Mott Simplified

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I give a simplified discussion of why cloud chamber tracks are linear. (Not for publication; for posting on the Talks+Memos section of my web page).

I. INTRODUCTION

Mott's paper [1] is confusing to read, in part because there are some typos, for example, the $-\epsilon^2$ in front of the final two potential terms in Eq. (2) should be $+2\epsilon^2$, and there are confusions about whether the first atom with which the alpha particle interacts is labeled 1 or 2. It is not necessary to consider two atoms to make his point, and in the next section I give the core of Mott's calculation using just one atom.

II. AN ALPHA PARTICLE EXCITING ONE ATOM

Consider an alpha particle emitted at the origin in a spherical wave, and an atom at location $\vec{a}$. We want the amplitude for the alpha particle to scatter to a plane wave with four vector $\vec{k}$, conditional on the atom being excited from an initial state ψ_0 to an excited state ψ_S. The Born approximation (dropping overall constants) is proportional to

$$f(\vec{k}) \propto \int \psi^*_{final} V \psi_{initial} \quad . \tag{1}$$

In our case we have, for an alpha particle with coordinate $\vec{R}$ and an atomic electron with coordinate $\vec{r}$, and assuming the atom excitation energy is negligible,

$$\begin{aligned} \int =& \int d^3R d^3r \quad , \\ \psi^*_{final} \propto& e^{-i\vec{k}\cdot\vec{R}} \psi^*_S(\vec{r}) \quad , \\ V \propto& 1/|\vec{R}-\vec{r}| \quad , \\ \psi_{initial} \propto& \big(e^{ik|R|}/|R|\big)\psi_0(\vec{r}) \quad . \end{aligned} \tag{2}$$

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Subsituting into Eq. (1) we get

$$f(\vec{k}) \propto \int d^3R R^{-1} e^{ikR(1-\hat{k}\cdot\hat{R})} V_{0S}(\vec{R}) \quad , \tag{3}$$

where $\vec{k} = k\hat{k}$, $\vec{R} = R\hat{R}$, and where following Mott we have defined

$$V_{0S}(\vec{R}) = \int d^3r \psi_S^*(\vec{r})\psi_0(\vec{r})/|\vec{R} - \vec{r}| \quad . \tag{4}$$

Since $V_{0S}(\vec{R})$ is significantly different from 0 only for $\vec{R} \simeq \vec{a}$, we can approximate Eq. (3) by making the replacement $\hat{R} \simeq \hat{a}$, where $\vec{a} = |\vec{a}|\hat{a}$, giving

$$f(\vec{k}) \propto \int d^3R R^{-1} e^{ikR(1-\hat{k}\cdot\hat{a})} V_{0S}(\vec{R}) \quad . \tag{5}$$

Mott then notes that the coefficient of R in the exponent is rapidly oscillating except when $1 \simeq \hat{k}\cdot\hat{a}$, that is when the outgoing wave vector is nearly parallel to the vector from the alpha emitter to the first atom. Thus a second atom can be excited only if it lies in a small cone about a vector $\hat{a}$ pointing from the first atom, since the fact that the first atom was excited means that the subsequent scattered wave has its origin at this atom. Mott's more detailed treatment calculates the form of the wave scattered by the first atom, but the magnitude is governed by the amplitude of Eq. (5) above.

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[1] N.F. Mott, Proc. Roy. Soc. A **126**, 79 (1929).