INVESTIGATIONS ON GAUGED RARITA-SCHWINGER THEORY

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- S. L. Adler, Phys. Rev. D92, 085022 + 085023 (2015)
- S. L. Adler, M. Henneaux, P. Pais, Phys. Rev. D96, 085005 (2017)
- S. L. Adler, arXiv 1711.00907

THESE PAPERS FOCUS ON MASSLESS RARITA-SCHWINGER
(EARLY PAPERS: JOHNSON + SUDARSHAN, VELO + ZWANZIGER DEALT WITH MASSIVE CASE)

MOTIVATIONS

① GRAND UNIFICATION: CAN GAUGE ANOMALIES CANCEL BETWEEN SPIN 1/2 AND SPIN 3/2 FERMIONS?

② ANOMALY LITERATURE: SPIN 3/2 CHIRAL ANOMALY CALCULATED FOR GENERAL GAUGE GROUPS

BUT ONLY GAUGED SUPERGRAVITIES ARE O(N)

③ SPECIFIC MODEL: S.L. ADLER INT. J. MOD. PHYS. A29, 1450130 (2014)

- SU(8) REPRESENTATION & SPIN 3/2 + SPIN 1/2 TO CANCEL ANOMALIES
- GROUP CONTENT FORBIDS GENERATION OF A SPIN 3/2 MASS TERM → MASSLESS SPIN 3/2

RARITA - SCHWINGER ACTION

$$S(\psi_\mu) = \int d^4x \, \bar{\psi}_\mu R^\mu$$

$$R^\mu = i \, \varepsilon^{\mu\eta\nu\rho} \gamma_5 \gamma_\eta D_\nu \psi_\rho = -\gamma^{\mu\nu\rho} D_\nu \psi_\rho$$

$$D_\nu = \partial_\nu + g A_\nu \qquad \bar{\psi}_\mu = \psi_\mu^\dagger \, i \gamma^0$$

$$[D_\mu, D_\nu] = g F_{\mu\nu}$$

- WHEN $g = 0$, $S(\psi_\mu)$ HAS FERMIONIC GAUGE INVARIANCE $\psi_\mu \to \psi_\mu + D_\mu \varepsilon$
- WHEN $g \neq 0$, THIS INVARIANCE BROKEN BY $F_{\mu\nu}$

EXTENDED MODEL HAS FERMIONIC GAUGE INVARIANCE FOR $g \neq 0$ (FORMULATED IN MY FIRST PHYS. REV. PAPER STUDIED IN DETAIL BY ADLER, HENNEAUX, PAIS)

- ADD A DIMENSION 1/2 SPIN 1/2 FIELD Λ COUPLED TO THE GAUGE FIELDS AND RARITA-SCHWINGER FIELD

- USE LEFT CHIRAL REDUCTION

$$\psi_\mu \to \Psi_0, \vec{\Psi}$$

$$\gamma^\nu \to 1, \vec{\sigma} \qquad \text{PAULI MATRICES}$$

$\sigma_a \sigma_b = \delta_{ab} + i \epsilon_{abc} \sigma_c$ ITERATION OF THIS GIVES NEEDED IDENTITIES

ACTION, IN HAMILTONIAN FORM:

$$S = \int dt\, L \qquad L = \int d^3x \,(\text{TIME DERIVATIVES} + \text{CONSTRAINTS}) - H$$

$\leftarrow$ HAMILTONIAN

- TIME DERIVATIVES $= \frac{1}{2} \vec{\Psi}^\dagger \cdot \left(-\vec{\sigma} \times \partial_0 \vec{\Psi} - \frac{1}{2} i g \Lambda^\dagger \vec{\sigma} \cdot \vec{B}\, \partial_0 \Lambda \right)$

- CONSTRAINTS $= -\Psi_0^\dagger K - K^\dagger \Psi_0$

$$K = \frac{1}{2} \vec{\sigma} \cdot (\vec{D} \times \vec{\Psi}) - \frac{1}{2} i g\, \vec{\sigma} \cdot \vec{B}\, \Lambda$$

$$K^\dagger = -\frac{1}{2} \vec{\Psi}^\dagger \cdot (\vec{D} \times \vec{\sigma}) + \frac{1}{2} i g\, \Lambda^\dagger \vec{\sigma} \cdot \vec{B}$$

