New Phases of QCD_3 and QCD_4

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IAS

D. Gaiotto, A. Kapustin, Z. Komargodski, and NS, arXiv:1703.00501;

Z. Komargodski, and NS, arXiv:1706.08755;

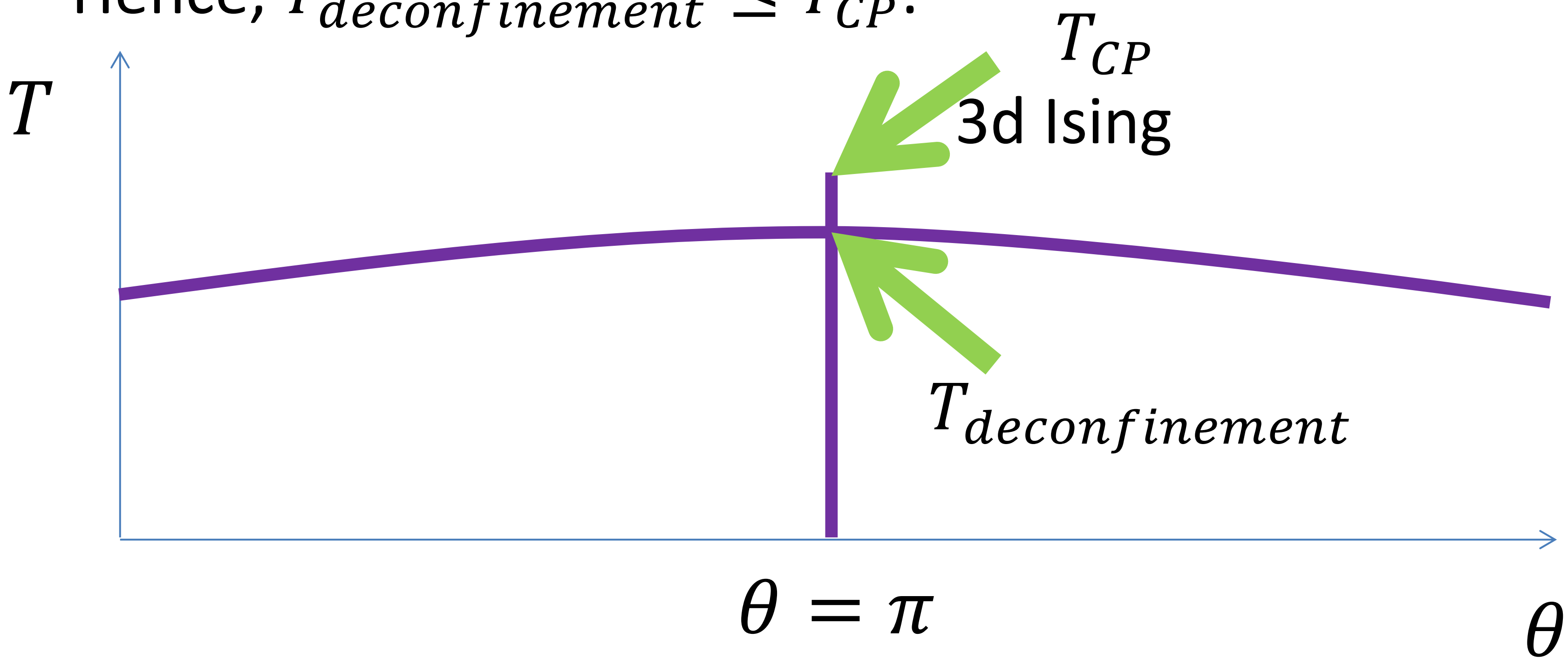
D. Gaiotto, Z. Komargodski, and NS, arXiv:1708.06806

4d pure gauge $SU(N)$

- Large N: 1st order transition at $\theta = \pi$.
  - CP (or T) is spontaneously broken there [Witten]
- Finite N: a one-form global symmetry associated with the center of the gauge group [Gaiotto, Kapustin, NS, and Willett].
  - At $\theta = \pi$: mixed 't Hooft anomaly between it and CP.
  - 't Hooft anomaly matching: cannot move continuously from confinement at $\theta = 0$ to confinement at $\theta = 2\pi$.
  - Specifically, at $\theta = \pi$ deconfinement, or broken *CP*, or TQFT
  - Simplest scenario (as at large N): a single 1st order phase transition at $\theta = \pi$.
  - Assume it (more exotic scenarios are in the paper).

