

Dynamics of 2+1d Quantum Field Theories

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IAS

NS, E. Witten, 1602.04251;

NS, T. Senthil, C. Wang, E. Witten, 1606.01989;

P.-S. Hsin, NS, 1607.07457;

O. Aharony, F. Benini, P.-S. Hsin, NS, 1611.07874;

F. Benini, P.-S. Hsin, NS, 1702.07035;

D. Gaiotto, A. Kapustin, Z. Komargodski, NS, 1703.00501;

Z. Komargodski, NS, 1706.08755;

D. Freed, Z. Komargodski, NS, 1707.05448;

D. Gaiotto, Z. Komargodski, NS, 1708.06806;

J. Gomis, Z. Komargodski, NS, 1710.03258;

C. Cordova, P.-S. Hsin, NS, 1711.10008;

C. Cordova, P.-S. Hsin, NS, 1712.08639 .

New elements (will not review here)

- Better understanding of the fermion determinant [Witten] and the role of the spin-charge relation [NS, Witten]
- Boson/fermion duality
 - SUSY dualities in 3d [many papers]
 - Condensed matter motivation [many papers]
 - AdS₄ and large N [...; Giombi, Minwalla, Prakash, Trivedi, Wadia, and Yin; ...].
 - Finite N version of it [Aharony]
 - Detailed analysis for $N = 1$ [NS, Senthil, Wang, Witten; Karch, Tong]

New elements (will not review here)

- Better understanding of level-rank duality in Chern-Simons TQFT [Hsin, NS; Aharony, Benini, Hsin, NS; Cordova, Hsin, NS]
 - Subtleties in the spin structure dependence, the global part of the group, and coupling to background fields and their counterterms

$$SU(N)_k \leftrightarrow U(k)_{-N}$$

$$U(N)_{k, k \pm N} \leftrightarrow U(k)_{-N, -k \mp N}$$

$$Sp(N)_k \leftrightarrow Sp(k)_{-N}$$

$$SO(N)_k \leftrightarrow SO(k)_{-N}$$

New elements (will not review here)

- Careful analysis of global symmetries and 't Hooft anomalies
 - Gravitational and time-reversal anomalies [many condensed matter papers, Witten]
 - Higher form global symmetries and their anomalies [Kapustin, NS; Gaiotto, Kapustin, NS, Willett; Gaiotto, Kapustin, Komargodski, NS]
 - Global part of the global symmetry group [Benini, Hsin, NS]
 - Many mixed anomalies [many papers]

Steps in analyzing QFT (short distance, kinematics)

- Find the global symmetry (make sure it acts faithfully)
 - Continuous symmetries, e.g. $SU(N_f)$
 - Discrete symmetries, e.g. charge conjugation
 - Time-reversal
 - Supersymmetry
- Find the 't Hooft anomalies
 - Global part of the global symmetry group is essential
 - Gravitational anomalies
- ...

Steps in analyzing QFT (semiclassical dynamics)

- Look for semiclassical limits, where the analysis is easy
 - Large Chern-Simons coefficient k
 - Large mass m
 - Large N_f
- Relate theories using semiclassical reasoning
 - Give masses to some of the quarks and integrate them out
 - Change the scalar potential

Steps in analyzing QFT (quantum)

- Various limits
 - Large N and holography
- Global symmetry breaking?
- 't Hooft anomaly matching conditions
- Dualities
 - including level-rank duality (rigorous)
- Is supersymmetry broken?
- Relate to other theories
 - On domain walls and interfaces of higher dimensional theories (will not discuss here)

