

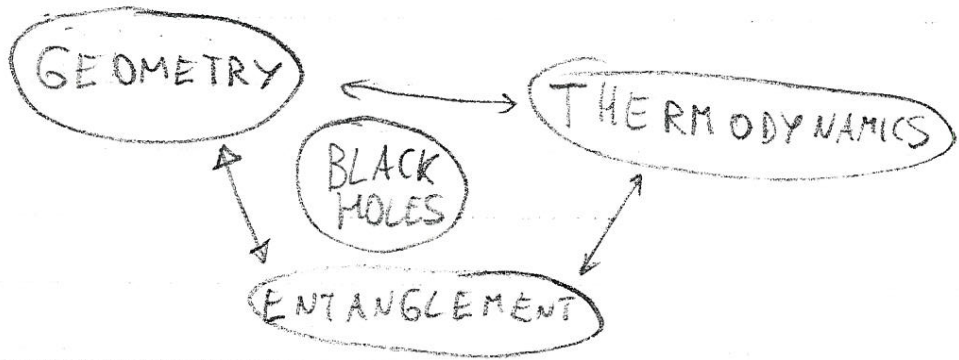
PITP 2013

ENTANGLEMENT &

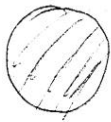
GEOMETRY

JUAN MALDACENA

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BLACK HOLES



$$ds^2 = -\left(1 - \frac{r_s}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)} + r^2 d\Omega_2^2$$

$r = r_s \leftarrow$ HORIZON.

NEAR HORIZON: $r = r_s + \frac{\rho^2}{4r_s}$

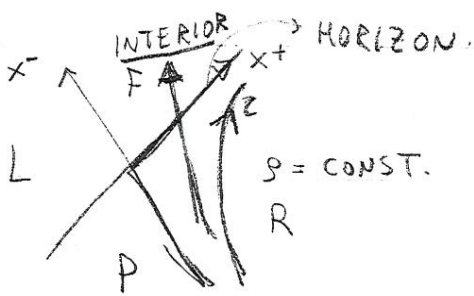
$$ds^2 = -\frac{\rho^2}{(2r_s)^2} dt^2 + d\rho^2 + \dots r_s^2 d\Omega_2^2$$

$$\tau = \frac{t}{2r_s}$$

$$= -\rho^2 dz^2 + d\rho^2 + \dots$$

$$= -dx^+ dx^- + \dots$$

$$x^\pm = \pm \rho e^{\pm \tau}$$



- NOTHING SPECIAL AT $\rho = 0$!

R \rightarrow EXTERIOR OF BLACK HOLE

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RINDLER SPACE,

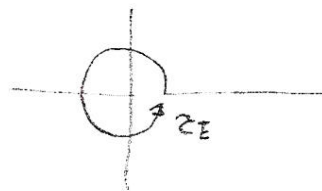
• OBSERVER AT $\rho = \text{CONST}$ \rightarrow ACCELERATED

\Rightarrow SEES A TEMPERATURE

$$ds^2 = -\rho^2 d\tau^2 + d\rho^2$$

LET US MAKE $\tau \rightarrow i\tau_E$

$$ds^2 = \rho^2 d\tau_E^2 + d\rho^2$$



IF $\tau_E = \tau_E + 2\pi$ \rightarrow USUAL PLANE,

\Rightarrow USUAL PLANE $\Rightarrow \tau_E$ IS PERIODIC

\Rightarrow FINITE TEMPERATURE $\beta = 2\pi$

$$T = \frac{1}{\beta} = \frac{1}{2\pi}$$

LOCAL TEMPERATURE AT ρ .

$$T_{\text{PROPER}} = \frac{1}{2\pi} \times \frac{1}{\rho}$$

- IN A GRAVITATIONAL FIELD, IN THERMAL EQUILIBRIUM

\rightarrow LOCAL PROPER TEMPERATURE DEPENDS ON THE REDSHIFT FACTOR (OR ON $g_{00}(r)$)

MINKOWSK
OBSERVER
SEES
BREMSTRALLUNG



ρ ALSO SETS PROPER ACCELERATION.

$$a = \frac{1}{\rho} \sim \frac{1}{\lambda} \quad \lambda = \text{WAVELENGTH OF RADIATION.}$$

\uparrow
NOT A LOCAL EFFECT \rightarrow IN GRAV. FIELD OF THE EARTH WE COULD BE AT $T=0$

EX: COMPUTE T FOR $a=g$ & COMPUTE ρ

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NEAR HORIZON GEOMETRY

$$ds^2 = - \rho^2 \frac{dt^2}{\underbrace{(2r_s)^2}_{d\tau^2}} + d\rho^2$$

$$\Rightarrow \beta = T^{-1} = 2\pi \cdot 2r_s \quad \Rightarrow \lambda \sim r_s$$

- HOW BIG A BH EMITTING VISIBLE LIGHT?

