTIVATIONS

arXiv: 1306.0482

"TRACE DYNAMICS" IN CLASSICAL $g_{\mu\nu}$

- DYNAMICS OF CLASSICAL, MATRIX VARIABLES
- CANONICAL ENSEMBLE AVERAGE GIVES EFFECTIVE EMERGENT QUANTUM THEORY

$$\rho = Z^{-1} e^{-\tau \underset{\sim}{H}} + \cdots$$

$$\underset{\sim}{H} = \int d^3x \, ({}^{(4)}g)^{1/2} \, \mathrm{Tr}(T_0^{\ 0})$$ TRACE HAMILTONIAN

$$\Delta S_{g;\,\mathrm{INDUCED}} = \int d^4x \, ({}^{(4)}g)^{1/2} \, \mathrm{Tr}(\langle \mathcal{L}(x) \rangle_{AV}) / \mathrm{Tr}(1)$$

$$\langle \mathcal{L}(x) \rangle_{AV} = \int d\mu \, \rho \, \mathcal{L}(x)$$

MOTIVATIONS - CONTINUED

$\underset{\sim}{H}$ USES $\int d^3x$ PICKS A FRAME
3 SPACE GEN. COORD. INVARIANT

WEYL SCALING: $g_{\mu\nu} \to \lambda^2 g_{\mu\nu}$

MATTER FIELDS: $q \to \lambda^{-w_q} q$ $p \to \lambda^{-w_p} p$

FORGER + RÖMER ANN PHYS **309**, 306 (2004)

$({}^{(4)}g)^{1/2} \mathcal{L}$ IS WEYL INVARIANT FOR MASSLESS SPIN 0, 1/2, 1 MATTER FIELDS

S.L.A. arXiv: 1306.04826

$({}^{(4)}g)^{1/2} T_\mu{}^\nu$ IS WEYL INVARIANT FOR MASSLESS MATTER FIELDS (λ = CONSTANT GLOBAL WEYL)

MOTIVATIONS - CONTINUED

TOGETHER, THESE $\Rightarrow \Delta S_{g,\text{INDUCED}}$ IS

WEYL SCALING INVARIANT

TO ZEROTH ORDER IN METRIC DERIVATIVES

$$\Delta S_g = \int d^4x \, ({}^{(4)}g)^{1/2} (g_{00})^{-2} A(g_{0i} \, g_{0j} \, g^{ij}/g_{00}, \dots)$$

FOR DIAGONAL METRICS

$$\Delta S_g = \int d^4x \, ({}^{(4)}g)^{1/2} (g_{00})^{-2} A$$

G 't Hooft arXiv: 1410.6675 "Local Conformal Symmetry: the Missing Symmetry Component for Space and Time"
First Award, Gravity Research Foundation Essays, 2015

RULES TO GET EQUATIONS OF MOTION

VARYING ΔS_g WITH RESPECT TO FULL $g_{\mu\nu}$ GIVES A $\Delta T_g^{\mu\nu}$ THAT DOES NOT OBEY $D_\mu \Delta T_g^{\mu\nu} = 0$

PROCEED AS FOLLOWS: ① $\delta / \delta g_{ij} \Rightarrow$

$$G^{ij} + 8\pi G \left(\Delta T_g^{ij} + T^{ij}_{\text{MATTER}} \right) = 0$$

② FROM $D_\mu \Delta T_g^{\mu\nu} = 0$, THEN INFER ΔT_g^{i0} AND ΔT_g^{00}. THIS GIVES CONSISTENT

$$G^{\mu\nu} + 8\pi G \left(\Delta T_g^{\mu\nu} + T^{\mu\nu}_{\text{MATTER}} \right) = 0$$