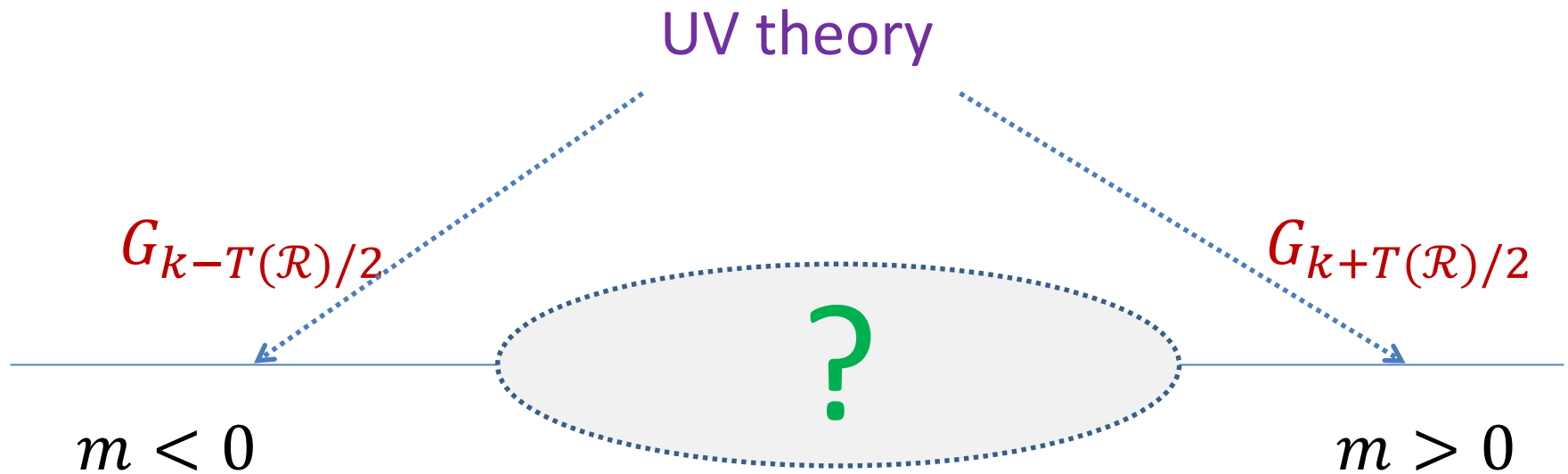
The theories

- Gauge group G
 - $U(1)$
 - $SU(N)$
 - $SO(N), Spin(N)$
 - $Sp(N)$
- Fermions in a representation \mathcal{R}
 - N_f fundamentals
 - Adjoint
 - Symmetric or anti-symmetric tensors
- Chern-Simons terms (and discrete θ -parameters)

G_k with fermions in a representation \mathcal{R}

Large mass $|m|$

- At low energy a TQFT $G_{k \pm T(\mathcal{R})/2}$. Need $k = (T(\mathcal{R})/2) \bmod 1$, where $T(\mathcal{R})$ is the index of \mathcal{R} .