- TEMPERATURE FAR AWAY

USE 1st LAW

$$TdS = dM$$

WE KNOW THE MASS.

$$dS = \frac{dM}{T}$$

$$2G_N M = r_s$$

\Rightarrow COMPUTE S.

$$S = \frac{4\pi r_s^2}{4G_N} = \frac{(\text{AREA OF HORIZON})}{4G_N} = \frac{\text{AREA}}{4 \ell_{\text{PLANCK}}^2}$$

$$S \propto M^2 \quad (\text{MORE THAN EXTENSIVE})$$

$G_N/r_s^2 \rightarrow$ PERTURBATIVE EXPANSION PARAMETER.

$S \rightarrow \infty$ IN THE CLASSICAL LIMIT.

MACROSCOPIC ENTROPY

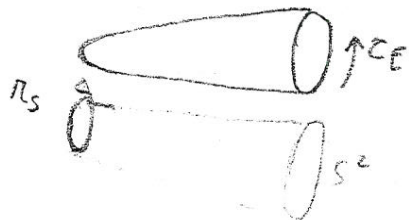
- MICROSCOPIC INTERPRETATION OF ENTROPY?

GEOMETRY & THERMODYNAMICS!

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EUCLIDEAN BLACK HOLES

$$t \rightarrow it_E, \quad t_E \sim \tau_E + \beta_H.$$



EUCLIDEAN TIME CIRCLE SHRINKS SMOOTHLY.

$$\mathcal{Z} = e^{-I} \quad I = \frac{-1}{16\pi G_N} \left[\int \sqrt{g} R + 2 \int_{\text{Bdy}} K \right]$$

$$\mathcal{Z} = e^{-\beta F}$$

$$S = (1 - \beta \partial_\beta) (\log \mathcal{Z}).$$

CLASSICAL GEOMETRY \rightarrow THERMODYNAMIC COMPUTATION!

REALLY?

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MICROSTATE COUNTING

(FOR SPECIAL BHs IN STRING THEORY)

- SUSY COMPACTIFICATIONS.

- EXTREMAL CHARGED BHs

$M \geq Q$, $M = Q$ = EXTREMAL

- WE CAN KEEP Q FIXED (IN QUANTIZED UNITS)

& VARY PARAMETERS OF THE COMPACTIFICATION

SUSY IMPLIES THAT NUMBER OF STATES

DOES NOT CHANGE (OR CHANGES IN A CONTROLLED WAY).



$$S = \frac{A}{4G_N}$$



g



SYSTEM OF BRANES.

WEAKLY COUPLED

↑
COUNT HERE

N_{STATES}

$$\log [N_{\text{STATES}}] = \frac{A}{4G_N} + \text{CORRECTIONS} + \dots$$

↑
LOOP CORRECTIONS

TO THE G-H COMPUTATION.

(LOTS OF FUN MATH...)

$\frac{A}{4G_N}$ = COUNTS THE NUMBER OF MICROSTATES OF THE B.H.

OR NUMBER OF STATES NECESSARY FOR THE OUTSIDE OBSERVER.

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ENTANGLEMENT

A
↑

B
↑

$$|\Psi\rangle = |\downarrow\rangle_A |\uparrow\rangle_B - |\uparrow\rangle_A |\downarrow\rangle_B$$

