

Symmetries, Duality, and the Unity of Physics

2017

Nathan Seiberg

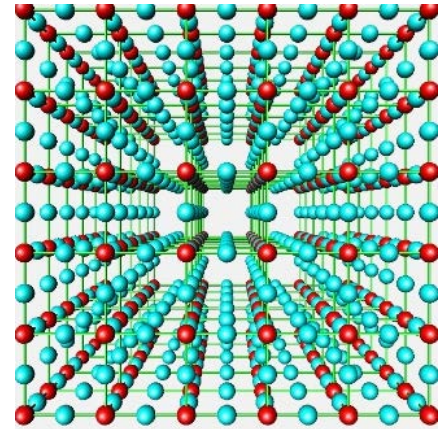
IAS

Symmetries



Physicists love symmetries

- Galileo, Lorentz, Poincare
- Crystallography
- Global symmetry (flavor)
 - e.g. Isospin $SU(2)$, $SU(3)$
- Gauge (local) symmetry
 - $A_\mu \rightarrow A_\mu + \partial_\mu \Lambda$ with $\Lambda(x)$ in Maxwell theory
 - $g_{\mu\nu} \rightarrow g_{\mu\nu} + \partial_\mu \xi^\rho g_{\rho\nu} + \partial_\nu \xi^\rho g_{\mu\rho} + \xi^\rho \partial_\rho g_{\mu\nu}$ with $\xi^\rho(x)$ in General Relativity
 - Color $SU(3)$, the standard model $SU(3) \times SU(2) \times U(1)$



Gauge symmetry is deep

- Largest symmetry (a group for each point in spacetime)
- Useful in making the theory manifestly Lorentz invariant, unitary, and local (and hence causal)
- Appears in
 - Maxwell theory, the Standard Model of particle physics
 - General Relativity
 - Many condensed matter systems
 - Deep mathematics (fiber bundles)

But

- Because of Gauss law the Hilbert space is gauge invariant.
- Hence: **gauge symmetry is not a symmetry.**
 - It does not act on anything.
 - All the operators are gauge invariant.
- A better phrase is **gauge redundancy.**

Gauge symmetry is not a symmetry

- Not surprising to mathematicians and some physicists
 - Manifestly true in many lattice constructions in condensed matter physics
 - Manifestly true in some low-dimensional continuum quantum field theories
- In particle physics, perhaps we should not look for a more complete theory with a larger gauge group
- Of course, this does not mean that gauge symmetry is not an extremely powerful concept and a useful tool.

Emergent symmetries

Global symmetries can emerge as accidental symmetries at long distance.

- Then they are approximate.
- Examples
 - Parity and time reversal symmetries are approximate symmetries at low energy, even though they are not symmetries of the standard model of particle physics
 - Approximate baryon number symmetry (and the related stability of the proton)

Emergent symmetries

Gauge symmetries (redundancies) can be emergent.

