SOFT PIONS, CURRENT ALGEBRAS, AND SUM RULES

Stephen L. Adler
Institute for Advanced Study
Princeton, NJ 08540

- Preliminaries
- Soft Pions
- Current Algebras
- Sum Rules
- Further Soft Pion Applications
- Deep Inelastic Scattering Sum Rules
- Anomalies and PCAC
- Historical Significance
- Updates and Revivals

For more details and citations: See "Commentaries" in my book "Adventures in Theoretical Physics", Vol. 37 in the World Scientific Series in 20th Century Physics, Chapters 2 and 3.

PRELIMINARIES

NOETHER'S THEOREM

$$\mathcal{L}(x) = \mathcal{L}[\Phi_i, \partial_\lambda \Phi_i]$$

$$\Phi_j(x) \to \Phi_j(x) + \Lambda(x)\, G_j[\Phi_i]$$

$$\delta\mathcal{L} = \frac{\delta \mathcal{L}}{\delta \Lambda}\,\Lambda + \frac{\delta \mathcal{L}}{\delta(\partial_\alpha \Lambda)}\,\partial_\alpha \Lambda$$

CURRENT $\quad J^\alpha = -\dfrac{\delta \mathcal{L}}{\delta(\partial_\alpha \Lambda)} = -\sum_j \dfrac{\delta \mathcal{L}}{\delta(\partial_\alpha \Phi_j)}\, G_j$

OBEYS $\quad \partial_\alpha J^\alpha = -\delta\mathcal{L}/\delta\Lambda$

IF $\mathcal{L}$ IS INVARIANT FOR CONSTANT Λ, THEN $\delta\mathcal{L}/\delta\Lambda = 0$ AND THE CORRENT IS CONSERVED

IF $\delta\mathcal{L}/\delta\Lambda$ IS $\neq 0$ BUT SMALL, THE CURRENT IS PARTIALLY CONSERVED

APPLICATION TO THE DIRAC LAGRANGIAN

$$\mathcal{L} = \bar{\psi}\gamma^\mu \partial_\mu \psi + im\bar{\psi}\psi \qquad \bar{\psi} = \psi^\dagger \gamma^0$$

① $\psi \to e^{i\theta}\psi$

$\mathcal{L}$ INVARIANT $\Rightarrow$ CONSERVED VECTOR CURRENT $V^\mu = \bar{\psi}\gamma^\mu\psi$

$$\partial_\mu V^\mu = 0$$

② $\psi \to e^{i\theta\gamma_5}\psi$ $\qquad [\gamma_5, \gamma^0\gamma^\mu] = 0$, $[\gamma_5, \gamma^0] \neq 0$

$\mathcal{L}$ INVARIANT ONLY WHEN $m = 0$

$\Rightarrow$ PARTIALLY CONSERVED AXIAL-VECTOR CURRENT $A^\mu = \bar{\psi}\gamma^\mu\gamma_5\psi$

$$\partial_\mu A^\mu = im\bar{\psi}\gamma_5\psi$$

SOFT PIONS

PHYSICAL REVIEW VOLUME 110, NUMBER 5 JUNE 1, 1958

Decay of the Pi Meson

M. L. GOLDBERGER AND S. B. TREIMAN
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received February 10, 1958)

GOLDBERGER-TREIMAN RELATION

f_π = CHARGED PION DECAY CONSTANT

g_A = NUCLEON AXIAL-VECTOR COUPLING CONSTANT

g_r = PION-NUCLEON COUPLING CONITANT

M_N = NUCLEON MASS

$$f_\pi = \frac{\sqrt{2}\, M_N g_A}{g_r}$$

(THIS IS PDG f_π = ADLER-DASHEN f_π/M_π^2)

AMAZING - GOOD TO AROUND 6 % (NOW 2.3 %)

REINTERPRETATION AS {PARTIALLY CONSERVED / PION POLE DOMINATED} AXIAL-VECTOR CURRENT

$$\partial_\lambda A^\lambda_{1+i2} = C\,\phi^\dagger_{\pi^+} = C\,\phi_{\pi^-} \qquad C \equiv \frac{\sqrt{2}\,M_N g_A}{g_A(0)} \approx \frac{\sqrt{2}\,M_N g_A}{g_V} \approx f_\pi$$

$$\left(\equiv \mathcal{F}^{S\lambda}_{1+i2}\right)$$

(NAMBU; BERNSTEIN, FUBINI, GELL-MANN + THIRRING; GELL-MANN + LÉVY; BERNSTEIN, GELL-MANN + MICHEL; CHOU ALL 1960)