- $$H = -\frac{1}{2} \int d^3x \Big[\vec{\Psi}^\dagger \cdot \big(\vec{D} \times \vec{\Psi} - \vec{\sigma} \times g A_0 \vec{\Psi}\big) - i g \vec{\Psi}^\dagger \cdot \vec{C} \Lambda + i g \Lambda^\dagger \vec{C} \cdot \vec{\Psi} + i g \Lambda^\dagger \vec{C} \cdot \vec{D} \Lambda - i g^2 \Lambda^\dagger \vec{\sigma} \cdot \vec{B} A_0 \Lambda \Big]$$

$$\vec{C} = \vec{B} + \vec{\sigma} \times \vec{E}$$

- S INVARIANT UNDER: $\Psi_0 \to \Psi_0 + D_0 \varepsilon$, $\vec{\Psi} \to \vec{\Psi} + \vec{D} \varepsilon$, $\Lambda \to \Lambda - \varepsilon$

CANONICAL MOMENTA, BRACKETS

$$\vec{P} = \frac{1}{2}\vec{\Psi}^{\dagger} \times \vec{\sigma} \qquad \mathcal{L} = \frac{1}{2} i g \Lambda^{\dagger} \vec{\sigma} \cdot \vec{B}$$

UNEXTENDED R-S THEORY

$$\vec{K} = \frac{1}{2}\vec{\sigma} \cdot \vec{D} \times \vec{\Psi} \qquad \vec{K}^{\dagger} = -\frac{1}{2}\vec{\Psi}^{\dagger} \cdot (\vec{D} \times \vec{\sigma})$$

$$[K(\vec{x}), K^{\dagger}(\vec{y})] = -\frac{1}{2} i g\, \vec{\sigma} \cdot \vec{B}(\vec{x})\, \delta^3(\vec{x} - \vec{y})$$

$g = 0 \quad [K, K^{\dagger}] = 0 \quad$ FIRST CLASS

2 DEGREES OF FREEDOM $= \frac{1}{2}(\underset{12}{N} - 2\underset{4}{F} - \underset{0}{S})$

$g \neq 0 \quad [K, K^{\dagger}] \neq 0 \quad$ SECOND CLASS

4 DEGREES OF FREEDOM $= \frac{1}{2}(\underset{12}{N} - 2\underset{0}{F} - \underset{4}{S})$

EXTENDED R-S THEORY

$[K(\vec{x}), K(\vec{y})] = 0 \quad$ 4 DEG FREEDOM $= \frac{1}{2}(\underset{12+4}{N} - 2\underset{4}{F} - \underset{0}{S})$

FIRST CLASS FOR $g \neq 0$

ADD CONSTRAINTS TO BREAK GAUGE INVARIANCE

$\phi_{1,2}$ INVOLVE ONLY $\vec{\Phi}, \Lambda$

$\chi_{1,2}$ INVOLVE ONLY $\vec{\Phi}^\dagger, \Lambda^\dagger$

$$M_{ab}(\vec{x}-\vec{y}) = [\phi_a(\vec{x}), \chi_b(\vec{y})]$$

$$[F(\vec{\Phi},\Lambda), G(\vec{\Phi},\Lambda,\vec{\Phi}^\dagger,\Lambda^\dagger)]_D$$ ← DIRAC BRACKET

$$= [F,G] - \sum_a \sum_b [F,\chi_a] M^{-1}_{ab} [\phi_b, G]$$

QUANTIZATION:

(ANTI-) COMMUTATOR $= i \cdot$ DIRAC BRACKET

$\Lambda = 0$ GAUGE (UNEXTENDED R-S)

$$\phi_1 = \Lambda \, , \quad \phi_2 = \vec{\sigma} \times \vec{D} \cdot \vec{\Phi} - i g \, \vec{\sigma} \cdot \vec{B} \, \Lambda$$

$$\chi_1 = 2 (\vec{p} \cdot \vec{D} + \ell) \, , \quad \chi_2 = \ell$$

DIRAC BRACKETS

$$[\Lambda(\vec{x}), \Lambda(\vec{y})]_D = [\Lambda(\vec{x}), \Phi_j(\vec{y})]_D = 0$$

$$[\Phi_i(\vec{x}), \Phi_j^{\dagger}(\vec{y})]_D = -2i \Big[\big(\delta_{ij} - \tfrac{1}{2} \sigma_i \sigma_j \big) \delta^3(\vec{x} - \vec{y}) - \vec{D}_{x_i} \frac{\delta^3(\vec{x} - \vec{y})}{g \, \vec{\sigma} \cdot \vec{B}} \overleftarrow{D}_{y_j} \Big]$$