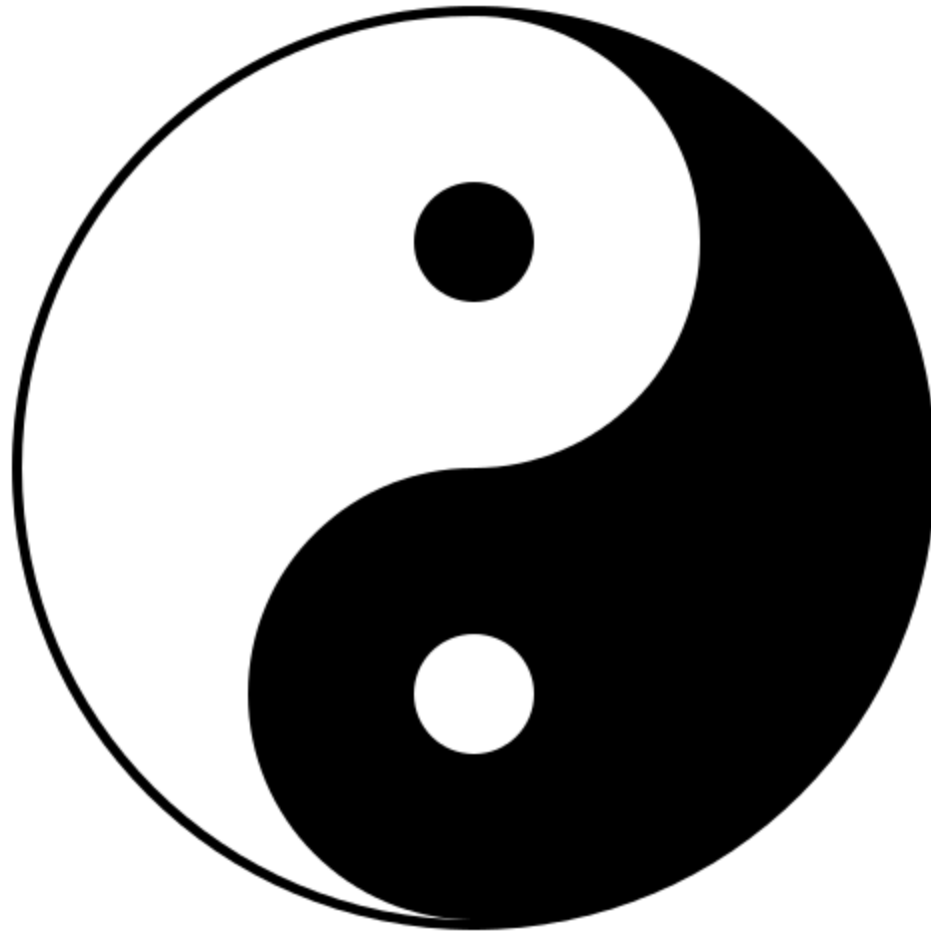
- They must be exact because they are not symmetries.
- First examples
 - lattice systems (mostly in condensed matter)
 - FQHE
 - continuum quantum field theory in low dimensions
 - These systems do not have a massless spin one particle reflecting the emergent gauge symmetry.
- We will soon discuss examples in higher dimensions

Symmetries in a theory of gravity

General considerations based on black holes strongly suggest that in any theory of quantum gravity

- There cannot be any exact **global symmetry**
 - Approximate, accidental global symmetries are possible.
- **Gauge symmetries** are possible.
 - The spectrum must include excitations with all electric and magnetic charges that are compatible with the symmetry.
 - Actually, this follows from the previous point (notion of generalized global symmetries).

Duality



Simple examples of duality

Simple dualities in free (solvable) theories (easy to establish)

Harmonic oscillator

$$H = \frac{1}{2m} p^2 + \frac{k}{2} q^2$$

Map H to itself:

- Classically, a canonical transformation ($q \rightarrow \frac{p}{\sqrt{mk}}$,
 $p \rightarrow -\sqrt{mk}q$)
- Quantum mechanically, a Fourier transform

Duality and emergent gauge symmetry

Again, easy to establish (essentially Fourier transform):

- A free scalar field in $2+1d$ is dual to Maxwell theory

$$F_{\mu\nu} \sim \epsilon_{\mu\nu\rho} \partial^\rho \phi$$

- The vector potential A_μ is related in a non-local way to ϕ .
- The equation of motion of $F_{\mu\nu}$ is trivial in terms of ϕ .
- The Bianchi identity of $F_{\mu\nu}$ is the equation of motion of ϕ .

Duality and emergent gauge symmetry

Again, easy to establish (essentially Fourier transform):

- Maxwell theory in $3+1d$ is dual to a magnetic Maxwell theory ($E \sim \tilde{B}$, $B \sim -\tilde{E}$)

$$F_{\mu\nu} \sim \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \tilde{F}^{\rho\sigma}$$

- The equation of motion of $F_{\mu\nu}$ is the Bianchi identity of $\tilde{F}_{\mu\nu}$ and vice versa.
- The new vector potential \tilde{A}_μ is related in a non-local way to the original vector potential A_μ .
- Its gauge symmetry is emergent (not present in the original formulation).

Simple examples of duality

In all these examples duality is achieved by a Fourier transform.

Small fluctuations in q \leftrightarrow large fluctuations in p

Small fluctuations in p \leftrightarrow large fluctuations in q

Both descriptions are correct, but depending on the problem one description can be more useful than another.

More examples of duality

These examples involve more subtle (interacting) theories

- Kramers–Wannier duality in the Ising model and its generalizations in higher dimensions including gauge systems
- Many examples in $1+1d$, especially in the context of bosonization, conformal field theory, etc.