EARLY APPLICATIONS

PHYSICAL REVIEW VOLUME 125, NUMBER 4 FEBRUARY 15, 1962

Chirality Conservation and Soft Pion Production*

Y. NAMBU AND D. LURIÉ
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, University of Chicago, Chicago, Illinois
(Received September 25, 1961)

χ = NUCLEON + PION AXIAL CHARGE $= -i\int J_4\, d^3x$

$\langle \alpha^{in}|\chi|\alpha^{in}\rangle = \langle \alpha^{out}|\chi|\alpha^{out}\rangle$

PHYSICAL REVIEW VOLUME 135, NUMBER 4B 24 AUGUST 1964

Tests of the Conserved Vector Current and Partially Conserved Axial-Vector Current Hypotheses in High-Energy Neutrino Reactions*

STEPHEN L. ADLER†
Princeton University, Princeton, New Jersey
(Received 8 April 1964)

(CVC) (PCAC)

RELATES $\frac{d\sigma}{d\Omega}(\nu + \alpha \to l + \beta)$ FORWARD l TO CROSS SECTION FOR $\pi + \alpha \to \beta$

THAT IS, $\nu\bar{l}$ AT $k^2 \simeq 0$ IS LIKE π BY PCAC

SOFT PION THEOREMS AS A PRECISION TOOL

PHYSICAL REVIEW VOLUME 137, NUMBER 4B 22 FEBRUARY 1965

Consistency Conditions on the Strong Interactions Implied by a Partially Conserved Axial-Vector Current

STEPHEN L. ADLER*
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey and Bell Telephone Laboratories, Murray Hill, New Jersey
(Received 15 September 1964)

$A^{\pi N(+)}$ = SYMMETRIC ISOSPIN PION-NUCLEON SCATTERING AMPLITUDE

$A^{\pi N(+)}(\nu = \nu_B = 0) \approx \frac{g_A^2}{M_N}$ GOOD TO ~ 10%

$A^{\pi\pi}(s = t = u = -M_\pi^2, k^2 = 0) = 0$ "ADLER ZERO"

3 PIONS ON-SHELL, q^μ PION OFF-SHELL

PHYSICAL REVIEW VOLUME 139, NUMBER 6B 20 SEPTEMBER 1965

Consistency Conditions on the Strong Interactions Implied by a Partially Conserved Axial-Vector Current. II

STEPHEN L. ADLER*
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts
(Received 26 March 1965)

GENERAL EXTERNAL LINE INSERTION RULES

NUCLEON LINE : NONZERO INSERTION
PION LINE : ZERO INSERTION

CURRENT ALGEBRAS

Physics Vol. 1, No. 1, pp. 63–75, 1964. Pergamon Press, Inc. Printed in the United States.

THE SYMMETRY GROUP OF VECTOR AND AXIAL VECTOR CURRENTS*

MURRAY GELL-MANN

California Institute of Technology, Pasadena, California

(*Received* 25 *May* 1964)

$$F_i(t) = -i\int \mathscr{F}_{i4}\, d^3x, \qquad F_i^5(t) = -i\int \mathscr{F}_{i4}^5\, d^3x,$$

F_j, F_j^5 SU(3) OCTETS

$$[F_i(t), F_j(t)] = if_{ijk}F_k(t). \qquad [F_i(t), F_j^5(t)] = if_{ijk}F_k^5(t).$$

(c) The commutation rules of the operators $F_i^5(t)$ close the algebraic system by giving

REVOLUTIONARY!

$$[F_i^5(t), F_j^5(t)] = if_{ijk}F_k(t).$$

Physics Vol. 1, No. 4, pp. 229-247, 1965. Physics Publishing Co. Printed in Great Britain.