4d pure gauge $SU(N)$ at finite T

- Because of the anomaly, cannot move continuously from confinement at $\theta = 0$ to confinement at $\theta = 2\pi$.
- Hence, $T_{deconfinement} \leq T_{CP}$.

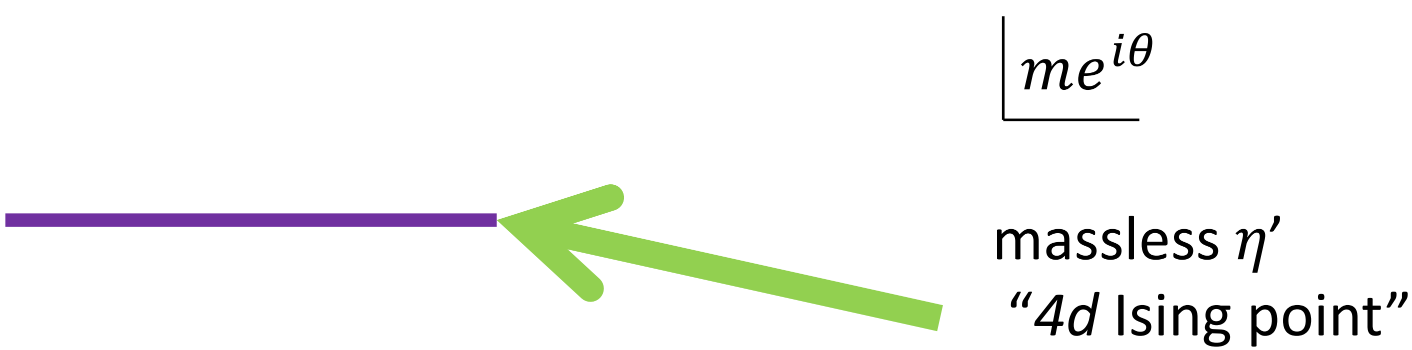


QCD_4 with one quark ($N_f = 1$)

- No chiral symmetry for massless quarks, but at infinite N a massless η' $\quad m^2_{\eta'} = \frac{1}{N}\Lambda^2 + O\left(\frac{1}{N^2}\right)$ [Witten]
- With massive quarks

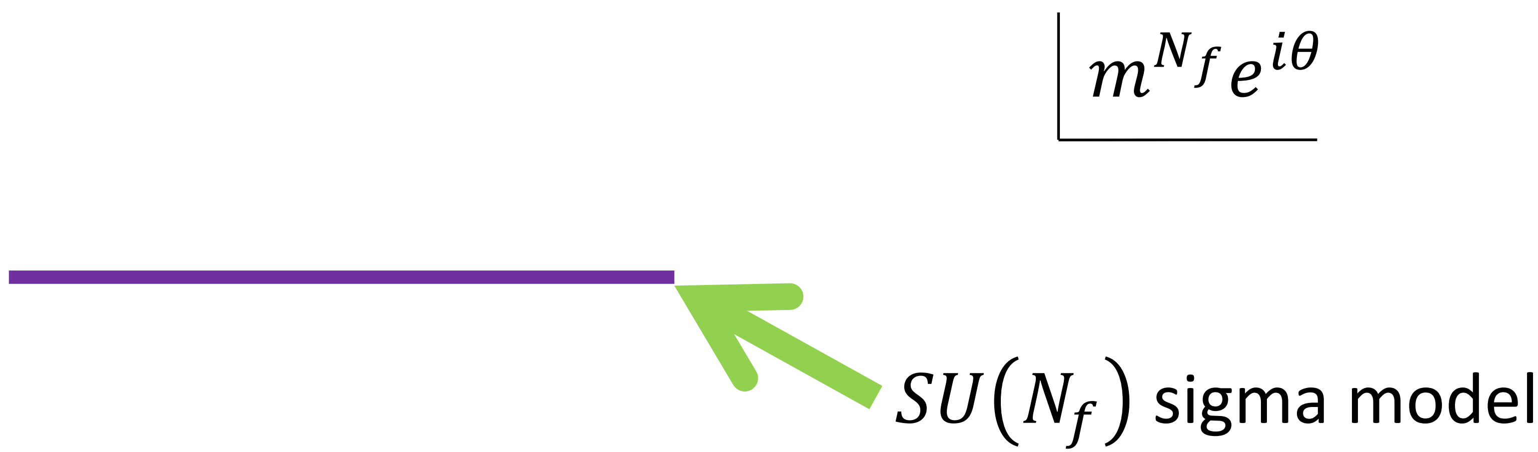
$$m^2_{\eta'} = \frac{1}{N}\Lambda^2 + Re(m)\,\Lambda + O(1/N^2, m^2, m/N)$$