MORE THAN CLASSICAL CORRELATIONS

(BELL INEQUALITY, TESTED IN BB OSCILLATIONS)
QUANT-PH/0310192

• THE QFT VACUUM IS VERY ENTANGLED

- THIS IS PARTICULARLY CLEAR IF WE VIEW IT AS ARISING FROM A REGULARIZING LATTICE -

• QUANTIFIED BY ENTANGLEMENT ENTROPY

2 SYSTEMS A & B $\mathcal{H} = \mathcal{H}_A \times \mathcal{H}_B$

$|\Psi\rangle \in \mathcal{H}$

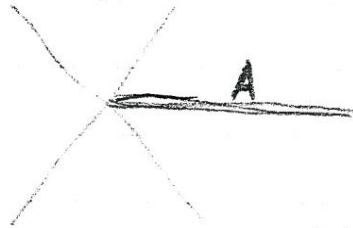
$$\rho_A = \text{Tr}_{\mathcal{H}_B} [|\Psi\rangle\langle\Psi|]$$

$$S_A = -\text{Tr} [\rho_A \log \rho_A]$$

• QFT



e.g. HALF OF SPACE



$$ds^2 = -p^2 dt^2 + dp^2$$

$$\rightarrow T \sim \frac{1}{2\pi p}$$

ENTROPY DENSITY: $\mathcal{D} \sim \frac{1}{p^3}$

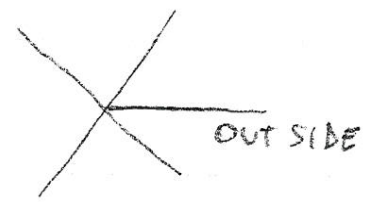
$$S = \int d^3V \mathcal{D} = \text{AREA} \times \int_{\epsilon}^{\infty} \frac{dp}{p^3} \sim \frac{\text{AREA}}{\epsilon^2}$$

$\epsilon = UV$ CUTOFF

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- WE CAN SUBTRACT UV DIVERGENCES & GET "RENORMALIZED" ENTANGLEMENT ENTROPIES WHICH ARE USEFUL FOR SOME THEORETICAL PURPOSES, LIKE COUNTING DOF, FINDING QUANTITIES THAT INCREASE UNDER R.G.

- IS BH ENTROPY RELATED TO ENTANGLEMENT ?



IS $E \sim \ln p$ & DOES NOT GIVE THE COEFFICIENT.

- CONSERVATIVE : 1-LOOP CORRECTION & DIV IS CANCELLED BY COUNTERTERMS.

GRAVITY REGULATES THIS DIV. HOW ?

ENTROPY $(1-p/p_c)$ & Z



CONE

IN GRAV WE FIX B.C. FAR AWAY, OR AT $p=p_c$.

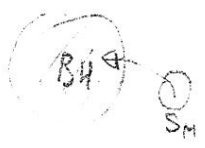


SMOOTH SPACE $\rightarrow I(p) \sim \text{CONST.} \sim \frac{1}{16\pi} \int \sqrt{g} R \sim \frac{\text{AREA}}{16\pi}$

BUT THIS CHANGE OF SPACE AS A FUNCTION OF β \rightarrow REMOVES THE STATISTICAL INTERPRETATION.

BLACK HOLES & THE 2nd LAW.

(2nd LAW DISCOVERED 1st BUT PROVEN LATER...)



SEND SOME ENTROPY

LOSE ENTROPY OUTSIDE?

$$S_{GEN} = \frac{Area}{4G\hbar} + S_{OUTSIDE} \quad \text{ALWAYS INCREASES}$$

⇒ WHEN WE SEND ENTROPY WE SHOULD SEND ENOUGH ENERGY TO AS TO INCREASE AREA

⇒ BEK: $S_{MATTER} < E \cdot R_{size}$



NO G_N!

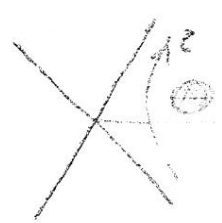
NEW PROPERTY OF QFT?

CASINI

- MORE PRECISE

$$\Delta S \leq 2\pi \langle \Delta K \rangle$$

← RINDLER HAMILT $\propto \hbar \omega \frac{\partial}{\partial x}$



$$S_{ENT} - S_{VAC}$$

$$K - K^{VAC}$$

↑ UV DIV CANCELS

• PROVEN USING GENERAL QUANTUM INFO ENTROPY BOUNDS

$$S(\rho|G) = -\text{Tr}[\rho \log \rho/G] \geq 0$$

• CONSISTENCY OF THERMODYNAMICS ⇔ PROPERTIES OF ENTANGLEMENT IN QFT

→ THIS IMPLIES 2nd LAW (WALL)