AGREES WITH ZERO MASS SPECIALIZATION OF JOHNSON + SUDARSHAN, VELO + ZWANZIGER

NON-PERTURBATIVE! $g \to 0$ SINGULAR

EXTENDED GAUGE COVARIANT RADIATION GAUGE

$$\phi_1 = \vec{D}\cdot\vec{\Psi} - g\,\hat{\sigma}\cdot\vec{B}\,\Lambda \quad , \quad \phi_2 = \hat{\sigma}\times\vec{D}\cdot\vec{\Psi} - i g\,\hat{\sigma}\cdot\vec{B}\,\Lambda$$

$$\chi_1 = 2(\vec{\ell}\cdot\hat{D} + \ell) \quad , \quad \chi_2 = \hat{\ell}\cdot(\hat{\sigma}\times\hat{D}) - i\,\ell$$

$$M_{ab}(\hat{x}-\hat{y}) = [\phi_a(\hat{x}), \chi_b(\hat{y})] = 2\,\mathbb{1}_{ab}(\vec{D}^2_{\hat{x}} + g\,\hat{\sigma}\cdot\vec{B})\,\delta^3(\hat{x}-\hat{y})$$

DEFINE INVERSE

$$M^{-1}_{ab}(\hat{x}-\hat{y}) = \frac{1}{2}\mathbb{1}_{ab}\underbrace{\mathcal{O}(\hat{x}-\hat{y})}_{(\vec{D}^2_{\vec{x}} + g\,\hat{\sigma}\cdot\vec{B})^{-1}}$$

INVERTIBLE FOR $g=0$

DIRAC BRACKETS

$$[\Psi_i(\hat{x}), \Psi^\dagger_j(\hat{y})]_D \quad , \quad [\Psi_i(\hat{x}), \Lambda^\dagger(\hat{y})]_D$$

INVERTIBLE AT $g=0$

$$[\Lambda(\hat{x}), \Lambda^\dagger(\hat{y})]_D = \underline{\frac{2i}{g}\,\frac{\hat{\sigma}\cdot\vec{B}}{\vec{B}^2}}\,\delta^3(\hat{x}-\hat{y}) - 3i\,\mathcal{O}(\hat{x}-\hat{y})$$

SINGULAR

SO THE $g=0$ SINGULARITY HAS BEEN MOVED TO THE Λ BRACKET IN THE EXTENDED THEORY, BUT NOT ELIMINATED

CAN ALSO SHOW THAT $i \cdot [\Psi_i, \Psi_j^\dagger)$ IS NON-POSITIVE-SEMIDEFINITE $\Rightarrow$ NEED AN INDEFINITE METRIC HILBERT SPACE TO QUANTIZE

CONCLUSION: MASSLESS GAUGED R-S IS INHERENTLY NON-PERTURBATIVE

$\Rightarrow$ LOOK AT HOW R-S APPEARS IN MY SO(8) MODEL

COUPLING OF R-S IN THE SU(8) MODEL

FIELDS	ψ_μ^α	$\alpha = 1, \ldots, 8$	8	R-S
	$\lambda_{[\alpha\beta]}$	$\alpha, \beta = 1, \ldots, 8$	$\overline{28}$	SPIN 1/2
	$\phi_{[\alpha\beta\gamma]}$	$\alpha, \beta, \gamma = 1, \ldots, 8$	56	SCALAR

- NO YUKAWA COUPLING $\bar{\psi}_{\mu\alpha} \psi^{\alpha\beta} \phi^*[\gamma\delta\varepsilon]$

  CANNOT MAKE SU(8) INVARIANT

  $\Rightarrow$ NO R-S MASS

- ALLOWED COUPLING $\bar{\lambda}^{[\alpha\beta]} \gamma^\nu \psi_\nu^\gamma \phi^*_{[\alpha\beta\gamma]} + \dagger$

