IMPLICATIONS OF A FRAME DEPENDENT DARK ENERGY FOR THE SPACETIME METRIC AND "HUBBLE TENSION"

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OUTLINE

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STANDARD VERSUS WEYL SCALING INVARIANT DARK ENERGY ACTION

STANDARD COSMOLOGICAL ACTION

$$S_{cosm} = -\frac{\Lambda}{8\pi G}\int d^4x \, ({}^{(4)}g)^{1/2}$$

$$\Lambda = 3 H_0^2 \Omega_\Lambda$$

- FOUR-SPACE GENERAL COORDINATE INVARIANT
- "VACUUM ENERGY" - ALL FIELDS CONTRIBUTE

ALTERNATIVE DARK ENERGY ACTION

$$S_{eff} = -\frac{\Lambda}{8\pi G}\int d^4x \, ({}^{(4)}g)^{1/2} (g_{00})^{-2}$$

- IN FRW WITH $g_{00}=1$, MIMICS S_{cosm}
- THREE-SPACE GENERAL COORDINATE INVARIANT
- WEYL SCALING INVARIANT $g_{\mu\nu} \to \lambda g_{\mu\nu}$
- FRAME DEPENDENT

FRAME DEPENDENT PHYSICS IS ALLOWED - CMB DIPOLE PICKS A PREFERRED FRAME $\Big\} \to$

PHENOMONOLOGY TO DISTINGUISH BETWEEN THEM

$$S_\Lambda = (1-f)\, S_{\text{corm}} + f\, S_{\text{eff}} = -\frac{\Lambda}{8\pi G} \int d^4x \left({}^{(4)}g\right)^{1/2} \left[1 - f + f\,(g_{00})^{-2}\right]$$

$f = 0$ ONLY S_{corm}

$f = 1$ ONLY S_{eff}

FOR FRW $g_{00} \equiv 1$ f DROPS OUT

f ONLY ENTERS AT LEVEL OF METRIC PERTURBATIONS

MOTIVATIONS

- **HISTORICAL**

WEYL PROPOSED INVARIANCE $g_{\mu\nu}(x) \rightarrow \lambda(x)\, g_{\mu\nu}(x)$

$\lambda(x)$ = CONFORMAL FACTOR

CALLED THIS A "GAUGE TRANSFORMATION", SCALE A LOCAL PROPERTY OF METRIC

- **RECENT FOLLOW-UPS**

hep-th/0307199 FORGER + RÖMER - DETAILED FIELD THEORY ANALYSIS

1410.6675 'T HOOFT - GRAVITATIONAL ESSAY ADVOCATING WEYL INVARIANCE AS "THE MISSING SYMMETRY COMPONENT FOR SPACE AND TIME"

- **MY INVOLVEMENT**

1306.0482 S.L.A. – "INCORPORATING GRAVITY INTO TRACE DYNAMICS: THE INDUCED GRAVITATIONAL ACTION"

BASIC IDEA: QUANTUM FIELD THEORY AS THERMODYNAMICS
CANONICAL ENSEMBLE AVERAGE OF UNDERLYING DYNAMICS

$$\rho \propto e^{-\tau \underset{\sim}{H}} = e^{-\tau \int d^3x \, ({}^{(4)}g)^{1/2} \, \mathrm{Tr}\, T_0{}^0(t, \vec{x})}$$

- HAMILTONIAN $\underset{\sim}{H}$ $\Rightarrow$ FRAME DEPENDENCE
- THREE-SPACE GENERAL COORDINATE INVARIANT
- FOR MASSLESS UNDERLYING FIELDS, $T_0{}^0$ GLOBAL WEYL INVARIANT

$\Rightarrow$ INDUCED GRAVITATIONAL ACTION INHERITS THESE PROPERTIES
TO ZEROTH ORDER IN DERIVATIVES $\quad S[g] \propto \int d^4x \, ({}^{(4)}g)^{1/2} (g_{00})^{-2}$