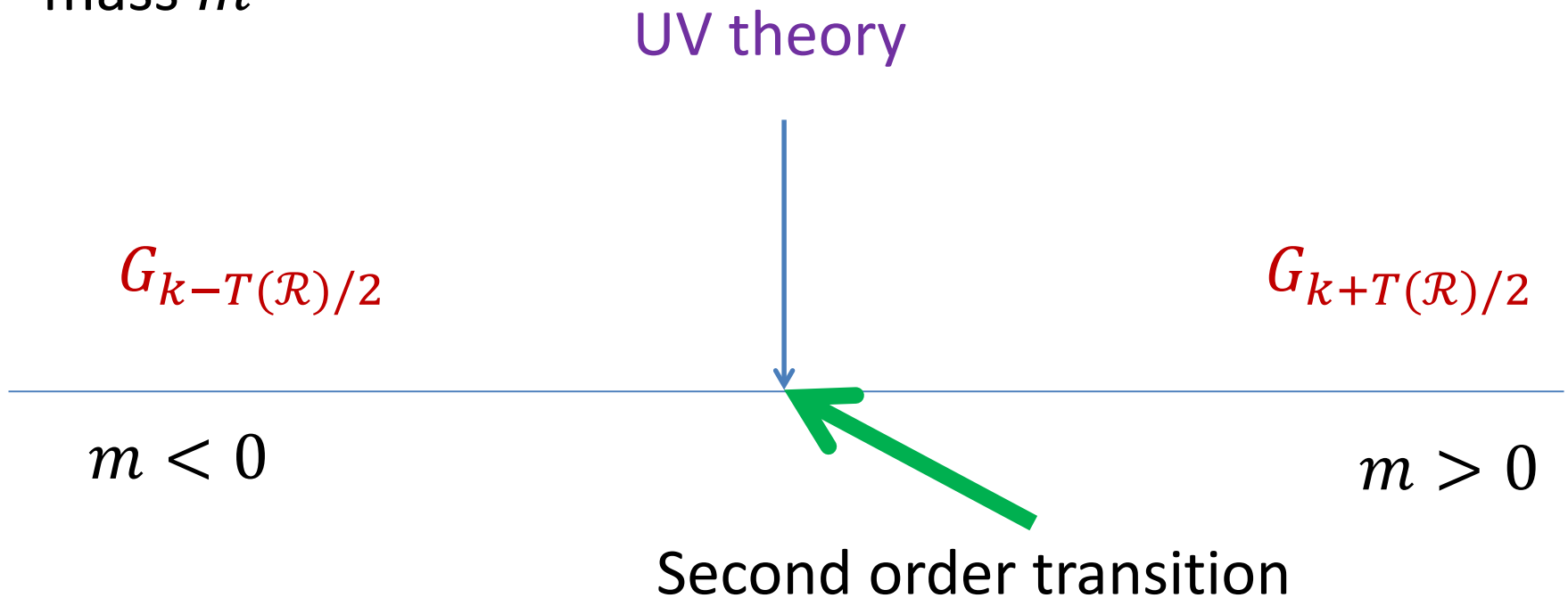


– What happens at small mass?

G_k with fermions in a representation \mathcal{R}

Large level k

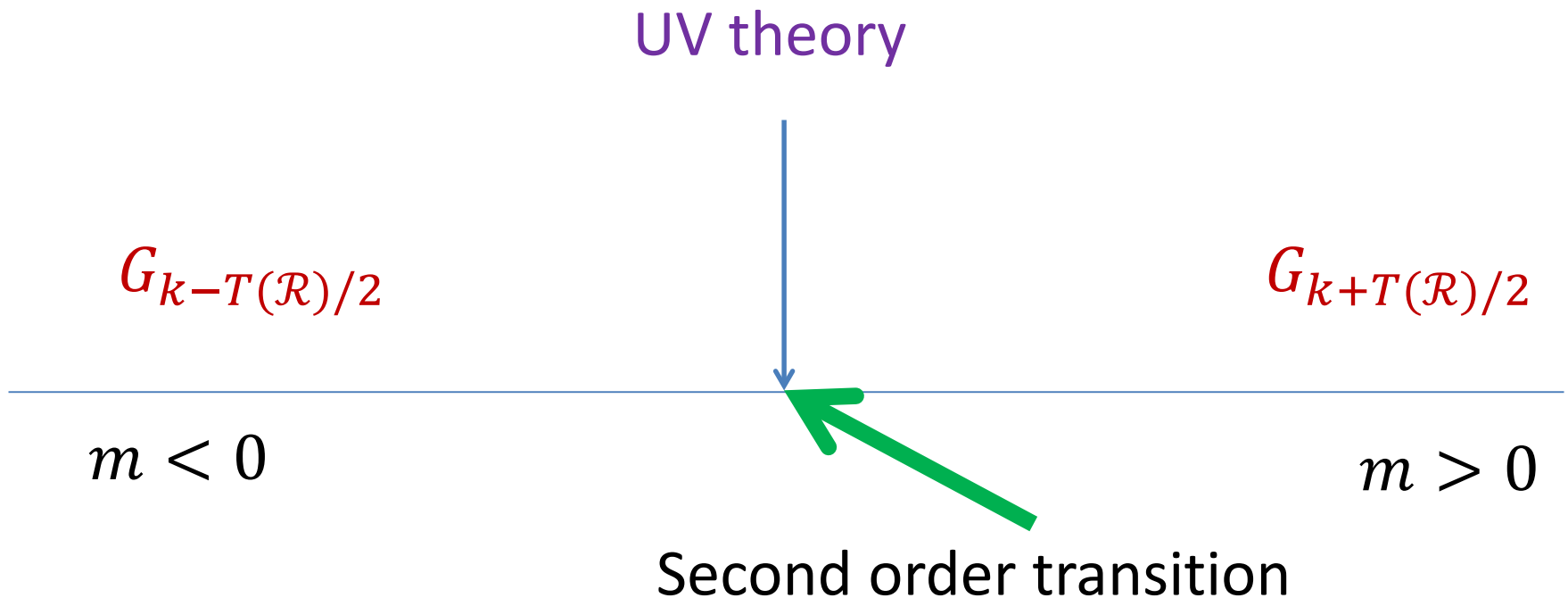
- Essentially free fermions with a Gauss law constraint
- Second order transition as a function of the fermion mass m