More examples of duality

In these more subtle (interacting) examples, the mapping
large fluctuations \leftrightarrow small fluctuations
in the simple examples becomes

strong coupling \leftrightarrow weak coupling

large \hbar \leftrightarrow small \hbar

- Useful in solving complicated systems
- Duality is an intrinsically quantum mechanical phenomenon.

Duality in $3+1d$ $N = 4$ supersymmetry

- This is a scale invariant interacting theory of gluons characterized by a gauge symmetry G and coupling constant α (like the fine structure constant)
- The theory with G and α is equivalent to the theory with another group \tilde{G} and $\frac{1}{\alpha}$
 - Exchanges strong and weak coupling – large and small fluctuations
 - The gluons of \tilde{G} are magnetic monopoles of G . As above, the map between them is non-local.

Duality in $3+1d$ $N = 4$ supersymmetry

$$G, \alpha \leftrightarrow \tilde{G}, \frac{1}{\alpha}$$

- The duality is an exact equivalence of theories. Like the Fourier transform above, but cannot be performed explicitly (cannot prove it)
 - Same spectrum of states
 - Same spectrum of operators
 - Same correlation functions

Duality in $3+1d$ $N = 4$ supersymmetry

- The gauge symmetry of the dual description is emergent!
- Which of the two gauge symmetries is fundamental?
- Perhaps neither gauge symmetry is fundamental.
- Which set of gluons is elementary?
- Notion of “elementary particle” is ill-defined.

Another notion of duality

The previous examples are **exact dualities** – two different descriptions of the same physics.

An alternate notion: two theories with the **same long distance (low energy) physics**

Hamiltonian at short distances (e.g. many electrons with some interactions)

Effective Hamiltonian describing the long distance physics (typically different degrees of freedom)



Another notion of duality

Many known examples

- Sometimes we can identify the long distance degrees of freedom using the short distance variables; e.g.
 - Ginzburg–Landau theory of superconductivity
 - QCD at short distances; pions at long distances
- Sometimes the relation is not so easy – it might be non-local
 - The examples of emergent gauge symmetry
 - Particle-vortex duality in $2+1d$
 - Emergent gauge fields in the FQHE

Interacting gauge theories

Start at short distance with a gauge group G . Depending on the details we end up at long distance with:

- IR freedom – a free theory based on G (same theory)
- A nontrivial fixed point. Interacting scale invariant (conformal) field theory – no notion of particles.
- An approximately free (IR free) theory of bound states
- An empty theory – gap (possibly topological order)

All these options are realized in QCD for various numbers of flavors. (The approximately free theory of bound states is a theory of pions.)

Duality in interacting field theories

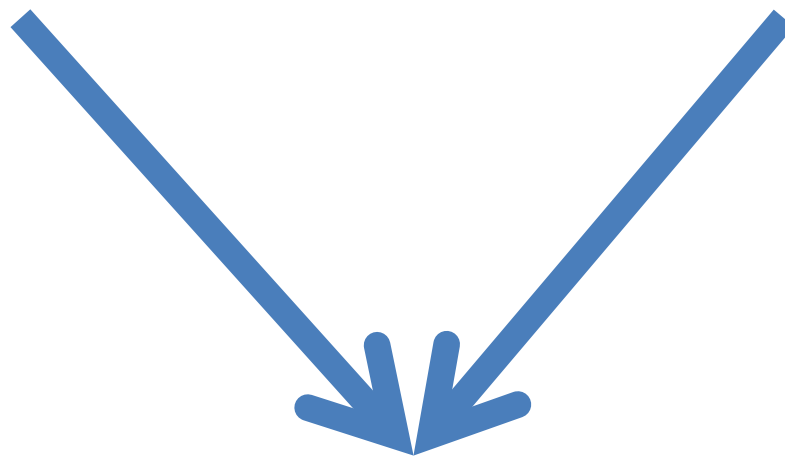
$N = 1$ supersymmetry

Electric theory

G

Magnetic theory

\tilde{G}



Non-trivial fixed point at long distances

Duality in interacting field theories

$N = 1$ supersymmetry

Another option:

Electric theory

Based on G



Approximately free theory (IR free)