THIS IS ALSO LOCAL WEYL INVARIANT

NEW INSIGHTS ON OLD PROBLEMS

- THE COSMOLOGICAL CONSTANT PROBLEM $\hbar = c = 1$

S_{COSM} INTERPRETS Λ AS VACUUM ENERGY DENSITY

$$\rho^{\Lambda}_{VAC} = \frac{\Lambda}{8\pi G} = (2.2 \cdot 10^{-3}\ eV)^4$$

PLANCK SCALE ZERO POINT ENERGIES CORRESPOND TO

$$\rho^{QM}_{VAC} \sim M^4_{PL} = (1.2 \cdot 10^{28}\ eV)^4$$

SO $\frac{\rho^{\Lambda}_{VAC}}{\rho^{QM}_{VAC}} \sim 1.1 \cdot 10^{-123}$ TREMENDOUS FINE TUNING FOR PARTICLE VACUUM ENERGIES TO CANCEL DOWN TO ρ^{Λ}_{VAC}

- SUPPOSE UNDERLYING PHYSICS REQUIRES $S[g]$ TO BE WEYL SCALING INVARIANT $S_{COSM} \propto \int d^4x\,({}^{(4)}g)^{1/2}$ IS NOT

$\Rightarrow$ AN EXACT SUM RULE REQUIRING VACUUM ENERGIES TO SUM TO 0

$\int d^4x\,({}^{(4)}g)^{1/2}\, g_{00}^{-2}$ IS ALLOWED, BUT IS NOT VACUUM ENERGY

Λ STILL SMALL, BUT NO LARGE DENOMITOR M^4_{PL} TO COMPARE IT TO

- THE BLACK HOLE INFORMATION "PARADOX"

- WHAT HAPPENS TO INFORMATION THAT FALLS THROUGH BLACK HOLE HORIZON WHEN BLACK HOLE EVAPORATES VIA HAWKING RADIATION?

  $S_{cosm} + S_{\text{EINSTEIN-HILBERT}} \Rightarrow$ SCHWARZSCHILD DE SITTER BLACK HOLE
  STILL HAS NEAR EVENT HORIZON

- WHAT ABOUT $S_{CE} + S_{\text{EINSTEIN-HILBERT}}$?

  1308.1448 S.L.A. & FETHI RAMAZANOGLU

  ANALYTIC AND NUMERICAL STUDY OF SPHERICALLY SYMMETRIC VACUUM SOLUTIONS: NO HORIZON $g_{00} \neq 0$ FOR $r > 0$

  IN ISOTROPIC COORDINATES, SOLUTION IS SMOOTH TO $r=0$ SINGULARITY

  OUTSIDE $r_{\text{SCHWARZSCHILD}} + 10^{-17}\left(\frac{M_{\text{HOLE}}}{M_\odot}\right)^2$ cm, UP TO COSMOLOGICAL DISTANCES, SOLUTION CLOSE TO SCHWARZSCHILD SOLUTION, SO EXPECT ASTROPHYSICS UNALTERED. WHAT ABOUT INFORMATION PARADOX?

IMPLICATIONS FOR COSMOLOGY - GENERAL

1905.08228 S.L.A.

- HOMOGENEOUS, ISOTROPIC, ZERO SPATIAL CURVATURE LINE ELEMENT WITH PHYSICS INVARIANT UNDER THREE SPACE GENERAL COORDINATE ~~TRANSFORMATIONS~~

$$ds^2 = \alpha^2(t)\,dt^2 - \psi^2(t)\,d\vec{x}^2 = g_{00}\,dt^2 + g_{ij}\,dx^i\,dx^j$$

$$g_{00} = \alpha^2(t) \qquad g_{ij} = -\delta_{ij}\,\psi^2(t)$$

- $f \neq 0$ CASE CANNOT BE REDUCED TO $f=0$ BY REDEFINING THE TIME VARIABLE

DEFINE PROPER TIME τ BY $d\tau = \alpha(t)\,dt$, $\tau = \int_0^t du\,\alpha(u)$

$\Rightarrow$ $ds^2 = d\tau^2 - \psi^2[\tau]\,d\vec{x}^2$ WHERE $\psi(t(\tau)) \equiv \psi[\tau]$