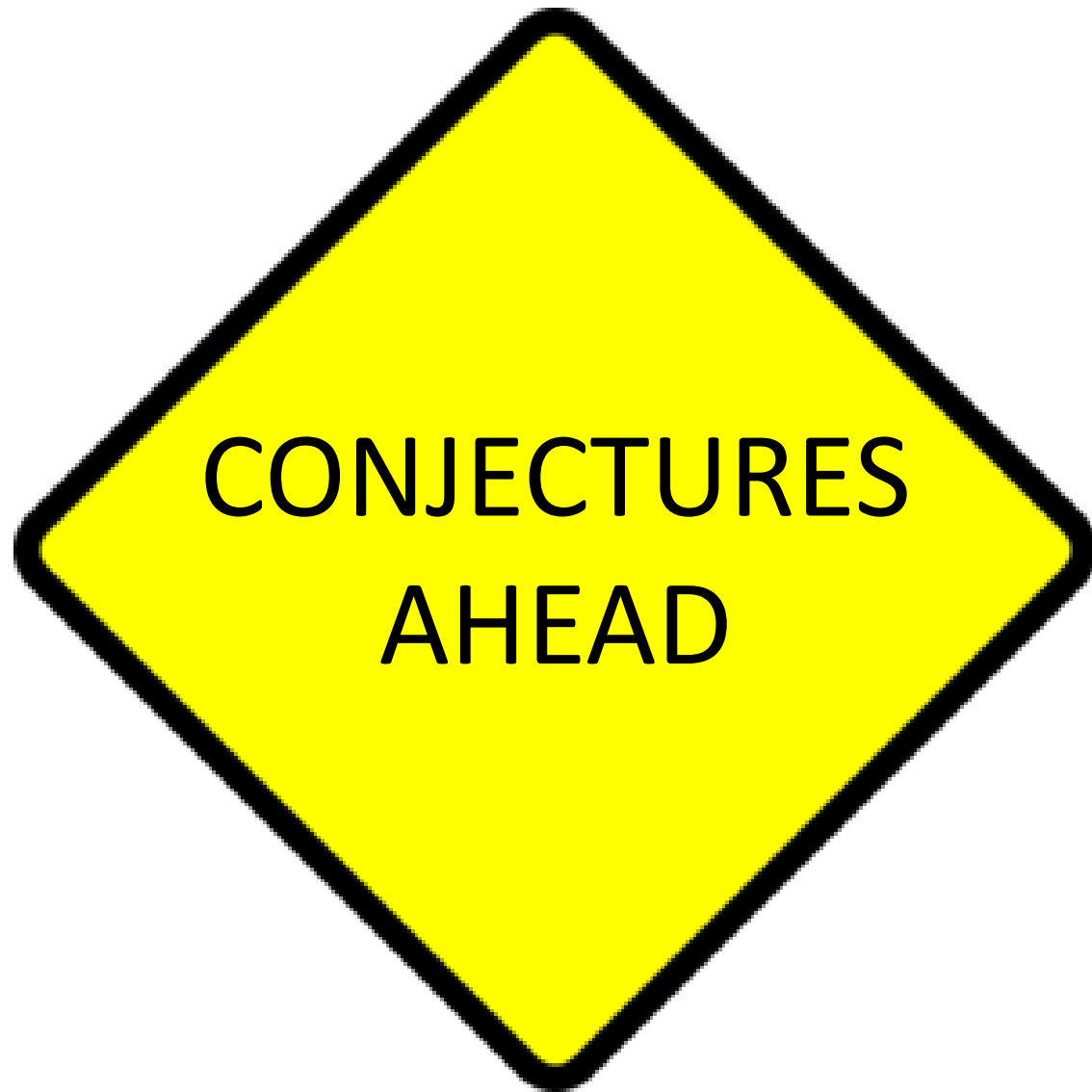
– What happens at small k ?