Based on \tilde{G}

Duality in interacting field theories

$N = 1$ supersymmetry

At short distances an asymptotically free theory based on G

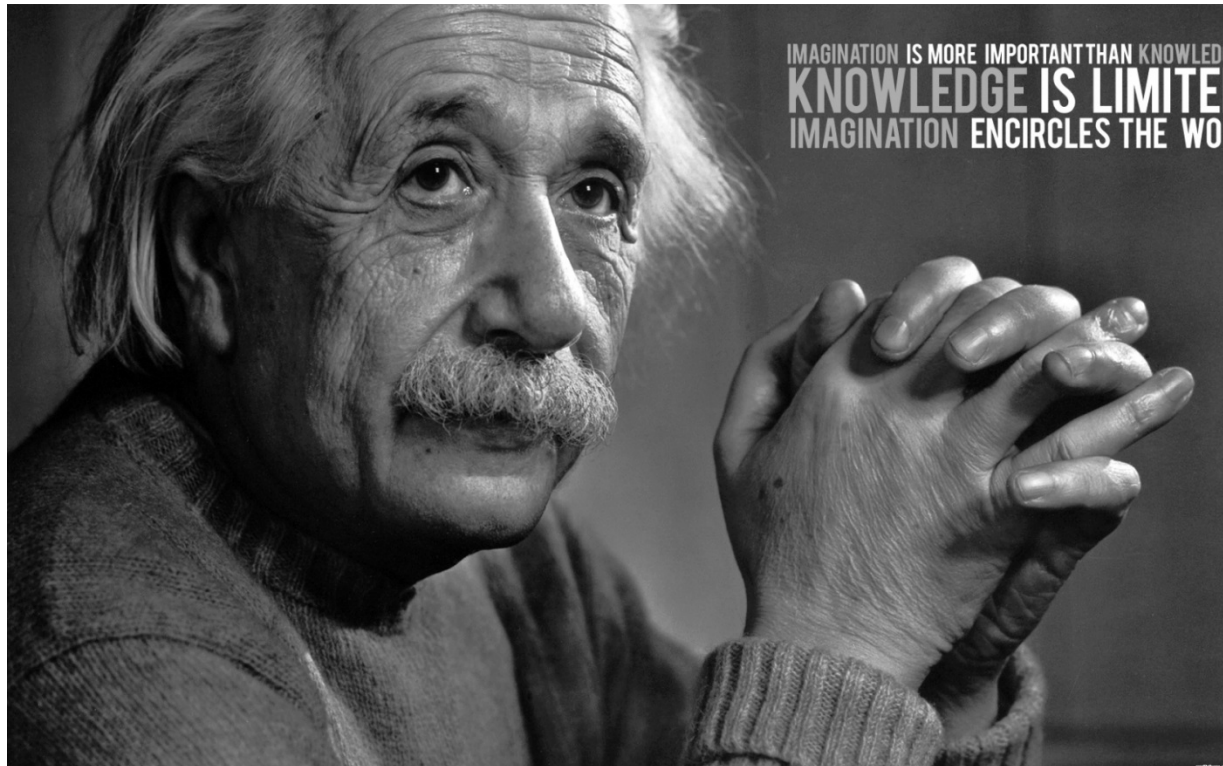
At long distances an almost free theory based on \tilde{G}

Unlike ordinary QCD , at long distances this theory has a non-Abelian gauge theory – \widetilde{QCD} .

- The quarks and gluons of \widetilde{QCD} are composite. They can be thought of as magnetic monopoles of G
- Their gauge symmetry is emergent.

Many more examples. Duality and emergent gauge symmetries are ubiquitous.

Other Dualities and the Unity of Physics



Emergent general covariance and emergent spacetime

- So far we discussed duality between two field theories
- String-string duality
 - T-duality
 - S-duality
 - U-duality
- String-fields duality
 - Matrix models for low dimensional string theories
 - BFSS M(atr ix) model
 - AdS/CFT
 - More generally gauge-gravity duality

Simple Boson/Fermion dualities

- Bosnization in $1+1d$
 - A theory of fermions is equivalent to a theory of bosons
 - No notion of spin of particles in $1+1d$ (only spin of operators)
- Spin and statistical transmutation in $2+1d$
 - Massive particles coupled to gauge fields with special interactions (Chern-Simons coupling) can be transmuted **Bosons** \leftrightarrow **Fermions**
 - Flux attachment in the FQHE

New Boson/Fermion dualities

Many references. These are some of the recent ones.