FOR $\alpha(t)$, $\psi(t)$ BOTH POSITIVE, $({}^{(4)}g)^{1/2} = \alpha(t)\,\psi^3(t)$ $\Rightarrow$

$$S_\Lambda = \frac{-\Lambda}{8\pi G}\int dt\,d^3x\,\alpha(t)\,\psi^3(t)\left[1-f+f\,\alpha(t)^{-4}\right] = \frac{-\Lambda}{8\pi G}\int d\tau\,d^3x\,\psi^3[\tau]\left[1-f+f\,\alpha[\tau]^{-4}\right]$$

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- WORKS OUT THE f CONTRIBUTION TO METRIC PERTURBATIONS AROUND UNIFORM FRW BACKGROUND

  WITH $f \neq 0$ STILL NO SCALAR PROPAGATING GRAVITATIONAL WAVES

- FRAME DEPENDENCE OF f TERM $\Rightarrow \frac{\delta S_\Lambda}{\delta g_{ij}}$ GIVES T_f^{ij}

  TO GET FULL $T_f^{\mu\nu}$ IMPOSE COVARIANT CONSERVATION

$$D_\mu T_f^{\mu\nu} = 0$$

- BLACK HOLE WITH SPHERICAL SYMMETRY : ALGEBRAIC CONDITION
- FRW PERTURBATIONS : ORDINARY DIFFERENTIAL EQUATIONS
- GENERAL METRIC : PARTIAL DIFFERENTIAL EQUATIONS

NOTATIONS

- INTRODUCE $\theta(t)$, $\Phi(t)$, $\Psi(t)$

$$\alpha(t) = 1 + \Phi(t)$$

$$\Psi(t) = a(t)\,\theta(t)/\theta(0)$$

$$\theta(t) = 1 - \Psi(t)$$

$a(t) =$ STANDARD FRW EXPANSION FACTOR

- FOR MATTER-DOMINATED ERA

$$a(t) \simeq \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/3} \left(\sinh(x)\right)^{2/3} \qquad \Omega_m = 1 - \Omega_\Lambda$$

$x = \frac{3}{2}\sqrt{\Omega_\Lambda}\, H_0 t$ DIMENSIONLESS TIME VARIABLE

PRESENT ERA $a(t_0) = 1 \Rightarrow x_0 = \operatorname{arcsinh}\left(\sqrt{\frac{\Omega_\Lambda}{\Omega_m}}\right) \simeq 1.169$

$\Rightarrow H_0 t_0 = \frac{2}{3} x_0/\sqrt{\Omega_\Lambda} = 0.946$ $\qquad \Omega_\Lambda = 0.679 \quad \Omega_m = 0.321$

WILL SEE LATER THAT $H_0 = \alpha(0)\, H_0^{PL}$

- HUBBLE PARAMETER $H(t)$ DEFINED BY STANDARD FRW IS

$$H(t) = \frac{da(t)/dt}{a(t)} = \frac{\dot{a}(t)}{a(t)}$$

$$= H_0 \sqrt{\Omega_\Lambda} \coth(x) = H_0 \left[\Omega_m (1+z)^3 + \Omega_\Lambda \right]^{1/2} \qquad 1+z = \frac{1}{a(t)}$$

- HUBBLE PARAMATER ARISING FROM $f \neq 0$ MODIFIED DYNAMICS IS

$$H_{eff}[\tau] = \frac{d\psi[\tau]/d\tau}{\psi[\tau]} = H_{eff}(t) = \frac{d\psi(t)/dt}{\alpha(t)\,\psi(t)}$$