G_K with fermions in a representation \mathcal{R} Large $T(\mathcal{R})$ (e.g. large N_f)

- Second order transition as a function of the fermion mass m [Appelquist, Nash]



– What happens at small N_f ?



G_k with fermions in a representation \mathcal{R}

Large k

Conjectured duality

UV theory

Dual theory

$G_{k-T(\mathcal{R})/2}$

$G_{k+T(\mathcal{R})/2}$

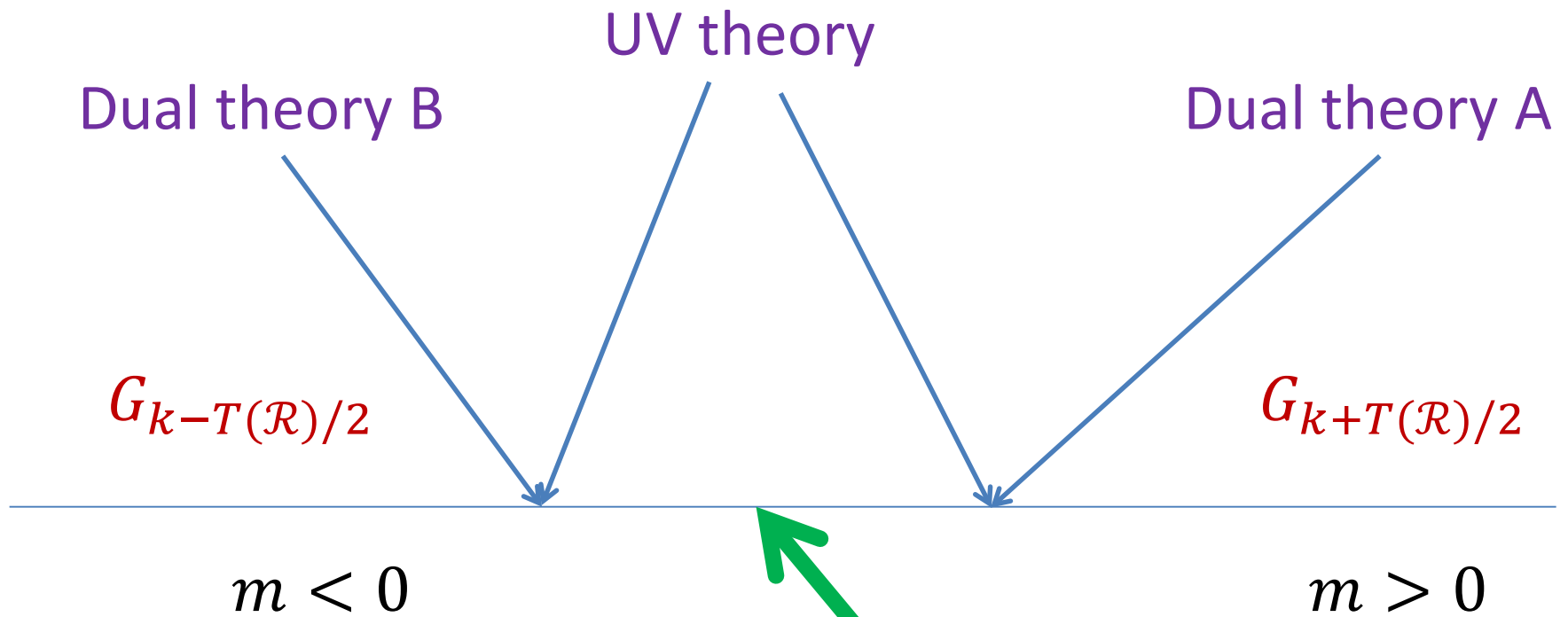
$m < 0$

$m > 0$

Second order transition

In the dual theory descriptions $G_{k\pm T(\mathcal{R})/2}$ are replaced by dual TQFTs (e.g. level-rank duality).

