# Recent Advances in 2+1d QFT

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IAS

NS, E. Witten, 1602.04251;

NS, T. Senthil, C. Wang, E. Witten, 1606.01989;

P.-S. Hsin, NS, 1607.07457;

O. Aharony, F. Benini, P.-S. Hsin, NS, 1611.07874;

F. Benini, P.-S. Hsin, NS, 1702.07035;

D. Gaiotto, A. Kapustin, Z. Komargodski, NS, 1703.00501;

Z. Komargodski, NS, 1706.08755;

D. Freed, Z. Komargodski, NS, 1707.05448;

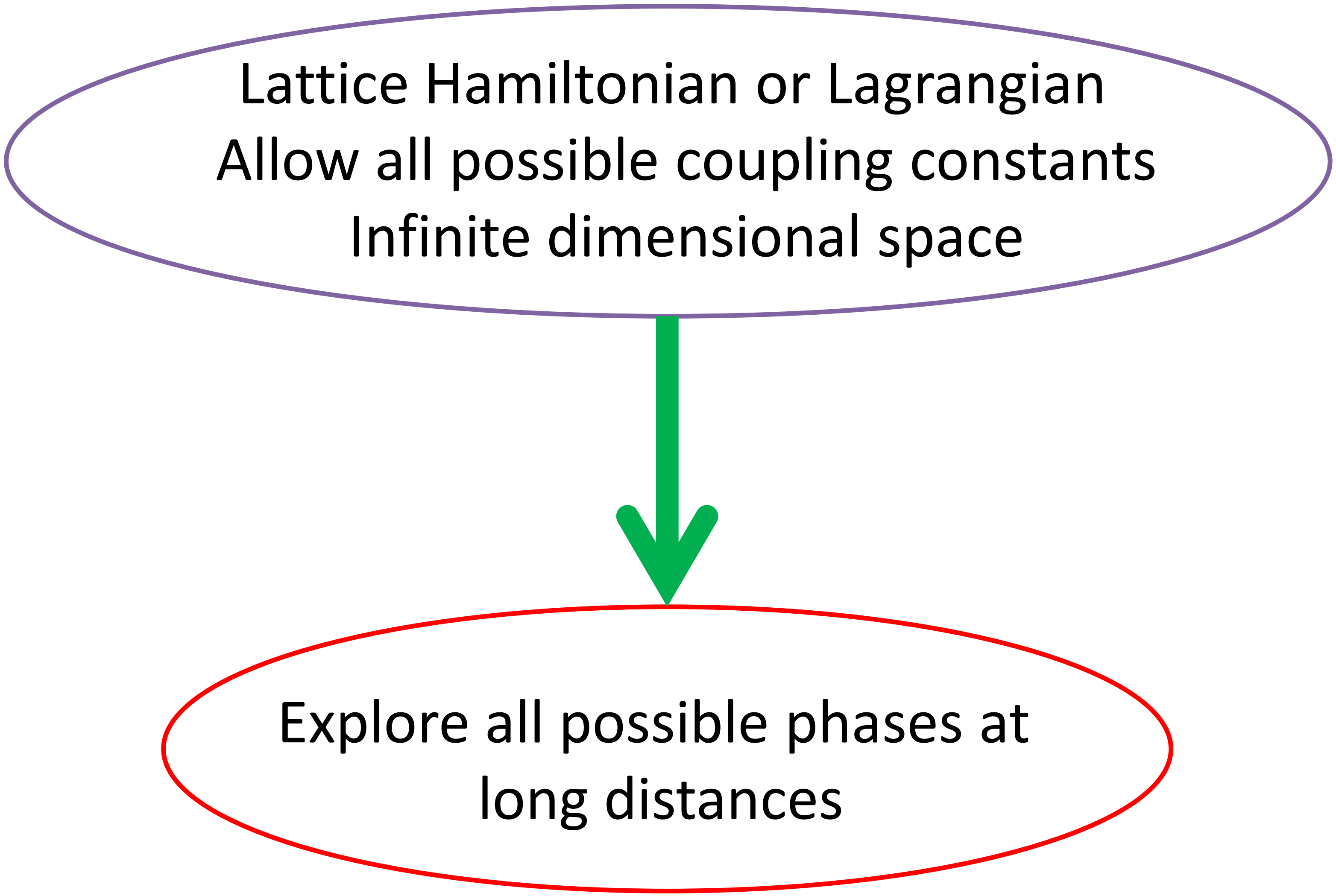
D. Gaiotto, Z. Komargodski, NS, 1708.06806;