METRIC PERTURBATION ANALYSIS

- MATTER DOMINATED ERA: $p = \delta p = 0 \qquad \delta\rho \neq 0$

  TAKE LINEAR COMBINATION OF PERTURBATION EQUATIONS
  THAT ELIMINATES ρ TO GET

$$\ddot{\Phi} + 4\frac{\dot{a}}{a}\dot{\Phi} + \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)\Phi = 2\Lambda f\,\Phi$$

($f=0$ CASE IN MUKHANOV TEXT)

- STEPS TO ANALYZE THIS EQUATION

(1) CHANGE TO DIMENSIONLESS VARIABLE x TO GET

$$\frac{d^2\Phi}{dx^2} + \frac{8}{3}\coth(x)\frac{d\Phi}{dx} = \frac{4}{3}(2f-1)\,\Phi$$

(2) LARGE x BEHAVIOR $\qquad \coth(x) \to 1$

GET EQUATION WITH CONSTANT COEFFICIENTS, SOLVED BY

$$\Phi(x) = C_1 e^{\mu_+ x} + C_2 e^{\mu_- x}$$

$$\mu_\pm = -\frac{2}{3}\left[2 \pm (6f+1)^{1/2}\right]$$

CROSSOVER IN BEHAVIOR AT $f = 1/2$

$f < \frac{1}{2}$ BOTH EXPONENTS NEGATIVE

$f > \frac{1}{2}$ ONE EXPONENT POSITIVE, $\Phi \to \infty$ AS $x \to \infty$

$\Rightarrow$ FOR $f = 1$, THE CASE OF SCALE INVARIANT DARK ENERGY ACTION, METRIC PERTURBATION GROWS WITH TIME

(3) SMALL X BEHAVIOR

AS $x \to 0$, LEFT HAND SIDE DOMINATES, SO EQUATION IS APPROXIMATED BY

$$\frac{d^2\Phi}{dx^2} + \frac{8}{3}\frac{1}{x}\frac{d\Phi}{dx} = 0$$

SOLVED BY

$$\Phi(x) = C_3 + C_4\, x^{-5/3}$$

SO GET A UNIQUE SOLUTION BY REQUIRING REGULARITY AT $x = 0$, AND GIVING $\Phi(0)$, THE SOLE PARAMETER OF THE MODEL

(4) NUMERICAL SOLUTION

(4a) INTEGRAL EQUATION

DEFINE NORMALIZED PERTURBATION $\hat{\Phi}(x) = \Phi(x)/\Phi(0)$

OBEYS

$$\hat{\Phi}(x) = 1 + \frac{4}{3}(2f-1)\int_0^x dw\,\left(\sinh(w)\right)^{-8/3}\int_0^w du\,\left(\sinh(u)\right)^{8/3}\hat{\Phi}(u)$$

ITERATE STARTING FROM INITIAL ASSUMPTION $\hat{\Phi}(x) = 1$

(4b) FORWARD STEPPING

EQUATION IS INVARIANT UNDER $x \leftrightarrow -x$, SO REGULAR SOLUTION WILL HAVE $\left.\frac{d\Phi}{dx}\right|_{x=0} = 0 \Rightarrow$ STARTING FROM GIVEN $\Phi(0)$ AND $\Phi'(0) = 0$,

WRITE EQUATION AS $\Phi'' = G(\Phi', \Phi) = -\frac{8}{3}\coth(x)\,\Phi' + \frac{4}{3}(2f-1)\Phi$