- Therefore, even at finite N can find an exactly massless η'.
- For large $|m|$ the same behavior as in the pure gauge system.
- First order transition in the complex $me^{i\theta}$ plane



QCD_4 with N_f quarks for $1 < N_f < N_{CFT}$

- For $N_f \geq N_{CFT}(N)$ a CFT or lack of asymptotic freedom
- For $1 < N_f < N_{CFT}$ a massless theory – $SU(N_f)$ sigma model
- Turn on equal masses. As for $N_f = 1$, a first order transition line at $\theta = \pi$, which ends at the massless point [Dashen]



QCD$_3$ $SU(N)_k$ with N_f quarks

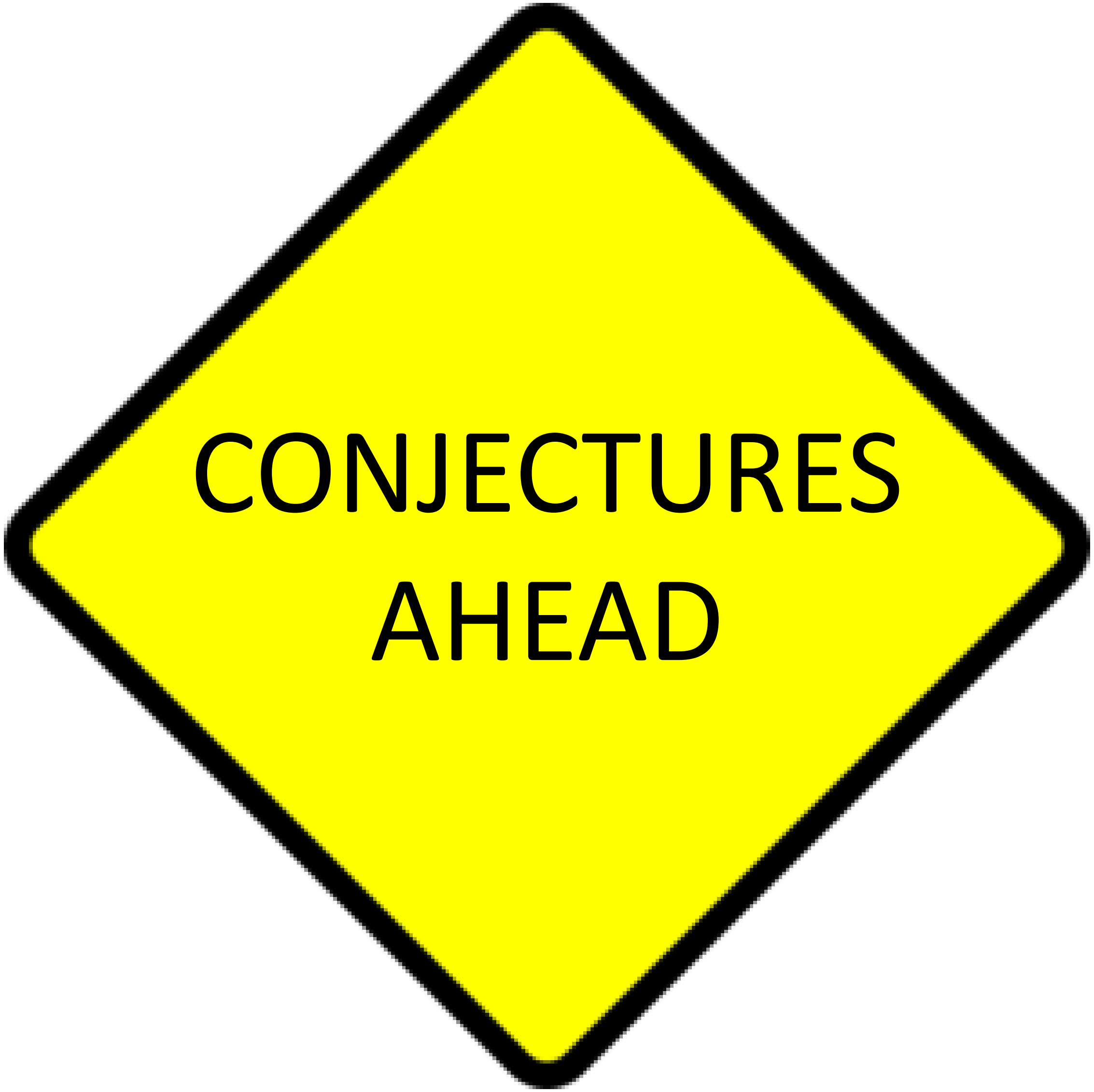
- Now we can add a Chern-Simons term with coefficient k.
- For large N_f a non-trivial fixed point [Appelquist and Nash]
- Assume that this remains the case for $N_f \geq N^*(N,k)$
- Topological phases at large $|m|$