APPLICATION 1: SPHERICALLY SYMMETRIC SCHWARZSCHILD-LIKE SOLUTIONS

arXiv: 1308.1448

POLAR COORDINATES $(\Delta T_g \equiv \Delta T)$

$$(ds)^2 = B(r)(dt)^2 - A(r)(dr)^2 - r^2\left[(d\theta)^2 + \sin^2\theta\,(d\phi)^2\right]$$

$$\Delta T^{ij} = \frac{\Lambda}{8\pi G}\, g^{ij} / B(r)^2 \quad , \quad \Delta T_{ij} = \frac{\Lambda}{8\pi G}\, g_{ij} / B(r)^2$$

VARYING $g_{ij} \Rightarrow$

$$G_{rr} - \frac{\Lambda A(r)}{B(r)^2} = 0 \qquad G_{\theta\theta} - \frac{\Lambda r^2}{B(r)^2} = 0$$

COVARIANT CONSERVATION CONDITION IS ALGEBRAIC

$$\Delta T_{tt} = -\frac{3\Lambda}{8\pi G B} \Rightarrow G_{tt} - \frac{3\Lambda}{B} = 0 \quad , \quad R = 0$$

ANALYTICAL AND NUMERICAL RESULTS

① FOR $10^{-17}\left(\frac{M}{M_\odot}\right)^2 \text{cm} < r - 2MG < H^{-1}$

SOLUTION APPROXIMATES SCHWARZSCHILD
HENCE ASTROPHYSICS OF BLACK HOLES BASICALLY UNCHANGED

② g_{00} NEVER VANISHES $\qquad x = \Lambda^{1/2} r$

$$B \simeq \alpha + 1.279\,(x-\alpha)^{1/2} \qquad \alpha \simeq 0.030135 \text{ FOR } M\Lambda^{1/2} = 10^{-2}$$

CUSP-LIKE SQUARE ROOT

THIS A COORDINATE SINGULARITY $\quad R_{\mu\nu}R^{\mu\nu}$ ETC FINITE

③ PHYSICAL SINGULARITY AT $r = \infty$
LIKELY BREAKDOWN OF STATIC ASSUMPTION

ISOTROPIC COORDINATES

$$(ds)^2 = \frac{B[r]^2}{A[r]^2}(dt)^2 - \frac{A[r]^4}{r^4}\left[(dr)^2 + r^2\left((d\theta)^2 + \sin^2\theta\,(d\phi)^2\right)\right]$$

$$G_{rr} = \frac{\Lambda A^8}{r^4 B^4} \qquad G_{\theta\theta} = \frac{\Lambda A^8}{r^2 B^4}$$

COVARIANT CONSERVATION GIVES

$$G_{tt} = \frac{3\Lambda A^2}{B^2}$$

SMOOTH SOLUTION FROM PHYSICAL SINGULARITY AT CENTER OF BLACK HOLE TO PHYSICAL SINGULARITY AT ∞

NO HORIZON, AND NO SQUARE ROOT CUSP

APPLICATION 2: PERTURBATIONS ON THE ROBERTSON-WALKER (RW) METRIC

arXiv: 1704.00388

TO SET UP A PHENOMENOLOGY TO DISTINGUISH BETWEEN ACTIONS I AND II FORM A LINEAR COMBINATION

$$S_\Lambda = (1-f)\, S_{\text{covar}} + f\, S_{\text{eff}}$$

$$= -\frac{\Lambda}{8\pi G} \int d^4x \, (-g)^{1/2} \left[1 - f + f (g_{00})^{-2}\right]$$

$f=0$ ONLY A STANDARD COSMOLOGICAL CONSTANT

$f=1$ ONLY A COSMOLOGICAL CONSTANT COMING FROM A FRAME-DEPENDENT ACTION

CONSERVING EXTENSION $T_\Lambda^{\mu\nu}$ OF T_Λ^{ij}

PERTURBED RW METRIC

$$g_{00} = 1 + h_{00}$$

$$g_{i0} = g_{0i} = h_{i0}$$

$$g_{ij} = -a^2(t)\,\delta_{ij} + h_{ij}$$

$$\delta/\delta g_{ij} \Rightarrow \qquad T_\Lambda^{ij} = \frac{\Lambda}{8\pi G}\left[g^{ij} + f\,t^{ij}\right]$$