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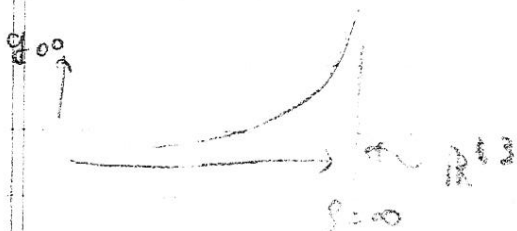
Gauge-Gravity Duality

- MOST SUSY QCD = $N=4$ SUPER YANG MILLS



SAME AS QUANTUM GRAVITY / STRING THEORY
IN $AdS_5 \times S^5$

$AdS_5 \rightarrow ds^2 = dp^2 + e^{2\sigma} dx_{4+1}^2$
↑
GRAV POTENTIAL



BLACK BRANES



BLACK BRANES \leftrightarrow QFT @ FINITE T

$\frac{AREA}{4G_N} \leftrightarrow$ ENTROPY

LONG DISTANCE WAVES ON HORIZON \leftrightarrow HYDRODYNAMICS

BLACK HOLE = ORDINARY QM SYSTEM

ANY QUESTION WE CAN ASK FROM OUTSIDE

\leftrightarrow SOME QUESTION IN THE FIELD THEORY

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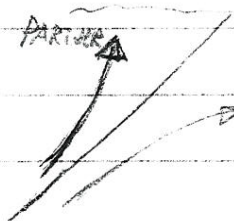
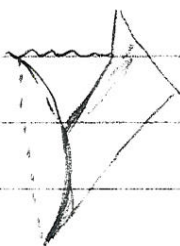
BLACK HOLE INFORMATION PROBLEM.

✓
✓
↑
↓
FORM
BH
IN PURE
STATE

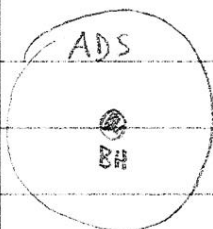


✓
✓
✓
✓
MIXED THERMAL
RADIATION ?

• IS RADIATION MIXED OR PURE ?



EACH STEP SEEMS TO GENERATE
SOME ENTROPY.



→ PROCESS IN A GAUGE TH
⇒ UNITARY QM PROCESS.

IF GAUGE/GRAV IS TRUE ⇒ BH FORMATION & EVAP.
(OR STABLE BLACK HOLES)
PRESERVE INFORMATION.

IN ADDITION WE KNOW, IN PRINCIPLE, WHAT COMPUTATION
WE NEED TO DO TO DESCRIBE OUTSIDE (ASYMPT.) OBSERV.
IN A UNITARY WAY

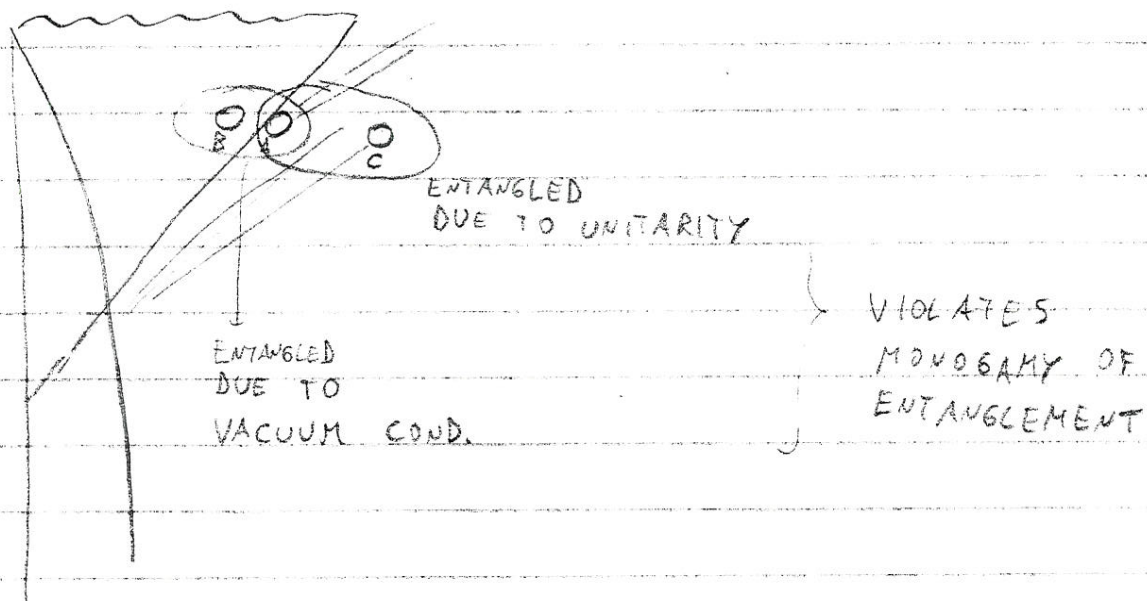
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• WE STILL DO NOT UNDERSTAND THE INTERIOR.