$SU(N)_{k-N_f/2}$ $SU(N)_{k+N_f/2}$

$m < 0$ $m > 0$

Phase transition, CFT

- What happens for $N_f < N^*(N,k)$?
- Use recently suggested dualities…


CONJECTURES
AHEAD

Dual descriptions

Many references. These are some of the recent ones.

…; Giombi, Yin; Aharony, Gur-Ari, Yacoby; Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin; Maldacena, Zhiboedov; Aharony, Giombi, Gur-Ari, Maldacena, Yacoby; Jain, Minwalla, Sharma, Takimi, Wadia, Yokoyama; Minwalla, Yokoyama; Yokoyama; Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama; Inbasekar, Jain, Mazumdar, Minwalla, Umesh, Yokoyama; Jain, Minwalla, Yokoyama; Gur-Ari, Yacoby; Son; Wang, Senthil; Metlitski, Vishwanath; Barkeshli, McGreevy; Radicevic; Aharony; Karch, Tong; NS, Senthil, Wang, Witten; Hsin, NS; Kachru, Mulligan, Torroba, Wang; Metlitski, Vishwanath, Xu; Aharony, Benini, Hsin, NS; Benini, Hsin, NS …

Dual descriptions

N_f fermions coupled to $\qquad$ N_f scalars at $|\Phi|^4$ point coupled to

$$SU(N)_k \quad \leftrightarrow \quad U\left(\frac{N_f}{2}+k\right)_{-N}$$

We take N positive and k non-negative.

The scalars are in a generalized Wilson-Fisher fixed point or a gauged version of it.

For $N_f > 2k$ apply time reversal and then $k \to -k$ to find another possible duality

$$SU(N)_k \quad \leftrightarrow \quad U\left(\frac{N_f}{2}-k\right)_{N}$$

Dual descriptions

N_f fermions coupled to $\quad\quad$ N_f scalars at $|\Phi|^4$ point coupled to

- $SU(N)_k \quad \leftrightarrow \quad U\left(\frac{N_f}{2}+k\right)_{-N}$
- $SU(N)_k \quad \leftrightarrow \quad U\left(\frac{N_f}{2}-k\right)_{N}$

Checks:

- For large N and k with fixed ratio explicit calculations and relation to AdS duals (only the first one)
- For $N_f = 0$ level/rank duality (only the first one)
- Relation to SUSY dualities of […; Giveon, Kutasov; …]
- Flow to fewer flavors
- Global symmetry and 't Hooft anomaly matching (for simplicity, limit to $N > 2$)