$$t^{ij} = \frac{2\,\delta_{ij}\,h_{00}}{a^2(t)}$$

CONSERVING EXTENSION OF g^{ij} IS $g^{\mu\nu}$

NEED EXTENSION OF t^{ij} WHICH IS FIRST ORDER

SOLVE $\quad 0 = D_\nu t^{\mu\nu} = \partial_\nu t^{\mu\nu} + \Gamma^\mu_{\nu\alpha} t^{\alpha\nu} + \Gamma^\nu_{\nu\alpha} t^{\mu\alpha}$

NEED ONLY UNPERTURBED RW CONNECTION Γ

GET DIFFERENTIAL EQUATIONS

$$0 = \partial_0 t^{l0} + \partial_j t^{lj} + 5\,\frac{\dot a}{a}\, t^{0l}$$

$$0 = \partial_0 t^{00} + \partial_j t^{0j} + a\dot a\, t^{mm} + 3\,\frac{\dot a}{a}\, t^{00}$$

INTEGRATING GET FROM $\quad t^{ij} = 2\,\delta_{ij}\, a^{-2}(t)\, h_{00}$

$$t^{l0} = -2\, a^{-5}(t) \int_{t_{init}}^{t} du\; a^3(u)\, \partial_l h_{00}(u) \qquad \begin{cases} (\vec{x} \text{ IMPLICIT}) \\ a(t_{init}) = 0 \end{cases}$$

$$t^{00} = -\, a^{-3}(t) \int_{t_{init}}^{t} du\; a^3(u) \left[\partial_l t^{0l}(u) + 6\, \frac{\dot a(u)}{a(u)}\, h_{00}(u) \right]$$

USING ZEROTH ORDER RW GET $t_{\mu\nu}$ FROM $t^{\mu\nu}$

WRITING $R_{\mu\nu} = R^{(0)}_{\mu\nu} + R^{(1)}_{\mu\nu}$

$$T_{\mu\nu\,\text{MATTER}} = T^{(0)}_{\mu\nu\,\text{MATTER}} + T^{(1)}_{\mu\nu\,\text{MATTER}}$$

THE PERTURBED EINSTEIN EQUATION BECOMES

$$0 = R^{(1)}_{\mu\nu} - \Lambda h_{\mu\nu} + 8\pi G \left[T_{\mu\nu\,\text{MATTER}} - \tfrac{1}{2} g_{\mu\nu} T_{\alpha}^{\alpha}{}_{\text{MATTER}} \right]^{(1)}$$

$$+ \Lambda f \left[t_{\mu\nu} - \tfrac{1}{2} g^{(0)}_{\mu\nu} t_{\alpha}^{\alpha} \right]$$

RESIDUAL GAUGE INVARIANCE WITH A FRAME-DEPENDENT EFFECTIVE ACTION

$$x^\alpha = x'^\alpha - \epsilon^\alpha(x') \qquad \epsilon^0 = 0$$

TO FIRST ORDER GIVES A GAUGE TRANSFORMATION

$$\delta_g h_{ij} = a^2(t)\left(\partial_j \epsilon^i + \partial_i \epsilon^j\right)$$

$$\delta_g h_{i0} = a^2(t)\,\partial_0 \epsilon^i$$

$$\delta_g h_{00} = 0 \rightarrow \delta_g t_{\mu\nu} = 0$$

SOME ALGEBRA GIVES

$$\delta_g \overline{T}^{(1)}_{\mu\nu\,\text{MATTER}} = -p\,\delta_g h_{\mu\nu}$$

$$\Rightarrow \tfrac{1}{2}(\rho - p)\,\delta_g h_{\mu\nu} = \delta_g\left[T_{\mu\nu\,\text{MATTER}} - \tfrac{1}{2} g_{\mu\nu} T^\alpha_{\alpha\,\text{MATTER}}\right]^{(1)}$$

$$\delta_g R_{\mu\nu} = \left[\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 \right] \delta_g h_{\mu\nu}$$