J. Gomis, Z. Komargodski, NS, 1710.03258;

C. Cordova, P.-S. Hsin, NS, 1711. 10008;

C. Cordova, P.-S. Hsin, NS, 1712.08639.

# QFT in Condensed Matter Physics



# QFT in High Energy Physics

Renormalizable continuum Lagrangian
Finite number of relevant coupling constants
Infinitesimal values of irrelevant couplings

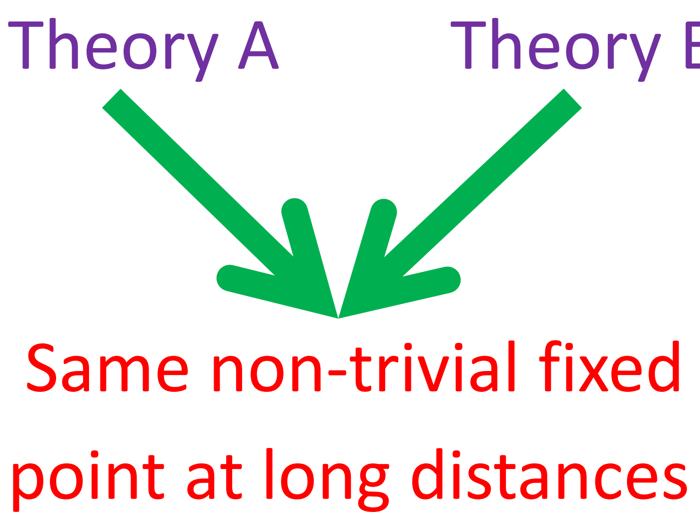
Much smaller set of couplings
More limited starting points

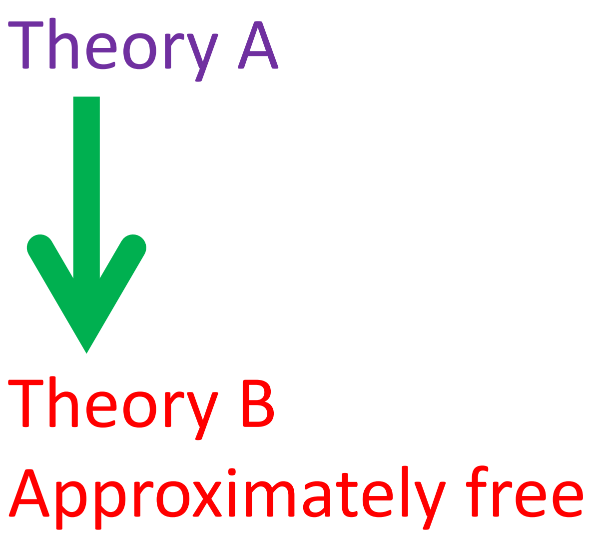
Explore all possible phases at long distances

# Duality in continuum QFT

Different kinds of duality including

- Exact duality as in the 1+1d Ising model or $\mathcal{N} = 4$ SUSY
- IR duality: two different theories with the same IR behavior, e.g. particle/vortex in 2+1d, $\mathcal{N} = 1$ SUSY. (Not merely universality.)



# Analyzing a continuum Lagrangian QFT

Semiclassical physics, mostly in the UV (reliable, straightforward, but can be subtle)

- Global symmetry and its ‘t Hooft anomalies
- Weakly coupled limits: flat directions, small parameters, …

Quantum physics in the IR (mostly conjectural)

- Consistency with the global symmetry (including ‘t Hooft anomalies) and the various semiclassical limits
- Approximate methods: lattice, bootstrap, $\epsilon$, $1/N$, …
- Integrability
- String constructions

# QED in 2+1d [Many references using various methods]

Simple, characteristic example, demonstrating surprising phenomena. Many applications.