Problem for $N_f > 2k$

N_f fermions coupled to | N_f scalars at $|\Phi|^4$ point coupled to

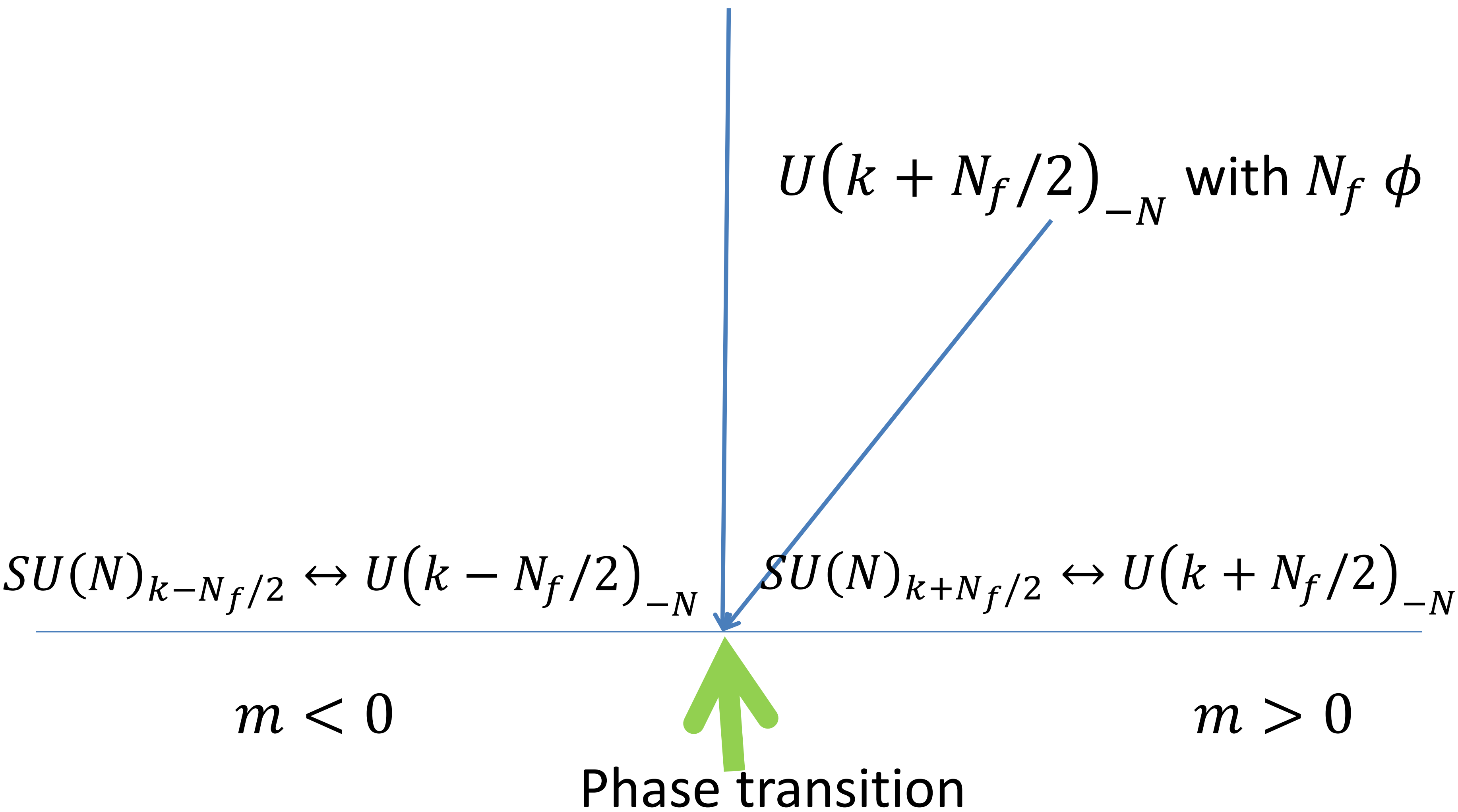
- $SU(N)_k \quad \leftrightarrow \quad U\left(\frac{N_f}{2}+k\right)_{-N}$
- $SU(N)_k \quad \leftrightarrow \quad U\left(\frac{N_f}{2}-k\right)_{N}$

A mass deformation in the fermionic theory leads to a gapped system $SU(N)_{k\pm N_f/2}$ (depending on the sign of the mass).