  AFTER ϕ SYMMETRY BREAKING $\phi^* \to \bar{\phi}^*$

$[\gamma_5, \gamma^\mu \gamma^\nu] = 0 \Rightarrow$ GIVES A "PROTO-MASS" TERM STILL CHIRAL

ABELIANIZED MODEL

- $S = S(\psi_\mu) + S(\lambda) + S_{\text{INTERACTION}}$

AS BEFORE (UP TO FACTOR OF 2)

$$S(\psi_\mu) = \int d^4x\, \bar{\psi}_\mu R^\mu \,, \quad R^\mu = i\,\varepsilon^{\mu\lambda\nu\rho}\gamma_5\gamma_\lambda D_\nu \psi_\rho$$
$$= -\gamma^{\mu\nu\rho} D_\nu \psi_\rho$$

$$D_\nu = \partial_\nu + g A_\nu \qquad \bar{\psi}_\mu = \psi_\mu^\dagger i\gamma^0$$

$$S(\lambda) = -\int d^4x\, \bar{\lambda}\gamma^\nu D_\nu \lambda \,, \quad \bar{\lambda} = \lambda^\dagger i\gamma^0$$

$$S_{\text{INTERACTION}} = m \int d^4x \left(\bar{\lambda}\gamma^\nu \psi_\nu - \bar{\psi}_\nu \gamma^\nu \lambda \right)$$

m REPLACES $\bar{\Phi}$

- EULER-LAGRANGE EQS:

$$R^\mu = m\gamma^\mu \lambda \,, \quad \gamma^\nu D_\nu \lambda = m\gamma^\nu \psi_\nu$$

CONSTRAINTS

- **PRIMARY** $\quad R^0 = m\gamma^0\lambda$

$$\Leftrightarrow \gamma^0 R^0 = i\gamma_5\gamma^0\varepsilon^{0nr}\gamma_e D_n\psi_r = -m\lambda$$

$m \to \infty$

$\Rightarrow \lambda = 0$

- **SECONDARY**

$$D_\nu\,(R^\nu = m\gamma^\nu\lambda) \Rightarrow$$

$$-g\,\gamma^{\mu\nu\rho}F_{\mu\nu}\psi_\rho = 2m^2\gamma^\nu\psi_\nu$$

(USED λ EQ OF MOTION AND $[D_\mu, D_\nu] = g(\partial_\mu A_\nu - \partial_\nu A_\mu) = g F_{\mu\nu}$)

- **IN LIMIT** $\quad g F_{\mu\nu}/m^2 \to 0$ THUS BECOMES

$$0 = \gamma^\nu\psi_\nu \quad \leftarrow \text{USUAL FREE R-S GAUGE CONSTRAINT}$$

- WILL USE COVARIANT EQS LATER FOR ANOMALY CALCULATION

LEFT CHIRAL REDUCTION

$\psi \to \Psi_0, \vec{\Psi}$
$\lambda \to \ell$

- $$S = S(\Psi_\mu) + S(\ell) + S_{\text{INTERACTION}}$$

$$S(\Psi_\mu) = \int d^4x \, \big(-\Psi_0^\dagger \vec{\sigma} \cdot \vec{D} \times \vec{\Psi} + \vec{\Psi}^\dagger \cdot \vec{\sigma} \times \vec{D} \, \Psi_0 + \vec{\Psi}^\dagger \cdot \vec{D} \times \vec{\Psi} - \vec{\Psi}^\dagger \cdot \vec{\sigma} \times D_0 \vec{\Psi} \big)$$

$$S(\ell) = \int d^4x \, \ell^\dagger (D_0 - \vec{\sigma} \cdot \vec{D}) \ell$$

$$S_{\text{INTERACTION}} = im \int d^4x \, \big(-\ell^\dagger \Psi_0 + \ell^\dagger \vec{\sigma} \cdot \vec{\Psi} + \Psi_0^\dagger \ell - \vec{\Psi}^\dagger \cdot \vec{\sigma} \ell \big)$$

- **EULER-LAGRANGE EQS**

$$0 = \vec{\sigma} \times \vec{D} \Psi_0 + \vec{D} \times \vec{\Psi} - \vec{\sigma} \times D_0 \vec{\Psi} - im \vec{\sigma} \ell$$

$$0 = (D_0 - \vec{\sigma} \cdot \vec{D}) \ell + m (\vec{\sigma} \cdot \vec{\Psi} - \Psi_0)$$