RENORMALIZATION EFFECTS FOR PARTIALLY CONSERVED CURRENTS*

S. FUBINI

Istituto de Fisica dell'Università-Torino and CERN, Geneva

G. FURLAN**

CERN, Geneva

(*Received* 23 *December* 1964)

$\langle N(\vec{p})|[F_{1+i2}, F_{1-i2}]|N(\vec{p})\rangle$

$p_z \to \infty$

INFINITE MOMENTUM FRAME

SUM RULES

VOLUME 14, NUMBER 25 PHYSICAL REVIEW LETTERS 21 JUNE 1965

CALCULATION OF THE AXIAL-VECTOR COUPLING CONSTANT RENORMALIZATION IN β DECAY

Stephen L. Adler*

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts
(Received 17 May 1965)

RENORMALIZATION OF THE WEAK AXIAL-VECTOR COUPLING CONSTANT*

William I. Weisberger

Stanford Linear Accelerator Center, Stanford University, Stanford, California
(Received 26 May 1965)

$$g_A^2 = 1 + f_\pi^2 \frac{2}{\pi} \int_{M_N+M_\pi}^{\infty} \frac{W\,dW}{W^2-M_N^2} \left[\sigma^{\pi^+ p}(W) - \sigma^{\pi^- p}(W)\right]$$

gives $g_A = 1.24$
expt $g_A = 1.272$

PHYSICAL REVIEW VOLUME 140, NUMBER 3B 8 NOVEMBER 1965

Sum Rules for the Axial-Vector Coupling-Constant Renormalization in β Decay*

STEPHEN L. ADLER†

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts
(Received 7 June 1965)

- $\pi\pi$ SUM RULE $1 = f_\pi^2 \frac{1}{\pi} \int_{2M_\pi}^{\infty} \frac{W\,dW}{W^2-M_\pi^2} \left[\sigma^{\pi^-\pi^+}(W) - \sigma^{\pi^+\pi^+}(W)\right]$

Need large $I=0$ S-wave $\pi\pi$ cross section at low energy to saturate sum rule

- SUM RULE FOR FORWARD LEPTON ($k^2 \approx 0$) INCLUSIVE νN SCATTERING

FURTHER SOFT PION APPLICATIONS (MANY!)

- CALLAN + TREIMAN $K \to \pi \ell \nu$ RELATED TO $K \to \ell \nu$
- WEINBERG $K \to 2\pi \ell \nu$ IMPORTANCE OF π POLE TERMS
- WEINBERG $\pi\pi$ SCATTERING LENGTHS, MULTIPLE π PRODUCTION

→ THESE ARE REPRINTED IN ADLER + DASHEN, "CURRENT ALGEBRAS AND APPLICATIONS TO PARTICLE PHYSICS", W.A. BENJAMIN (1968)

→ SEE ALSO COLEMAN, "ASPECTS OF SYMMETRY", CAMBRIDGE (1985), CHAPTER 2 ON "SOFT PIONS"

NAMBU-GOLDSTONE BOSONS

- CHIRAL LAGRANGIANS AS GENERATING FUNCTIONS FOR SOFT PION THEOREMS WERE SYSTEMATIZED BY WEINBERG; AND BY CALLAN, COLEMAN, WESS + ZUMINO
- CHIRAL $SU(3) \times SU(3)$ AS A STRONG INTERACTION SYMMETRY WAS DEVELOPED BY DASHEN; GELL-MANN, OAKES + RENNER

DEEP INELASTIC SCATTERING SUM RULES

PHYSICAL REVIEW VOLUME 143, NUMBER 4 MARCH 1966

Sum Rules Giving Tests of Local Current Commutation Relations in High-Energy Neutrino Reactions

STEPHEN L. ADLER*
CERN, Geneva, Switzerland and Lyman Laboratory, Harvard University, Cambridge, Massachusetts
(Received 6 October 1965)

$$2 = \int_0^\infty d\nu \left[W_2^{\bar{\nu}p}(q^2,\nu) - W_2^{\nu p}(q^2,\nu) \right]$$

$q = k_\nu - k_\ell$
$\nu = E_\nu - E_\ell$

$$\Rightarrow \lim_{E_\nu \to \infty} \left[\frac{d\sigma^{\bar{\nu}p}}{d(q^2)} - \frac{d\sigma^{\nu p}}{d(q^2)} \right] = \frac{G_F^2}{\pi}$$

VOLUME 16, NUMBER 10 PHYSICAL REVIEW LETTERS 7 MARCH 1966

INEQUALITY FOR ELECTRON AND MUON SCATTERING FROM NUCLEONS*

J. D. Bjorken
Stanford Linear Accelerator Center, Stanford University, Stanford, California
(Received 7 February 1966)

$$\lim_{E_e \to \infty} \frac{d(\sigma_p + \sigma_n)}{dq^2} > \frac{2\pi\alpha^2}{q^4}$$