• WHAT HAPPENS AS WE HIT THE SINGULARITY?

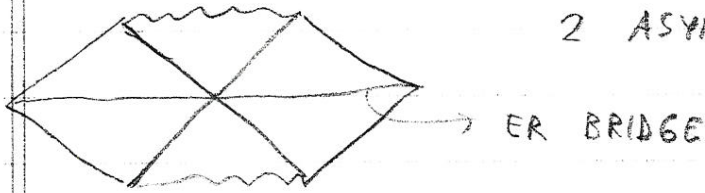
• MANY NEW PARADOXES (FIREWALL).

↪ "I SAY THERE IS A WALL AT THE HORIZON FOR MOST STATES - NOW PROVE ME WRONG!"



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ETERNAL BLACK HOLE



2 ASYMPTOTIC REGIONS.

→ THERMOFIELD DOUBLE

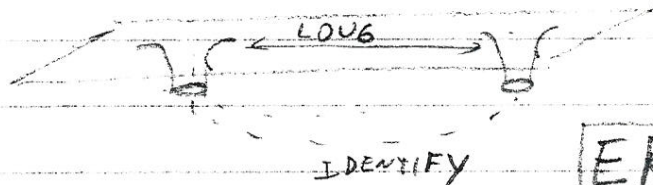
CPT CONJUGATE

$$|\Psi\rangle = \sum_m e^{-\beta E_m/2} |\bar{m}\rangle |m\rangle$$

ENTANGLED STATE IN 2 COPIES.

→ SUM OVER MICROSTATES (FINITE NUMBER OF RELEVANT TERMS)

- THIS SOLUTION CAN ALSO BE VIEWED AS 2 SEPARATED BHs IN THE SAME SPACETIME



$$\boxed{ER = EPR}$$

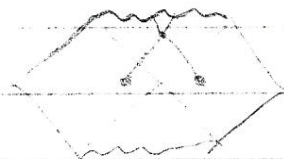
WORMHOLE ↔ ENTANGLED STATE

ENTANGLEMENT → CREATED A GEOMETRIC CONNECTION
 , PREPARED IN THIS VERY SPECIAL STATE.

ONE BH HERE & OTHER IN ANDROMEDA

• WE CANNOT SEND A SIGNAL.

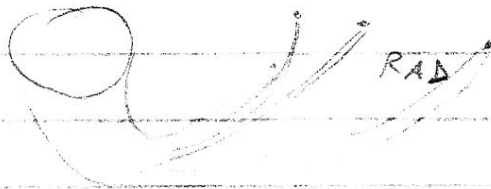
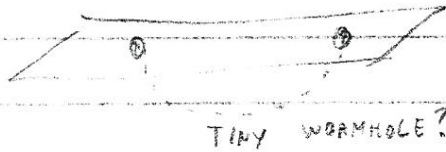
• WE COULD MEET AN ALIEN.



→ WHAT YOU SEE AS YOU FALL IN DEPENDS ON THE ALIEN'S DECISIONS.

→ INTERIOR OF AN ENTANGLED BH DEPENDS ON WHAT YOU DO WITH THE 2ND SYSTEM.

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- IS THE INTERIOR BUILT FROM MICROSTATES + THE RADIATION?

- HOW?

↑ EXERCISE.

SUMMARY:

- BLACK HOLES CONNECT
THERMODYNAMICS & ENTANGLEMENT & GRAVITY
IN INTERESTING & SURPRISING WAYS.

- UNDERSTANDING THIS BETTER WILL TEACH US MORE
ABOUT QUANTUM GRAVITY.

NOTE: ① INTERIOR ~ BIG CRUNCH COSMOLOGY

② HAWKING RAD & INFLATIONARY PERTURBATIONS
ARE INTIMATELY CONNECTED.