In the scalar theories for one sign it leads to a gapped theory $U\left(\frac{N_f}{2}+k\right)_{-N} \leftrightarrow SU(N)_{k+N_f/2}$, which is good.

But with the other sign the gauge symmetry is completely Higgsed and we end up with a massless theory.

$SU(N)_k$ with N_f quarks $\quad N_f \leq 2k$



$SU(N)_k$ with N_f quarks
$2k < N_f < N^*(N,k)$

$U\left(N_f/2-k\right)_N$ with N_f ϕ

$U\left(N_f/2+k\right)_{-N}$ with N_f ϕ

$SU(N)_{k-N_f/2} \leftrightarrow$
$U\left(N_f/2-k\right)_N$

$SU(N)_{k+N_f/2} \leftrightarrow$
$U\left(N_f/2+k\right)_{-N}$

$m < 0$

$$\frac{U(N_f)}{U\left(\frac{N_f}{2}+k\right)\times U\left(\frac{N_f}{2}-k\right)}$$
with $N\Gamma_{\mathrm{WZ}}$

$m > 0$

$SU(N)_k$ with N_f quarks
$2k < N_f < N^*(N,k)$

- Three phases
  - For large $|m|$ semiclassical physics – gapped, topological.
  - For small $|m|$ a new quantum phase with global symmetry breaking $U(N_f)/U\left(\frac{N_f}{2}+k\right) \times U\left(\frac{N_f}{2}-k\right)$
  - Each phase transition has a weakly coupled bosonic dual description
- The intermediate phase
  - Wess-Zumino term from the Chern-Simons term
  - For $k = 0$: $U(N_f) \rightarrow U(N_f/2) \times U(N_f/2)$ with a WZ term
  - Assuming this we can derive for other values of k
  - Skyrmions in the nonlinear model are monopoles in the bosonic theory and are the baryons in the fermionic theory

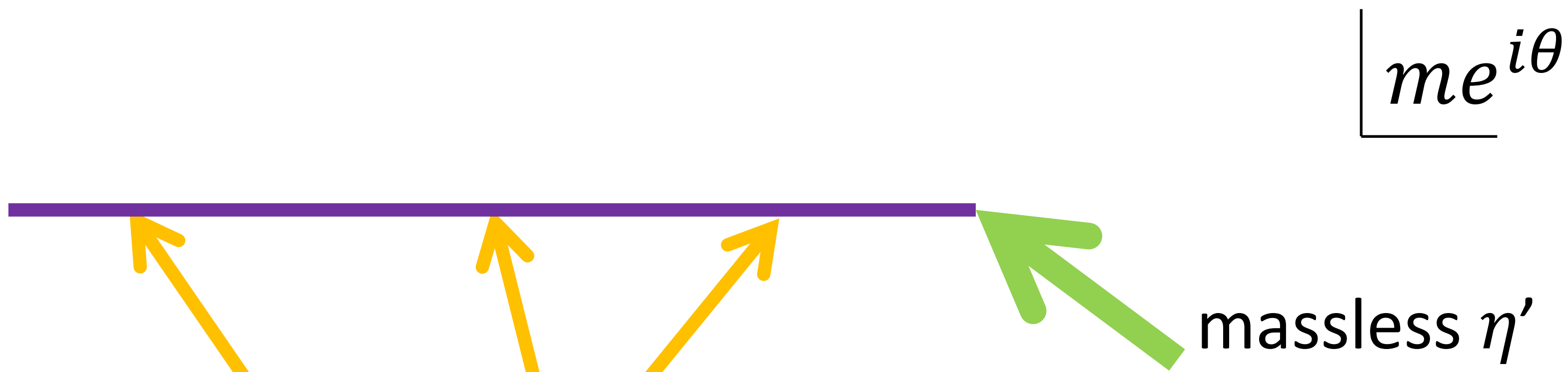
4d pure gauge $SU(N)$

Return to *4d.* Will soon relate to the *3d* story.

Study the domain wall at the first order transition point at $\theta = \pi$.

- The theory on the domain wall needs to account for the different 't Hooft anomalies between the two sides of the wall.
- $SU(N)_{-1}$ (Same as on the domain wall between neighboring vacua of $\mathcal{N} = 1$ SUSY $SU(N)$ pure gauge theory.)