- Cast of characters
  - $U(1)$ gauge field $a_\mu$. It can be an emergent field.
  - $N_f$ fermions $\psi^i$
- Parameters (because of renormalizability a finite number)
  - Fermion charges $q_i$ and masses $m_i$. (For simplicity, we'll take them equal.)
  - A bare Chern-Simons term…

# QED in 2+1d [Many references using various methods]

Bare Chern-Simons term

- It must be properly quantized
- Instead of using the quantized bare value, label the theory as $U(1)_k$ with a parameter $k$.
  - When all the fermions are massive, at low energies a TQFT $U(1)_{k_{low}}$ with

$$k_{low} = k + \frac{1}{2}\sum_i \operatorname{sign}(m_i) q_i^2$$

  - Since $k_{low} \in \mathbb{Z}$, $k + \frac{1}{2}\sum_i q_i^2 \in \mathbb{Z}$ .

# Global symmetries

- Charge-conjugation $\mathcal{C}$: $a_\mu \to -a_\mu$
  (with appropriate action on the fermions)
- $U(1)_0, m = 0$ time-reversal $\mathcal{T}$: $a_0(t,x) \to a_0(-t,x)$
  $$a_i(t,x) \to -a_i(-t,x)$$
  (with appropriate action on the fermions)
- Standard algebra on all the fundamental fields
  $$\mathcal{T}\mathcal{C} = \mathcal{C}\mathcal{T}$$
  $$\mathcal{T}^2 = (-1)^F$$
  $$(\mathcal{C}\mathcal{T})^2 = (-1)^F$$
- For equal charges and masses more symmetries, e.g. $SU(N_f)$.

# Global magnetic (topological) symmetry

- $U(1)_M$ symmetry: $j^\mu = \frac{1}{2\pi}\epsilon^{\mu\nu\rho}\partial_\nu a_\rho.$
- The charged operators are monopole operators (like a disorder operator).
  – Remove a point from spacetime and specify boundary conditions around it.
- Massless fermions have zero modes, which can lead to “funny” quantum numbers.

# Global magnetic (topological) symmetry

In many applications, the magnetic symmetry is approximate or absent

- In lattice constructions
- When the gauge $U(1)$ is embedded at higher energies in a non-Abelian gauge group
- When the gauge $U(1)$ is emergent
- In the generalization of the gauge $U(1) \cong SO(2)$ to $SO(N)$ with higher $N$ (only a magnetic $\mathbb{Z}_2$ symmetry).

It is natural to break $U(1)_M$ explicitly by adding a monopole operator to the Lagrangian.

# Example: $U(1)_k, N_f = 1$ with $q = 1$

$$k \in \mathbb{Z} + \frac{1}{2}$$

- Since $k$ cannot vanish, $\mathcal{T}$ is violated (parity anomaly).
- All gauge invariant polynomials in the fundamental fields are bosons.
- All gauge invariant monopole operators are bosons.
  - $U(1)_{\frac{1}{2}}$ simplest monopole operator has spin 0
  - $U(1)_{\frac{3}{2}}$ simplest monopole operator has spin 1

# Example: $U(1)_0, N_f = 2$ with $q = 1$

[Cordova, Hsin, NS]

- All gauge invariant polynomials in the fundamental fields are bosons with integer flavor (isospin) $SU(2)$.
- All gauge invariant monopole operators are bosons with flavor $SU(2)$ *isospin* $= \frac{M}{2} \mod 1$. Combined to $U(2)$.
- On a single monopole $\mathcal{T}^2 = -1$. More generally,

$$\mathcal{T}^2 = (\mathcal{CT})^2 = (-1)^M$$

rather than the standard $\mathcal{T}^2 = (\mathcal{CT})^2 = (-1)^F = +1$.
Related, but distinct statements in [Wang, T. Senthil; Metlitski, Fidkowski, Chen, Vishwanath; Witten]

# Examples: $U(1)_0, N_f = 1$ with $q = 2$

[Cordova, Hsin, NS]