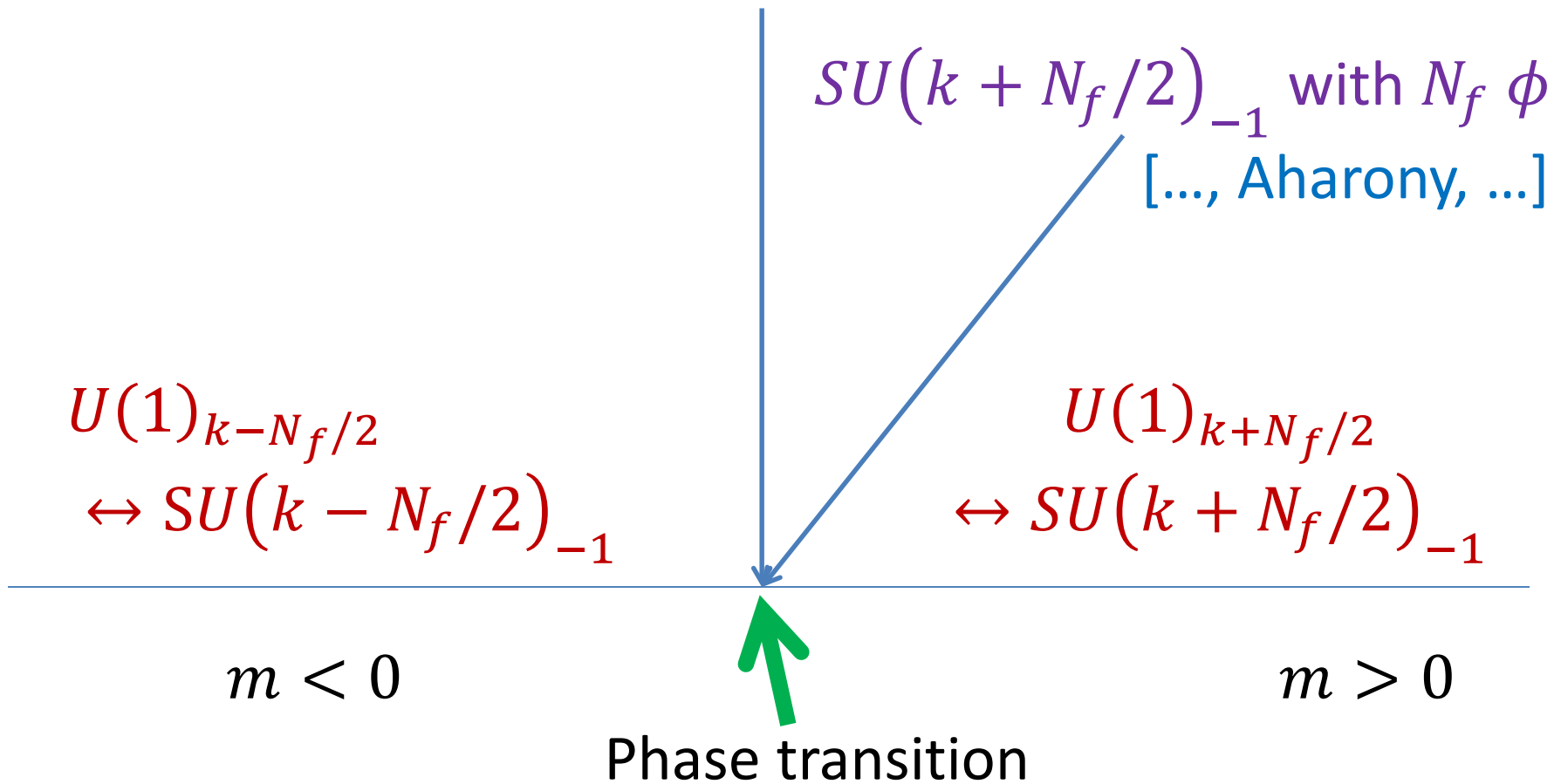
G_k with fermions in a representation \mathcal{R}
Small level k , small mass m , and small N_f
[Komargodski, NS]



Intermediate quantum phase.
It and the transitions are described in
the dual theories via weak coupling.