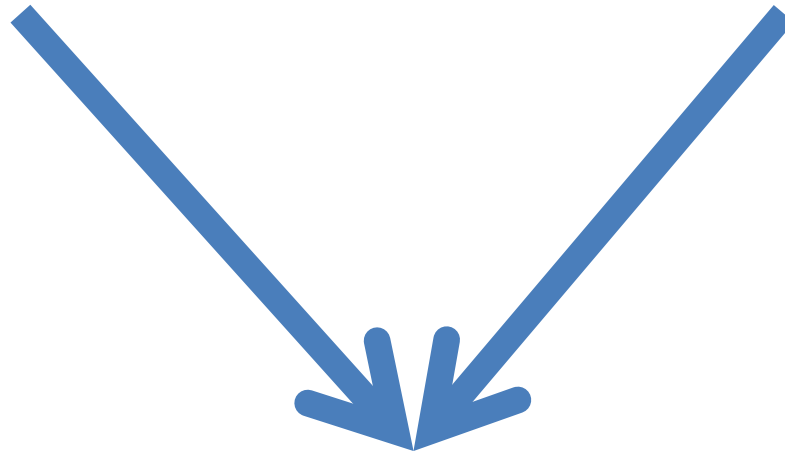
Giombi, Yin; Aharony, Gur-Ari, Yacoby; Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin; Maldacena, Zhiboedov; Aharony, Giombi, Gur-Ari, Maldacena, Yacoby; Jain, Minwalla, Sharma, Takimi, Wadia, Yokoyama; Minwalla, Yokoyama; Yokoyama; Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama; Inbasekar, Jain, Mazumdar, Minwalla, Umesh, Yokoyama; Jain, Minwalla, Yokoyama; Gur-Ari, Yacoby; Son; Wang, Senthil; Metlitski, Vishwanath; Barkeshli, McGreevy; Radicevic; **Aharony**; Karch, Tong; NS, Senthil, Wang, Witten; Hsin, NS; Kachru, Mulligan, Torroba, Wang ; Aharony, Benini, Hsin, NS.

New Boson/Fermion dualities

Many $2+1d$ examples of interacting theories of massless matter coupled to gauge fields

Fermions + gauge fields

Bosons + gauge fields



Non-trivial fixed point at long distances

New Boson/Fermion dualities

Many $2+1d$ examples of interacting theories of bosons at short distances flowing to free massless Fermions

Bosons + gauge fields



Free Fermions

Motivation (unity of physics)

- Well established particle/vortex dualities
- Previously found dualities in supersymmetric theories
 - Many tests
- String duality
 - Many tests
- AdS/CFT and large N
 - Same bulk theory is dual to different boundary theories
 - Checks at large N
- Some suggestions in the condensed matter literature

Checks and relations

- Can use these new dualities to derive well established dualities
- Relation to other conjectured dualities, which were subjected to many checks
- Relation to mathematics
 - Mirror symmetry
 - Level/rank duality
- Not independent: assuming some of these dualities are right we can derive others – **a web of dualities**

Applications in Condensed Matter Physics

- Fractional Quantum Hall Effect
- Physics of first Landau level at half filling
- Gapped phases of topological insulators and topological superconductors
- ...

Symmetries in Quantum Gravity and in Condensed Matter Physics

- No exact **global symmetries**
 - In gravity because of the physics of black holes
 - In condensed matter physics, if no exact global symmetry at short distances, all global symmetries at long distances are approximate
- Can have emergent **gauge symmetries** with emergent gauge fields.
 - Then there must be excitations with all allowed charges.

Conclusions

- **Symmetries** are common in physics
- Both in a theory of gravity and in condensed matter physics (unity of physics)
 - No exact global symmetries
 - Exact gauge symmetries can be emergent (all charges must be present)
- **Dualities** are common in physics: two (or more) different descriptions of the same physics

Conclusions

- **The unity of physics.** Insights from different branches of physics have recently converged leading to new dualities in $2+1d$.
 - Better understanding of known phenomena and mechanisms.
 - New phenomena and mechanisms.

Conclusions

- Gauge symmetry is not fundamental.
 - It is often convenient to use it to make the description manifestly Lorentz invariant, unitary, and local.
 - But there can be different such (dual) descriptions.
- Look for a reformulation of quantum field theory, which makes the duality manifest.
 - We should not be surprised by duality!