- **PRIMARY CONSTRAINT** $0 = \chi \equiv \vec{\sigma} \cdot \vec{D} \times \vec{\Psi} - im\ell$

SECONDARY CONSTRAINT + ℓ EULER EQ $\rightarrow$

- $\Phi_0 = \Phi_0[\vec{\Phi}] = (m^2 + g\,\hat{c}\cdot\hat{B})^{-1}[g(\hat{B} + \hat{c}\times\hat{E}) + m^2\hat{c}]\cdot\vec{\Phi}$

INVERTIBLE AT $g=0$!

$\xrightarrow{\text{ZERO FIELD}} \quad \hat{c}\cdot\vec{\Phi}$

$\left.\begin{array}{l}\vec{\Phi} \text{ EULER EQ.} \\ \ell \text{ EULER EQ.}\end{array}\right\}$ + PRIMARY CONSTRAINT

$\rightarrow \quad D_0\vec{\Phi} = \hat{D}\Phi_0 + i\,\hat{D}\times\vec{\Phi}$

- CONVERSELY, THIS + PRIMARY CONSTRAINT $\hat{c}\cdot\hat{D}\times\vec{\Phi} = i m \ell$

$\rightarrow$ ℓ EULER EQ.

CANONICAL MOMENTA

$\vec{p} = \vec{\Phi}^\dagger \times \vec{\sigma} \qquad \ell = -i\ell^\dagger$

CONSTRAINT BRACKETS

$$M(\vec{x},\vec{y}) \equiv [\chi(\vec{x}), \chi^\dagger(\vec{y})] = -i(m^2 + g\,\hat{\sigma}\cdot\vec{B}(\vec{x}))\,\delta^3(\vec{x}-\vec{y})$$

INVERTIBLE AT $g=0$!

$\Rightarrow \chi, \chi^\dagger$ SECOND CLASS FOR $g=0$
$g\neq 0$

NO CHANGE IN DEGREES OF FREEDOM AS GAUGE FIELD TURNED ON

DEGREES OF FREEDOM

$$\frac{1}{2}(\underbrace{N}_{N_{\vec{\Phi}}=12,\ N_\ell=4} - 2\underset{0}{\underset{\uparrow}{F}} - \underset{4}{\underset{\uparrow}{S}}) = 6 \text{ DEGREES OF FREEDOM}$$

← AGREES WITH COUNTING OF ZERO FIELD EIGENMODES

ZERO EXTERNAL FIELD SOLUTIONS

$$\vec{\Psi} = \hat{\vec{C}}\, e^{i\Omega t + iKz} \quad , \quad \ell = L\, e^{i\Omega t + iKz}$$

- $\vec{\Psi}$ EULER EQ. IS $\quad \Omega \hat{\vec{C}} = K \hat{z}\, \hat{\vec{\sigma}} \cdot \hat{\vec{C}} + iK \hat{z} \times \hat{\vec{C}}$

- PRIMARY CONSTRAINT IS $\quad iK\, \hat{\vec{\sigma}} \cdot \hat{z} \times \hat{\vec{C}} = imL$

  (THESE $\Rightarrow$ ℓ EULER EQ.)

- WRITING $\quad \frac{\Omega}{K}[C] = [W][C]$

$$(C) = [C_1^{\uparrow}\ \ C_1^{\downarrow}\ \ C_2^{\uparrow}\ \ C_2^{\downarrow}\ \ C_3^{\uparrow}\ \ C_3^{\downarrow}]$$

[W] NON-HERMITIAN $\Rightarrow$ REDUCE TO JORDAN CANONICAL FORM

FOUR TRUE EIGENVECTORS $\rightarrow$

$$\left.\begin{array}{ll} V_1 & \Omega/K = 1 \\ V_2 & \Omega/K = -1 \end{array}\right\} \text{LONGITUDINAL}$$

$$\left.\begin{array}{ll} V_3 & \Omega/K = 1 \\ V_4 & \Omega/K = -1 \end{array}\right\} \text{TRANSVERSE}$$

$$\left\{\begin{array}{lll} V_5 & [W]V_5 = V_5 + V_1 & \Omega/K = 1 \\ V_6 & [W]V_6 = -V_6 + V_2 & \Omega/K = -1 \end{array}\right\} \text{TWO JORDAN EIGENVECTORS}$$