AND STEPWISE ITERATE

$$\Phi'(i+1) = \Phi'(i) + G(\Phi'(i), \Phi(i))\,\Delta x$$

$$\Phi(i+1) = \Phi(i) + \Phi'(i)\,\Delta x$$

SOLUTION EVEN IN $x \Rightarrow$ HUBBLE DEVIATIONS WILL BE QUADRATIC IN x/x_0

CONNECTION TO H_0^{PL}, RESCALINGS

- NUMERICAL SOLUTION FOR $\hat{\Phi}(x)$ IS WELL APPROXIMATED BY

$$\hat{\Phi}(x) = 1 + \zeta \left(\frac{x}{x_0}\right)^2 \qquad \zeta = 0.244$$

SINCE $\dfrac{x_{\text{DECOUPLING}}}{x_0} \approx 3 \cdot 10^{-4}$, AT AND BEFORE DECOUPLING

$$\Phi = \Phi(0)\,\hat{\Phi} \approx \Phi(0) \quad \text{TO HIGH ACCURACY}$$

$\Phi(0)$ IS THE ONE PARAMETER OF THE MODEL

- RESCALINGS TO CONNECT TO PLANCK, WMAP FITS
  - RECALL $ds^2 = \alpha^2(t)\, dt^2 - a^2(t)\, d\vec{x}^2$
  - AT AND BEFORE DECOUPLING
    $\alpha(t) = 1 + \Phi(t) \approx \alpha(0) = 1 + \Phi(0)$
  - PROPER TIME $\tau = \alpha(0)\, t$ JUST A CONSTANT RESCALING OF t
  - $\Psi(t) = \Phi(t)$ TO FIRST ORDER $\Rightarrow$ $\theta(t) = 1 - \Psi(t) \approx 1 - \Phi(t) \approx 1 - \Phi(0) \approx \theta(0)$
  - $a[\tau] = a(t)\,\theta(t)/\theta(0) \approx a(t) = a(\tau/\alpha(0))$

- LINE ELEMENT FOR PLANCK ANALYSIS (ZEROTH ORDER) IS

$$ds^2 = d\tau^2 - \psi^2[\tau]\, d\vec{x}^2 = d\tau^2 - a^2(\tau/\alpha(0))\, d\vec{x}^2$$

- SO RELATION BETWEEN H_0^{PL} AND H_0 IS JUST A RESCALING BY $\alpha(0)$

$$H_0^{PL} = H_0/\alpha(0) \qquad \text{THAT IS} \qquad H_0^{PL}\tau = H_0 t$$

- SIMILARLY, τ_0^{PL} = AGE OF UNIVERSE MEASURED BY PLANCK IS RELATED TO t_0 BY $\tau_0^{PL} = t_0\,\alpha(0)$ THAT IS $H_0^{PL}\tau_0^{PL} = H_0 t_0$

- EFFECT OF RESCALINGS IS TO MAKE PHYSICAL PROCESSES DEPEND ON $\hat{\Phi}(t) - \hat{\Phi}(0) = C(t/t_0)^2 = C(X/X_0)^2$, SO IF C WERE ZERO THERE WOULD BE NO PHYSICAL EFFECTS

THIS IS ESSENTIAL BECAUSE WITH $C=0$ THE RELATION BETWEEN COORDINATE TIME t AND PROPER TIME τ IS A CONSTANT RESCALING, AND A CHANGE IN UNITS IN WHICH TIME IS MEASURED SHOULD HAVE NO EFFECT ON PHYSICAL CONSEQUENCES OF THE EQUATIONS

RESULTS FOR LATE TIME COSMOLOGY

• EFFECTIVE HUBBLE CONSTANT

• $$H_{eff}(t) = \frac{d\,\mathcal{N}(t)/dt}{\alpha(t)\,\mathcal{N}(t)} = \frac{d(\theta(t)\,a(t))/dt}{\alpha(t)\,\theta(t)\,a(t)} = \frac{1}{\alpha(t)}\left[\frac{\dot a(t)}{a(t)} + \frac{\dot\theta(t)}{\theta(t)}\right]$$

$$= \frac{1}{\alpha(t)} H(t) - \dot\Phi(t) \qquad \text{USING} \quad \alpha(t)\,\theta(t) \simeq 1$$