$\Rightarrow$

$$\delta_g [R^{(1)}_{\mu\nu} - \Lambda h_{\mu\nu}) = \left[\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 - \Lambda \right] \delta_g h_{\mu\nu} = 8\pi G \, \frac{1}{2}(\rho - p) \, \delta_g h_{\mu\nu}$$

$$= -8\pi G \, \delta_g \left[T_{\mu\nu\,\mathrm{MATTER}} - \frac{1}{2} g_{\mu\nu} T^{\alpha}_{\alpha\,\mathrm{MATTER}} \right]^{(1)}$$

$\Rightarrow$ FIRST ORDER PERTURBED EINSTEIN EQUATION IS GAUGE INVARIANT

SCALAR, VECTOR, TENSOR PERTURBATIONS

GAUGE FIXING

$$h_{00} = \underline{E}$$

$$h_{i0} = -a(\partial_i \underline{F} + G_i)$$

$$h_{ij} = -a^2(\underline{A}\,\delta_{ij} + \partial_i \partial_j \underline{B} + \partial_j C_i + \partial_i C_j + D_{ij})$$

$$\partial_i C_i = \partial_i G_i = \partial_i D_{ij} = D_{ii} = 0$$

TAKE $\quad \varepsilon^i = \frac{1}{2} \partial_i B$

$$\delta_g h_{ij} = a^2 \partial_i \partial_j B \qquad \text{ELIMINATES } B$$

MODIFIED SCALAR PERTURBATION EQUATIONS

B=0 GAUGE

ij)

$$0 = \delta_{ij} X + \partial_i \partial_j Y$$

$$X = 4\pi G a^2 \left(\rho^{(1)} - p^{(1)} - \nabla^2 \pi^S\right)$$

$$+ \left[a\ddot{a} + 2(\dot{a})^2\right]E + \frac{1}{2} a\dot{a}\dot{E} + \dot{a}\nabla^2 F - 3a\dot{a}\dot{A} - \frac{1}{2}a^2\ddot{A} + \frac{1}{2}\nabla^2 A$$

$$+ \Lambda f a^2 \left(\frac{1}{2} t_{00} - E\right)$$

$$Y = 8\pi G a^2 \pi^S$$

$$+ 2\dot{a}F + a\dot{F} + \frac{1}{2}(E+A)$$

MATTER PERTURBATIONS / METRIC TERMS / $\Lambda f\, t_{\mu\nu}$ PIECE FUNCTION OF $h_{00} = E$

MODIFIED ... CONTINUED

B=0 GAUGE

i0)

$$0 = -8\pi G (P+\rho)\partial_i u^{(1)} - \frac{\dot{a}}{a}\partial_i E + \partial_i \dot{A} + \Lambda f\, t_{0i}$$

00)

$$0 = 4\pi G(\rho^{(1)} + 3P^{(1)} + \nabla^2 \pi^S) - 3\frac{\ddot{a}}{a}E + 3\frac{\dot{a}}{a}\dot{A} + \frac{3}{2}\ddot{A} - \frac{3}{2}\frac{\dot{a}}{a}\dot{E} - \frac{\dot{a}}{a^2}\nabla^2 F - \frac{1}{a}\nabla^2\dot{F} - \frac{1}{2a^2}\nabla^2 E + \Lambda f\left(\frac{1}{2}t_{00} + 3E\right)$$