WAVE VELOCITY IN EXTERNAL GAUGE FIELD

- LOOK AT DISCONTINUITIES AROUND POINT WHERE $\vec{\Phi}$, $\vec{E}$, $\vec{B}$ CONTINUOUS

$$\partial_0 \hat{\Phi} = \vec{\nabla}\, \vec{R} \cdot \hat{\Phi} + i\, \hat{\nabla} \times \hat{\Phi}$$

$$\hat{c} \cdot \hat{\nabla} \times \hat{\Phi} = i m \ell$$

$\vec{R}$ FUNCTION OF $\hat{E}, \hat{B}, \hat{c}$

- **TRANSVERSE MODES** $\hat{C}_T \equiv C_1 \hat{x} + C_2 \hat{y} \neq 0$ LUMINAL
- **LONGITUDINAL MODES** $\hat{C}_T = 0$ $C_3 \neq 0$

$$-\frac{\Omega_+}{K}\,\frac{\Omega_-}{K} = 1 + \frac{E_T^2 + B_T^2}{m^4 - \hat{B}^2}$$

WHEN $m^4 > \hat{B}^2$ AT LEAST ONE IS SUPERLUMINAL

DIRAC BRACKETS

- $$[\Psi_i(\vec{x}), \Psi_j(\vec{y})]_D = -2i\left[(\delta_{ij} - \tfrac{1}{2}\sigma_i\sigma_j)\,\delta^3(\vec{x}-\vec{y}) - \vec{D}_{x_i}\,\frac{\delta^3(\vec{x}-\vec{y})}{m^2 + g\,\vec{\sigma}\cdot\vec{B}}\,\overleftarrow{D}_{y_j}\right]$$

  INVERTIBLE →

- $$[\lambda(\vec{x}), \lambda^+(\vec{y})]_D = -i\,\delta^3(\vec{x}-\vec{y})\,\frac{g\,\vec{\sigma}\cdot\vec{B}}{m^2 + g\,\vec{\sigma}\cdot\vec{B}}$$ ← VANISHES $m \to \infty$

- $$[\lambda(\vec{x}), \Psi_j^+(\vec{y})]_D = im\,\frac{\delta^3(\vec{x}-\vec{y})}{m^2 + g\,\vec{\sigma}\cdot\vec{B}}\,\overleftarrow{D}_{y_j}$$

- DIRAC BRACKETS GENERATE CORRECT HAMILTONIAN EQS. OF MOTION

PATH INTEGRAL QUANTIZATION

$$\langle \text{out} | S | \text{in} \rangle = \int d\mu \, e^{iS}$$

$$d\mu = (\det M)^{-1} \, d\Phi_\mu \, d\Phi^\dagger_\mu \, d\ell \, d\ell^\dagger$$

GHOST CONTRIBUTION TO CHIRAL ANOMALY

M COMES FROM BRACKET $(\chi, \chi^\dagger] \propto (\tilde{\chi}, \tilde{\chi}^\dagger]$

$\tilde{\chi} = -m\lambda + \cdots$ SAME CHIRALITY SPINOR - REMOVES DEGREE OF FREEDOM OF SAME CHIRALITY

- GHOST FOR THIS CONTRIBUTES $-1 \times$ STANDARD SPIN 1/2 ANOMALY

CONJECTURE:

$$M \propto \lim_{\delta \to 0} \left(m^2 + g \, \vec{\sigma} \cdot \vec{B} + \delta \, m \, \gamma^\mu D_\mu \right) \delta^4(x-y)$$

$$(\det M)^{-1} \propto \int d\theta \, d\bar{\theta} \, e^{i \int d^4x \, \bar{\theta} \left(\gamma^\mu D_\mu + m/\delta + \frac{g}{m\delta} \vec{\sigma} \cdot \vec{B} \right) \theta}$$

ANOMALY INDEPENDENT OF THIS

- **FEYNMAN RULES**

$$S = \frac{1}{(2\pi)^4} \int d^4k \, e^{ik\cdot x} \, S[k]$$

$$S[k] = \bar{\psi}_\mu(k) \, \bar{\lambda}(k) \, \mathcal{M} \begin{pmatrix} \psi_\rho(k) \\ \lambda(k) \end{pmatrix}$$

$$\mathcal{M} = \begin{bmatrix} -i\gamma^{\mu\nu\rho} k_\nu & -m\gamma^\mu \\ m\gamma^\rho & -i\not{k} \end{bmatrix}$$