- All gauge invariant polynomials in the fundamental fields are bosons.
- All gauge invariant monopole operators have *spin* $= \frac{M}{2} \bmod 1$, i.e. $(-1)^M = (-1)^F$
  – The basic monopole is a fermion. (In the previous example “fractional” isospin; here “fractional” spin.)
- Standard $\mathcal{T}$-symmetry

$$\mathcal{T}^2 = (-1)^F$$

- A $\mathbb{Z}_2$ one-form global symmetry [Gaiotto, Kapustin, NS, Willett] associated with a Wilson line of charge 1

# Break the magnetic $U(1)$ symmetry

[Cordova, Hsin, NS; Gomis, Komargodski, NS]

- Add to the Lagrangian a charge 2 monopole operator.
  - Cannot add a charge 1 monopole – it is a fermion.
- It breaks the
  - magnetic symmetry $U(1)_M \to \mathbb{Z}_2$: $(-1)^M$
  - $\mathcal{T}$-symmetry
- But it preserves another time-reversal symmetry (a subgroup of the original symmetry)

$$\mathcal{T}' = \mathcal{T} e^{\frac{i\pi}{2}M}$$

# Break the magnetic $U(1)$ symmetry

[Cordova, Hsin, NS; Gomis, Komargodski, NS]

$$\mathcal{T}' = \mathcal{T} e^{\frac{i\pi}{2}M}$$

- Its algebra is non-standard

$$\mathcal{T}'\mathcal{C} = \mathcal{C}\mathcal{T}'(-1)^M$$
$$(\mathcal{C}\mathcal{T}')^2 = (-1)^M = (-1)^F$$
$$(\mathcal{T}')^2 = 1 \neq (-1)^F$$

$\mathcal{C}$ and $\mathcal{T}'$ do not commute.

$\mathcal{T}'$ is not a conventional time-reversal symmetry, but $\mathcal{C}\mathcal{T}'$ is.

# What is the long distance behavior?

- What are the phases?
  - Gapped? Topological?
  - Symmetry breaking?
- What happens at the phase transitions? First or second order?
  - And if second order, free or interacting?
- It is clear for large $|m|$

CONJECTURES
AHEAD

# Vary $m$ at large $|k|$ or at large $N_f$

- Large $k$. Essentially free fermions with a modified Gauss law constraint
- Large $N_f$. Second-order transition as a function of the fermion mass $m$ [Appelquist, Nash, Wijewardhana]
- Conjecture that this is the case for all $k$, $q$, and $N_f$.

$U(1)_{k-\frac{1}{2}N_f q^2}$ $\qquad$ $U(1)_{k+\frac{1}{2}N_f q^2}$

$m < 0$ $\qquad$ $m > 0$

Second order transition at $m = 0$

# $U(1)_k, N_f = 1$ with charge one ($q = 1$)

$U(1)_{1/2}$ flows to the $O(2)$ Wilson-Fisher fixed point [Chen, Fisher, Wu; Barkeshli, McGreevy; NS, T. Senthil, Wang, Witten; Karch, Tong]

- The spin-zero monopole operator of the gauge theory is the order parameter of the $O(2)$ model.
- Emergent $\mathcal{T}$-symmetry in the IR

$U(1)_{3/2}$ flows to a fixed point with $SO(3)$ symmetry [Aharony, Benini, Hsin, NS; Benini, Hsin, NS]

- The spin-one monopole operator of the gauge theory becomes a conserved current in $\mathrm{O}(2) \to SO(3)$.
- Several dual bosonic and fermionic descriptions

# $U(1)_0, N_f = 2$ with $q = 1$

[Xu, You; Karch, Tong; Hsin, NS; Benini, Hsin, NS; Wang, Nahum, Metlitski, Xu, T. Senthil]

- Microscopically: global $U(2)$, $\mathcal{C}$ (they do not commute), and $\mathcal{T}$-symmetry
  - $\mathcal{T}$ and $\mathcal{CT}$ have a non-standard algebra