QCD$_4$ with $N_f = 1$

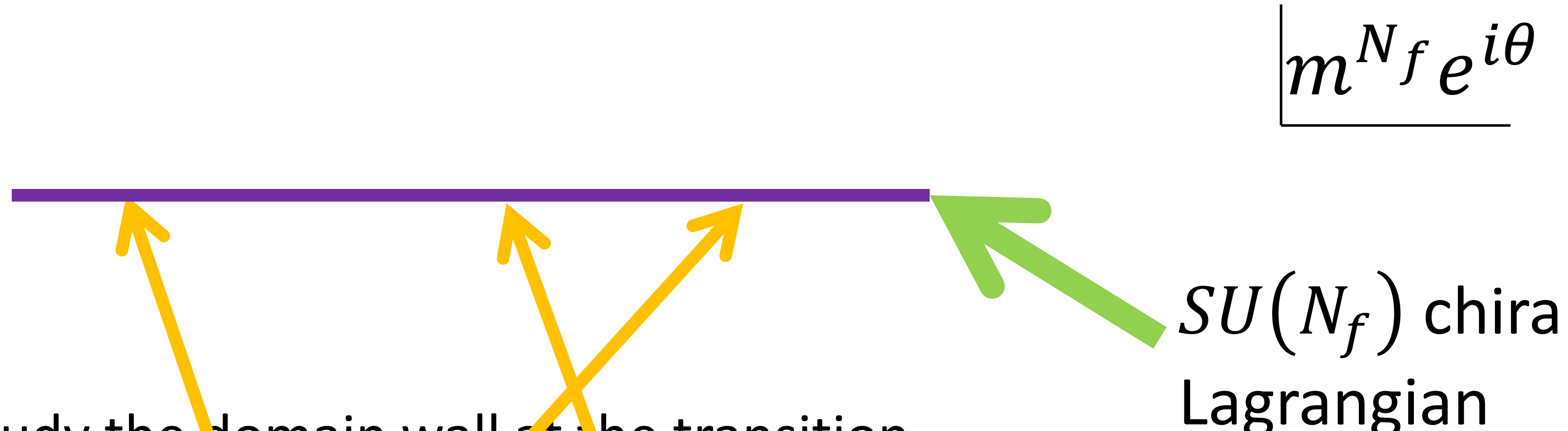


Study the domain wall at the transition.

- No anomaly argument
- For large negative $me^{i\theta}$, expect $SU(N)_{-1}$
- For small mass, should be trivial – use the η' theory
- There must be a phase transition on the domain wall.
- Same phases as in 3d

$$SU(N)_{-1/2} \text{ with } N_f = 1\ \psi \leftrightarrow U(1)_N \text{ with } N_f = 1\ \phi$$

QCD$_4$ with $1 < N_f < N_{CFT}$



Study the domain wall at the transition.

- For large negative $m^{N_f} e^{i\theta}$ expect $SU(N)_{-1}$
- For small mass, CP^{N_f-1} with $N\Gamma_{\mathrm{WZ}}$ – use the chiral Lagrangian
- There must be a phase transition on the domain wall
- Same phases as in 3d

$$SU(N)_{-1+N_f/2} \text{ with } N_f \text{ quarks} \leftrightarrow \; U(1)_N \text{ with } N_f \text{ scalars}$$

- Consistent with the intermediate phase in the *3d* discussion

Summary

QCD$_4$

- $N_f = 0$
  - New parity anomaly
  - Phase transition at $\theta = \pi$ for all N
  - $T_{deconfinement} \leq T_{CP}$
- $N_f = 1$
  - massless η' for all N
- All N_f
  - first order transition with domain walls

Summary

QCD$_3$ with a Chern-Simons term

- Large N_f: a second order transition separating two gapped topological phases
- Small N_f: same as large N_f, but with a bosonic dual
- Intermediate N_f: three phases. Two of them are gapped and topological. Intermediate phase with global symmetry breaking.

Consistent with the analysis of domain walls in QCD$_4$

Interesting generalization to $SO(N)/Spin(N)$ and $Sp(N)$ gauge theories – new insights about confinement.