- **INVERSE**

$$\mathcal{M}\mathcal{N} = \begin{bmatrix} \delta^\mu{}_\sigma & 0 \\ 0 & 1 \end{bmatrix} \qquad \mathcal{N} = \begin{bmatrix} N_{1\rho\sigma} & N_{2\rho} \\ N_{3\sigma} & N_4 \end{bmatrix}$$

$$N_{1\rho\sigma} = \frac{-i}{2k^2}\left[\gamma_\sigma \not{k} \gamma_\rho + 2\left(\frac{1}{m^2} - \frac{2}{k^2}\right) k_\rho k_\sigma \not{k}\right]$$

$$N_{2\rho} = \frac{\not{k}\, k_\sigma}{m k^2} \qquad N_{3\sigma} = -\frac{\not{k}\, k_\sigma}{m k^2}$$

$$N_4 = 0$$

- **AS $m \to \infty$**

$$\mathcal{N} \to \begin{bmatrix} \tilde{N} & 0 \\ 0 & 0 \end{bmatrix} \qquad \tilde{N}_{\rho\sigma} = \frac{-i}{2k^2}\left[\gamma_\sigma \not{k} \gamma_\rho - \frac{4}{k^2} k_\rho k_\sigma \not{k}\right]$$

$$\gamma^\rho \tilde{N}_{\rho\sigma} = \tilde{N}_{\rho\sigma}\gamma^\sigma = 0 \quad \leftarrow \text{FROM } \gamma^\sigma \psi_\sigma = 0$$

FEYNMAN RULES - CONTINUED

RELATION TO GAUGE FIXED PROPAGATOR FOR FREE R-S

GAUGE FIXING $y \bar{\Psi}_\mu \gamma^\mu \not{\partial} \gamma^\rho \psi_\rho$

$$-i\left[\left(\tfrac{1}{2}+y\right)\gamma^\mu \not{k} \gamma^\rho - \tfrac{1}{2}\gamma^\rho \not{k} \gamma^\mu\right] \hat{N}_{\rho\sigma} = \delta^\mu_\sigma$$

$$\hat{N}_{\rho\sigma}(k) = \frac{-i}{2k^2}\left[\gamma_\sigma \not{k} \gamma_\rho - \frac{1}{k^2}\left(4+\frac{2}{y}\right) k_\rho k_\sigma \not{k}\right]$$

SO $\tilde{N}$ IS $y \to \infty$ LIMIT OF $\hat{N}$

VECTOR VERTEX
$$V^\nu = \begin{bmatrix} -ig\gamma^{\mu\nu\rho} & 0 \\ 0 & -ig\gamma^\nu \end{bmatrix}$$

AXIAL VECTOR VERTEX
$$A^\nu = \begin{bmatrix} \gamma^{\mu\nu\rho} & 0 \\ 0 & \gamma^\nu \gamma_5 \end{bmatrix}$$

WARD IDENTITIES

SPIN 1/2 $\quad S(k) = i/\not{k}$ OBEYS

$$-i\gamma^{\mu} k_{\mu} = S^{-1}(p+k) - S^{-1}(p)$$

$$-i\gamma^{\mu}\gamma_5 k_{\mu} = S^{-1}(p+k)\gamma_5 + \gamma_5 S^{-1}(p)$$

SPIN 3/2 $\quad$ LINEARITY OF $\mathcal{M} = \mathcal{N}^{-1} \Rightarrow$

$$\begin{bmatrix} -i\gamma^{\mu\nu\rho} k_{\nu} & 0 \\ 0 & -i\gamma^{\nu} k_{\nu} \end{bmatrix} = \mathcal{N}^{-1}(k+p) - \mathcal{N}^{-1}(p)$$

$$\begin{bmatrix} -i\gamma^{\mu\nu\rho}\gamma_5 k_{\nu} & 0 \\ 0 & -i\gamma^{\nu}\gamma_5 k_{\nu} \end{bmatrix} = \mathcal{N}^{-1}(k+p)\gamma_5 + \gamma_5 \mathcal{N}^{-1}(p)$$

$$-i\gamma^{\mu\nu\rho} k_{\nu} = \tilde{N}^{-1}(k+p) - \tilde{N}^{-1}(p)$$

$$-i\gamma^{\mu\nu\rho}\gamma_5 k_{\nu} = \tilde{N}^{-1}(k+p)\gamma_5 + \gamma_5 \tilde{N}^{-1}(p)$$