$$\mathcal{T}^2 = (\mathcal{CT})^2 = (-1)^M$$

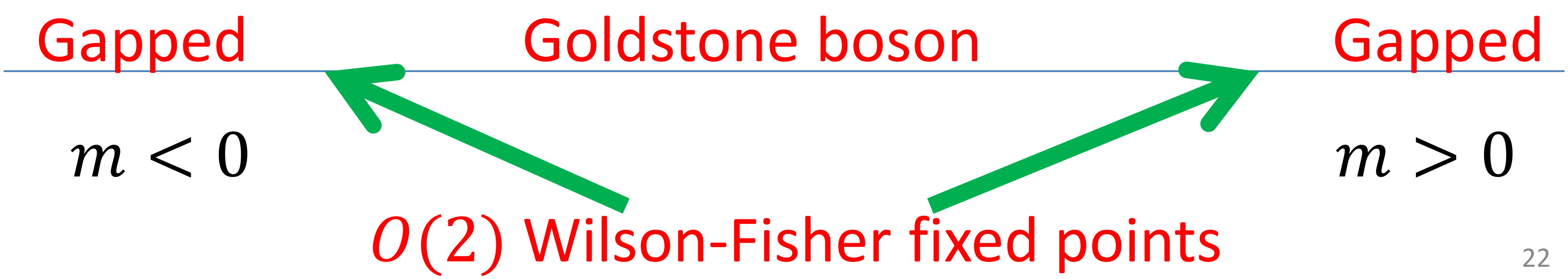
- Conjectured IR behavior: fixed point with enhanced global $O(4)$ symmetry
- Dual fermionic description: $U(1)_0$ with $N_f = 2$
  - Its $SU(2)$ flavor symmetry includes the original $U(1)_M$, and vice versa.

# $U(1)_0, N_f = 2$ with $q = 1$

[Benini, Hsin, NS; Komargodski, NS]

Break the magnetic symmetry by adding to the Lagrangian a double monopole operator

- This explicitly breaks the flavor $SU(2) \to U(1)$ and the magnetic $U(1)_M \to \mathbb{Z}_2$.
- For some range of $m$ the remaining flavor $U(1)$ symmetry is spontaneously broken.
  - The UV fermion is massive, but the IR theory is gapless!



# $U(1)_0, N_f = 1$ with $q = 2$

[Cordova, Hsin, NS]

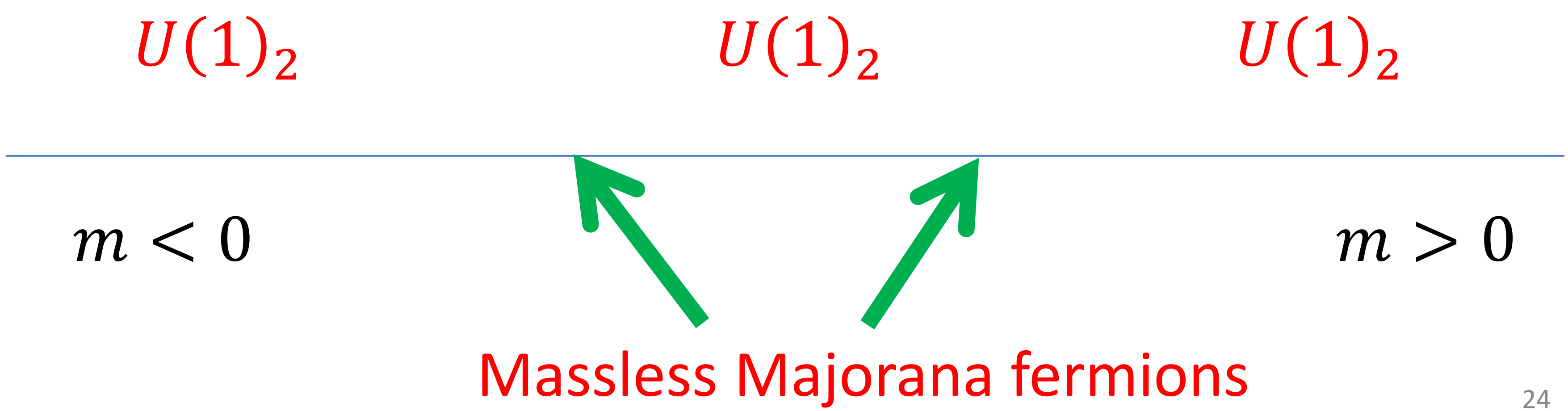
- Flows to a free Dirac fermion $\chi$ and a decoupled $U(1)_2$ TQFT
  - Similar (but not identical) to other fermion-fermion dualities [Son; Wang, T. Senthil; Metlitski, Vishwanath; NS, T. Senthil, Wang, Witten].
- The monopole operator in the UV becomes the free fermion $\chi$ in the IR.
- The Wilson line with charge 1 in the UV is described as the Wilson line of $U(1)_2$ (semion). It reflects the microscopic $\mathbb{Z}_2$ one-form global symmetry.