ABSENCE OF PROPAGATING SCALAR WAVES

FOURIER ANALYZE $e^{i\vec{k}\cdot a\vec{x}}$

$\partial_i \partial_j \propto k_i k_j$ INDEPENDENT FROM δ_{ij}

$\Rightarrow$ $X = 0$ $Y = 0$ DECOUPLE

WHEN MATTER PERTURBATIONS ARE ZERO

HAVE 4 HOMOGENEOUS EQUATIONS IN 3

UNKNOWNS E, A, F

OVERDETERMINED $\Rightarrow$ $E = A = F = 0$

EXPLICIT APPROXIMATE CALCULATION

$e^{i\vec{k}\cdot\lambda\vec{x}}$ SPATIAL $\qquad e^{-i\omega t}$ TIME

$\dot{\lambda} \equiv H\lambda \qquad \ddot{\lambda} \equiv H^2 Q \lambda$

H, Q, λ CONSTANT

$\partial_j \to i\lambda k_j \qquad \partial_t \to -i\omega \qquad \int dt \to (-i\omega)^{-1}$

GET EQUATIONS WITH CONSTANT COEFFICIENTS

REDUCE TO:
$$0 = E\left[\vec{k}^2 \alpha(\omega) + \beta(\omega)\right]$$
$$0 = E\left[\vec{k}^2 \gamma(\omega) + \delta(\omega)\right]$$

$$\alpha(\omega) = \gamma(\omega) = \frac{i\Lambda f H}{\omega^3} \Big/ \left(1 + \frac{iH}{\omega}\right)$$

$$\beta(\omega) = (Q-1)H^2 + 3i\Lambda f H/\omega \qquad \delta(\omega) = -3\beta(\omega)$$

INCONSISTENT

SUMMARY

- 0^{th} ORDER RW INSENSITIVE TO f

- SCHWARZSCHILD - LIKE BLACK HOLES
  - NO HORIZON WHERE $g_{00}=0$
  - CHANGE WITHIN $10^{-17}\,\text{cm}\left(\frac{M}{M_\odot}\right)^2$ OF NOMINAL HORIZON
  - ASTRO UNCHANGED
  - "INFORMATION PARADOX" ??

- RW PERTURBATIONS
  - NO CHANGE TO GRAVITATIONAL WAVE THEORY
  - POSSIBLE DETECTABLE EFFECTS IN EARLY UNIVERSE STRUCTURE FORMATION

RELATION TO GUBITOSI ET AL "EFFECTIVE THEORY OF DARK ENERGY"

$$S_{eff} = -\frac{\Lambda}{8\pi G}\int d^4x \, \left({}^{(4)}g\right)^{1/2}\left(-1 + 2 g^{00}\right) \qquad \text{TO FIRST ORDER}$$

$$\Rightarrow \quad \begin{aligned} \Lambda(t) &= -\Lambda \\ c(t) &= -2\Lambda \end{aligned} \qquad \text{CONSTANTS}$$

$$\boxed{f(t) = 1} \quad \text{EINSTEIN FRAME}$$

GUBITOSI ET AL VARY FULL $g^{\mu\nu}$ TO GET

$$G_{\mu\nu} M_*^2 + \left(c\,\delta g^{00} + \Lambda - c\right) g_{\mu\nu} - 2c\,\delta^0_\mu \delta^0_\nu = T_{\mu\nu}$$

NOT CONSISTENT WITH COVARIANT CONSERVATION AT 0^{th} ORDER: $\boxed{D_\nu\left(c\,\delta^\mu_0 \delta^\nu_0\right)} = \left(\dot{c} + 3\frac{\dot{a}}{a}c\right)\delta^\mu_0$

CORRECTIONS ARISING FROM NON-DIAGONALITY OF PERTURBED RW METRIC

$$\Delta S_g = \int d^4x \, ({}^{(4)}g)^{1/2} (g_{00})^{-2} \, A(g_{0i} \, g_{0j} \, g^{ij} / g_{00}, \ldots)$$

$$\frac{\delta}{\delta g^{ij}} A(g_{0i} \, g_{0j} \, g^{ij} / g_{00}, \ldots) = \frac{h_{0i} h_{0j}}{g_{00}} \, A_{,1}(\ldots)$$

SECOND ORDER CORRECTION TO t_{ij}

SO ONLY FIRST ORDER PERTURBATIONS ARE DETERMINED BY Λ