SIMILARLY FOR $\hat{N}$

CHIRAL ANOMALY CALCULATIONS

TO MINIMIZE γ MATRIX ALGEBRA, FOLLOW SHIFT METHOD IN JACKIW 1972 LECTURES

IN 1711.00907 DO THIS FOR SPIN 1/2
3/2 $\tilde{N}$
3/2 $\hat{N}$ $y \to \infty$

3/2 $\tilde{N}$ CALCULATION

$$\tilde{T}^{\nu}_{\sigma\tau} = \int \frac{d^4 r}{(2\pi)^4} (-1)\, \mathrm{tr} \Big[\tilde{N}(r+k_1) V_\sigma \tilde{N}(r) V_\tau \tilde{N}(r-k_2) A^\nu + \tilde{N}(r+k_2) V_\tau \tilde{N}(r) V_\sigma \tilde{N}(r-k_1) A^\nu \Big]$$

AXIAL DIVERGENCE $=0$

VECTOR DIVERGENCE $\neq 0$

$$k_1^\sigma \tilde{T}^{\nu}_{\sigma\tau} = g \int \frac{d^4 r}{(2\pi)^4} \mathrm{tr} \Big[\tilde{N}(r+k_1) V_\tau \tilde{N}(r-k_2) A^\nu - (k_1 \leftrightarrow k_2) \Big]$$

WOULD VANISH IF SHIFT $r \to r + k_2 - k_1$ LEGAL

$\tilde{N}$ CALCULATION CONTINUED

USE IDENTITY $\gamma^{\alpha\tau\beta} = \frac{1}{2}[\gamma^\alpha, \gamma^\tau]\gamma^\beta - \gamma^\alpha \eta^{\tau\beta} + \eta^{\alpha\beta}\gamma^\tau$

PROJECT TO 0 ON $\tilde{N}$

$\Rightarrow$ CAN REPLACE $V_\tau \to -ig\gamma_\tau$ $A^\nu \to \gamma^\nu \gamma_5$

FOR $k_1 - k_2$ SMALL

$$k_1^\sigma \tilde{T}^\nu_{\sigma\tau} \simeq -ig^2 (k_1 - k_2)_\kappa \int \frac{d^4 r}{(2\pi)^4} \frac{\partial}{\partial r_\kappa} \mathrm{tr}\left[\tilde{N}(r+k_2)\gamma_\tau \tilde{N}(r-k_1)\gamma^\nu \gamma_5\right]$$

STOKES THEOREM $\int_V d^4 r \frac{\partial}{\partial r_\kappa} f(r) = \int_S dS^\kappa f(r)$

+ WICK ROTATION $\Rightarrow$

$$k_1^\sigma \tilde{T}^\nu_{\sigma\tau} \simeq \frac{g^2}{(2\pi)^4} (k_1 - k_2)_\kappa \int_S dS^\kappa \, \mathrm{tr}\left[\tilde{N}(r+k_2)\gamma_\tau \tilde{N}(r-k_1)\gamma^\nu \gamma_5\right]$$

$= 5 \times$ STANDARD SPIN 1/2 ANOMALY

SUMMARY AND FUTURE DIRECTIONS

- COUPLED MODEL HAS NO CHANGE IN DEGREES OF FREEDOM FROM $g=0$ TO $g \neq 0$
- CONSTRAINT INVERTIBLE, CAN DO PERTURBATION EXPANSION IN g
- ANOMALY CALCULABLE

	GHOST LOOP	FERMION LOOP
$m=0$	$-2+1=-1$	$4+1=5$
$m \neq 0$	-1 (?)	5

- STILL HAS NON-POSITIVE ANTICOMMUTATORS FOR $|g\vec{B}| > m$ PROBLEM?
- STILL HAS LONGITUDINAL TACHYONS; ELIMINATED BY DYNAMICAL CHIRAL SYMMETRY BREAKING?