# Break the magnetic $U(1)$ symmetry

[Cordova, Hsin, NS; Gomis, Komargodski, NS]

Add to the Lagrangian a charge 2 monopole operator.

- It preserves $\mathcal{T}' = \mathcal{T} e^{\frac{i\pi}{2}M}$ with a nonstandard algebra
- In the IR it splits the fixed point with a massless Dirac fermion to two points with massless Majorana fermions



# Many generalizations

- Gauge group $G$
  - $U(1)$
  - $SU(N)$
  - $SO(N), Spin(N)$
  - $Sp(N)$
- Fermions in a representation $\mathcal{R}$
  - $N_f$ fundamentals
  - Adjoint
  - Symmetric or anti-symmetric tensors
- Chern-Simons terms (and discrete $\theta$-parameters)

# $SU(N)_k$ with $\lambda$ in adjoint for $N \leq 2k$

SUSY

$SU(N)_{k-\frac{N}{2}} \leftrightarrow U(k-N/2)_{-N}$

$SU(N)_{k+\frac{N}{2}} \leftrightarrow U(k+N/2)_{-N}$

$m < 0$

$m > 0$

Phase transition

No transition.  Supersymmetry is unbroken [Witten].

Holographic duals [Maldacena, Nastase] with $k - \frac{N}{2}$ branes.

# $SU(N)_k$ with $\lambda$ in adjoint for $0 \leq 2k < N$

[Gomis, Komargodski, NS]

$U\left(\frac{N}{2} - k\right)_{\frac{3N}{4}+\frac{k}{2},N}$ with $\hat{\lambda}$

$U\left(\frac{N}{2} + k\right)_{-\frac{3N}{4}+\frac{k}{2},-N}$ with $\tilde{\lambda}$

SUSY

$SU(N)_{k-\frac{N}{2}} \leftrightarrow$
$U(N/2 - k)_N$

$SU(N)_{k+\frac{N}{2}} \leftrightarrow$
$U(k + N/2)_{-N}$

$m < 0$

$m > 0$

$U(N/2 + k)_{-N/2+k,-N}$
$\leftrightarrow \;\; U(N/2 - k)_{N/2+k,N}$

Massless Goldstino [Witten].

Phase transitions

# $SU(N)_k$ with $\lambda$ in adjoint for $0 \leq 2k < N$

- Three phases
  - For large $|m|$ semiclassical physics – gapped, topological.
  - For small $|m|$ a new quantum phase
$$U\left(\frac{N}{2}+k\right)_{-\frac{N}{2}+k,-N} \leftrightarrow U\left(\frac{N}{2}-k\right)_{\frac{N}{2}+k,N}$$
  - Each phase transition has a fermionic dual description.
  - $m_\lambda = 0$ supersymmetry. Spontaneously broken with a massless Goldstino [Witten] in addition to the TQFT.
- For special $N$ and $k$ the picture simplifies.
- Many consistency checks