• $$\Phi(t) = \Phi(0)\left[1 + C\,\frac{t^2}{t_0^2}\right] \Rightarrow \quad \Phi(t_0) = \Phi(0)(1+C)$$

$$\dot\Phi(t_0) = 2\,\Phi(0)\,C/t_0$$

• $$H(t_0) = H_0 = \alpha(0)\,H_0^{PL} \qquad \frac{1}{\alpha(t_0)} = 1 - \Phi(t_0) = 1 - \Phi(0)(1+C)$$

$$H_{eff}(t_0) = [1 - \Phi(0) - \Phi(0)C]\,H_0 - 2\,\Phi(0)\,C/t_0$$

$$\approx H_0^{PL} - \Phi(0)C\,[H_0 + 2/t_0] \qquad \text{USING } (1-\Phi(0))H_0 = H_0^{PL}$$

$$\frac{H_{eff}(t_0)}{H_0^{PL}} \approx 1 - \Phi(0)\,C\,[1 + 2/H_0 t_0] \approx 1 - \Phi(0)\,C\,[1 + 3\sqrt{\Omega_\Lambda}/x_0]$$

$$\approx 1 - \Phi(0)\,0.76$$

RIESS $\quad \dfrac{H_{LOCAL}}{H_0^{PL}} = 1.100 \pm 0.023 \quad$ FIT BY $\Phi(0) = -0.133 \pm 0.031$

- REDUCTION IN AGE OF UNIVERSE

AGE τ_0 OF UNIVERSE FIXED IN OUR MODEL BY $\psi[\tau_0] = 1$

(ANALOG OF FRW $a(t_0) = 1$)

FIND

$$\tau_0 - \tau_0^{PL} = \frac{\Phi(0)\,\tau}{H_0^{PL}}\left(1 + \frac{2X_0}{9\sqrt{\Omega_\Lambda}}\right) \simeq \frac{1.315\,\Phi(0)\,\tau}{H_0^{PL}}$$

$$\tau_0^{PL} = 13.83 \text{ Gyr}, \quad \frac{1}{H_0^{PL}} = \frac{\tau_0^{PL}}{0.946} = 14.62 \text{ Gyr}, \quad \Phi(0) = -0.133 \pm 0.031$$

$$\Rightarrow \tau_0 - \tau_0^{PL} = -0.62 \pm 0.14 \text{ Gyr}$$

- MODEL IS LARGER THAN BAO HUBBLE PARAMETER AT $z \simeq 0.51$ BY 1 TO 1.4 STANDARD DEVIATIONS

SUGGESTIONS FOR FURTHER WORKS

- $\hat{\Phi}(x)$ DROPS FROM 1.244 AT $z = 0$ TO 1.122 AT $z \approx 0.35$

  DIVIDE H_0 MEASUREMENTS INTO: • "SMALL z" $z < 0.35$
  • "LARGE z" $z > 0.35$

  GET MEAN AND STANDARD DEVIATION FOR EACH GROUP

  WHAT IS STATICAL SIGNIFANCE OF $H_{0\,\text{SMALL}\,z} - H_{0\,\text{LARGE}\,z}$?

- EXISTENCE PROOF OF CONSERVING COMPLETION OF T_f^{ij} FOR GENERAL METRIC?

- BLACK HOLE - HOW IS "INFORMATION PARADOX" MODIFIED?

- REPEAT BAO, CAMBRIDGE GROUP (LEMOS ET AL.) ANALYSES WITH QUADRATIC PARAMETERIZATION SUGGESTED BY MY MODEL

- LONG LIVED STARS - HOW DOES "METHUSELAH STAR" ANALYSIS CHANGE?

- LENSING DISCREPANCY IN CMB - DOES FASTER LATE TIME EXPANSION HAVE ANY EFFECT?

- OTHER ASTROPHYSICS THAT COULD BE ALTERED BY ACCELERATED LATE TIME EXPANSION?