PHYSICAL REVIEW VOLUME 179, NUMBER 5 25 MARCH 1969

Asymptotic Sum Rules at Infinite Momentum*

J. D. BJORKEN
Stanford Linear Accelerator Center, Stanford University, Stanford, California
(Received 30 September 1968)

SCALING: $\nu W_2(q^2,\nu) \to F_2(x)$ $\quad x = q^2/(2M_N\nu)$

ANOMALIES AND PCAC

PHYSICAL REVIEW VOLUME 177, NUMBER 5 25 JANUARY 1969

Axial-Vector Vertex in Spinor Electrodynamics

STEPHEN L. ADLER
Institute for Advanced Study, Princeton, New Jersey 08540
(Received 24 September 1968)

IL NUOVO CIMENTO VOL. LX A, N. 1 1° Marzo 1969

A PCAC Puzzle: $\pi^0 \to \gamma\gamma$ in the σ-Model.

J. S. BELL
CERN - Geneva

R. JACKIW (*)
CERN - Geneva

Jefferson Laboratory of Physics, Harvard University - Cambridge, Mass.

(ricevuto l'11 Settembre 1968)

$$\partial^\mu \mathcal{J}^5_{3\mu} = \frac{f_\pi}{\sqrt{2}} \phi_{\pi^0} + S \frac{\alpha}{4\pi} F^{\xi\zeta} F^{\nu\rho} \varepsilon_{\xi\zeta\nu\rho} \qquad S = \sum_j g_j Q_j^2$$

$\pi^0 \to 2\gamma$ MATRIX ELMENT FOR CHARGE $2/3, -1/3$ QUARKS FACTOR OF 3 TOO SMALL

PHYSICAL REVIEW VOLUME 182, NUMBER 5 25 JUNE 1969

Absence of Higher-Order Corrections in the Anomalous Axial-Vector Divergence Equation

STEPHEN L. ADLER AND WILLIAM A. BARDEEN*
Institute for Advanced Study, Princeton, New Jersey 08540
(Received 24 February 1969)

S HAS NO HIGHER LOOP CORRECTIONS - DOES COUNT QUARKS

HISTORICAL SIGNIFICANCE

- QUANTUM FIELD THEORY, NOT RECIPROCAL BOOTSTRAP!
- PCAC → APPROXIMATE CHIRAL INVARIANCE OF STRONG INTERACTIONS
- CURRENT ALGEBRA → NON-ABELIAN (YANG-MILLS) GAUGE STRUCTURE OF ELECTROWEAK SECTOR
- DEEP INELASTIC SUM RULES, SCALING → REALITY OF QUARKS
- ANOMALY CALCULATION OF $\pi^0 \to 2\gamma$ → COLOR TRIPLING OF QUARKS

FOR AN EXCELLENT BOOK-LENGTH DISCUSSION, SEE:

TIAN YU CAO "FROM CURRENT ALGEBRA TO QUANTUM CHROMODYNAMICS: A Case for Structural Realism"
Cambridge University Press (2010)

UPDATES AND REVIVALS

PHYSICAL REVIEW D 75, 116002 (2007)

Evaluation of the axial-vector commutator sum rule for pion-pion scattering

Stephen L. Adler
Institute for Advanced Study, Einstein Drive Princeton, New Jersey 08540, USA

F. J. Ynduráin
Departamento de Física Teórica, C-XI, Universidad Autónoma de Madrid, Canto Blanco, E-28049, Madrid, Spain
(Received 10 April 2007; published 4 June 2007)

UPDATED EVALUATION OF A-W AND $\pi\pi$ SUM RULES AND G-T RELATION SATISFIED TO BETTER THAN 6%

Adler sum rule - Scholarpedia

Adler sum rule

Stephen L. Adler (2009), Scholarpedia, 4(6):8653.

UPDATED DERIVATION USING THE FULL 3 FAMILY CKM MATRIX FOR WEAK MIXING

THE AXION – VERY LIGHT PSEUDO-NAMBU-GOLDSTONE BOSON
NEUTRAL PSEUDOSCALAR PION ANALOG
ARISES (WEINBERG; WILCZEK) FROM SPONTANEOUS BREAKING OF PECCEI-QUINN SYMMETRY INTRODUCED TO SOLVE STRONG CP PROBLEM

POSSIBLE LIGHT DARK MATTER CANDIDATE