# $SU(N)_0$ with $\lambda$ in adjoint [Gomis, Komargodski, NS]

$U\left(\frac{N}{2}\right)_{\frac{3N}{4},N}$ with $\hat{\lambda}$

SUSY

$U\left(\frac{N}{2}\right)_{-\frac{3N}{4},-N}$ with $\tilde{\lambda}$

$SU(N)_{-\frac{N}{2}} \leftrightarrow$ $U(N/2)_N$

$SU(N)_{\frac{N}{2}} \leftrightarrow$ $U(N/2)_{-N}$

$m < 0$

$m > 0$

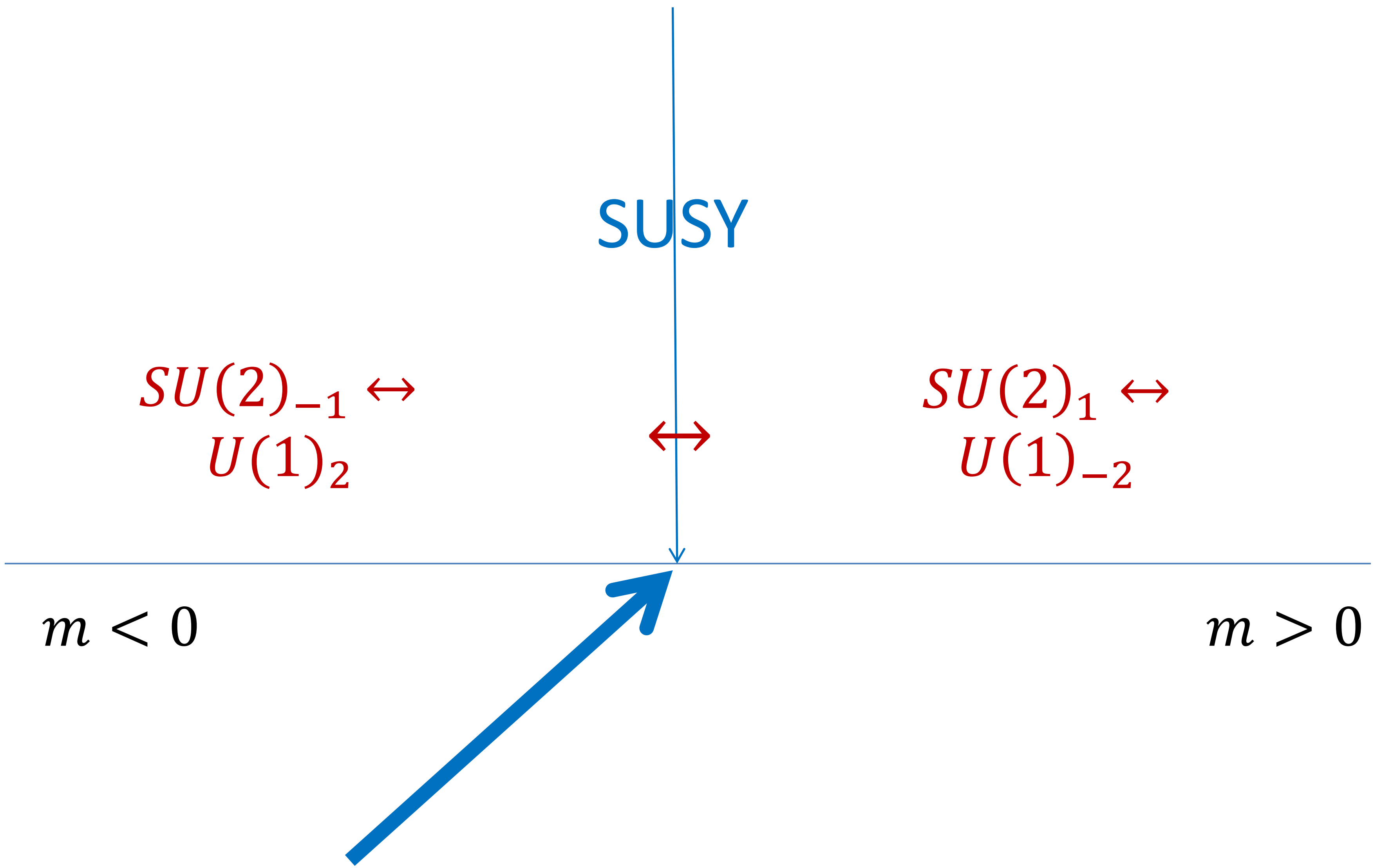
$U(N/2)_{-N/2,-N} \leftrightarrow U(N/2)_{N/2,N}$

$\mathcal{T}$ and $\mathcal{CT}$ invariant. Same anomaly $\nu$ as in the UV.

Massless Goldstino [Witten].

Phase transitions

# $SU(2)_0$ with $\lambda$ in adjoint [Gomis, Komargodski, NS]




Massless Goldstino [Witten].

# More

- Many other cases have been analyzed.
  - They exhibit many new phenomena.
- Many more generalizations have not yet been analyzed.

QFT is fun