$$U(1)_k \text{ with } N_f \text{ electrons}$$

$$\left(k = \frac{N_f}{2} \bmod 1\right) \text{ with } k > \frac{N_f}{2}$$



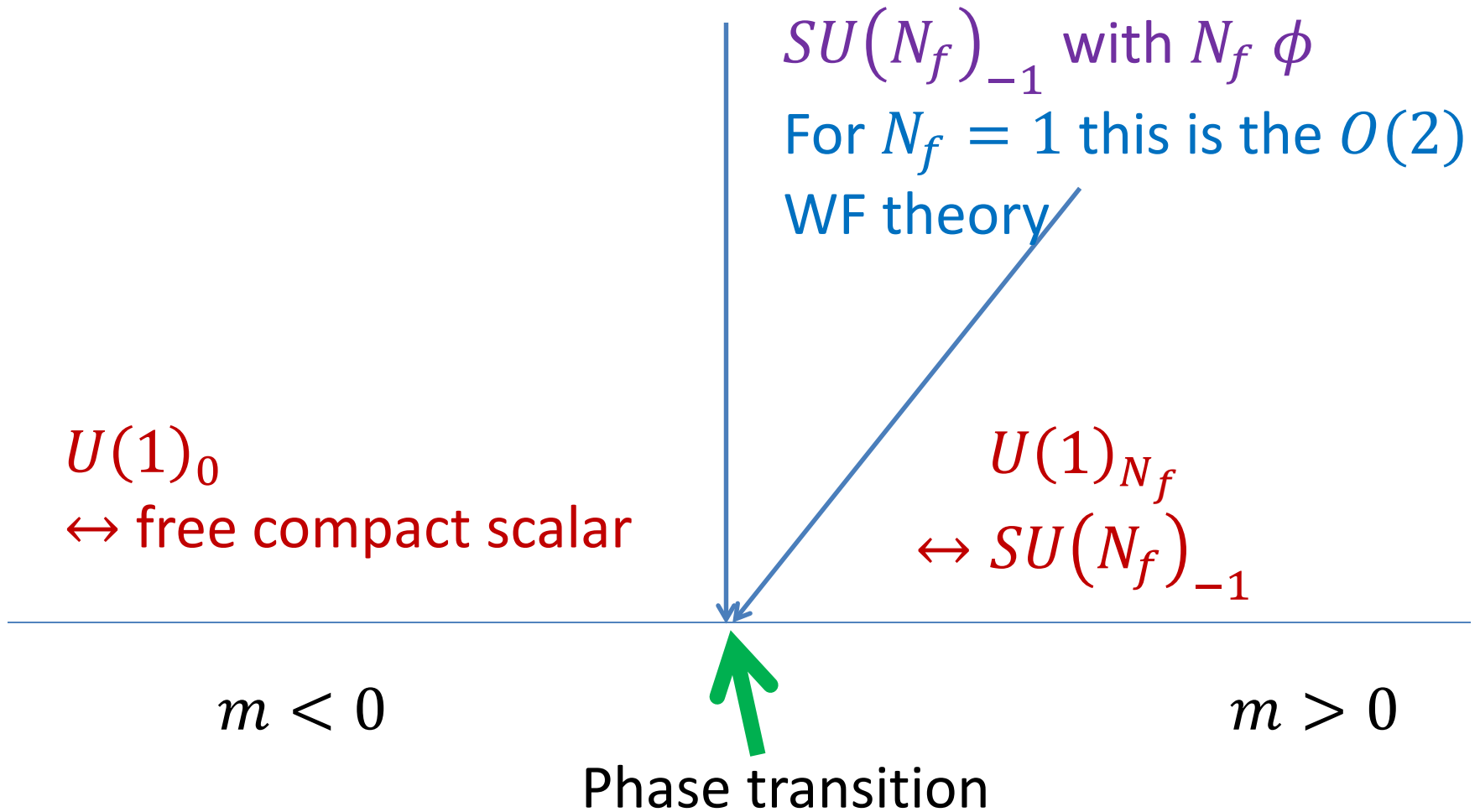
$U(1)_k$ with N_f electrons

- Did not prove it
- Did not determine whether the transitions are first or second order
- At large k the transition is second order. Conjecture that it is second order for all k .
- The duality originated from another duality at large N and k with $N_f = 1$ by [Giombi, Minwalla, Prakash, Trivedi, Wadia, and Yin] (we have $N = 1$)
- This statement for generic N , k , and N_f was found by Aharony and was made more precise in [Hsin, NS]

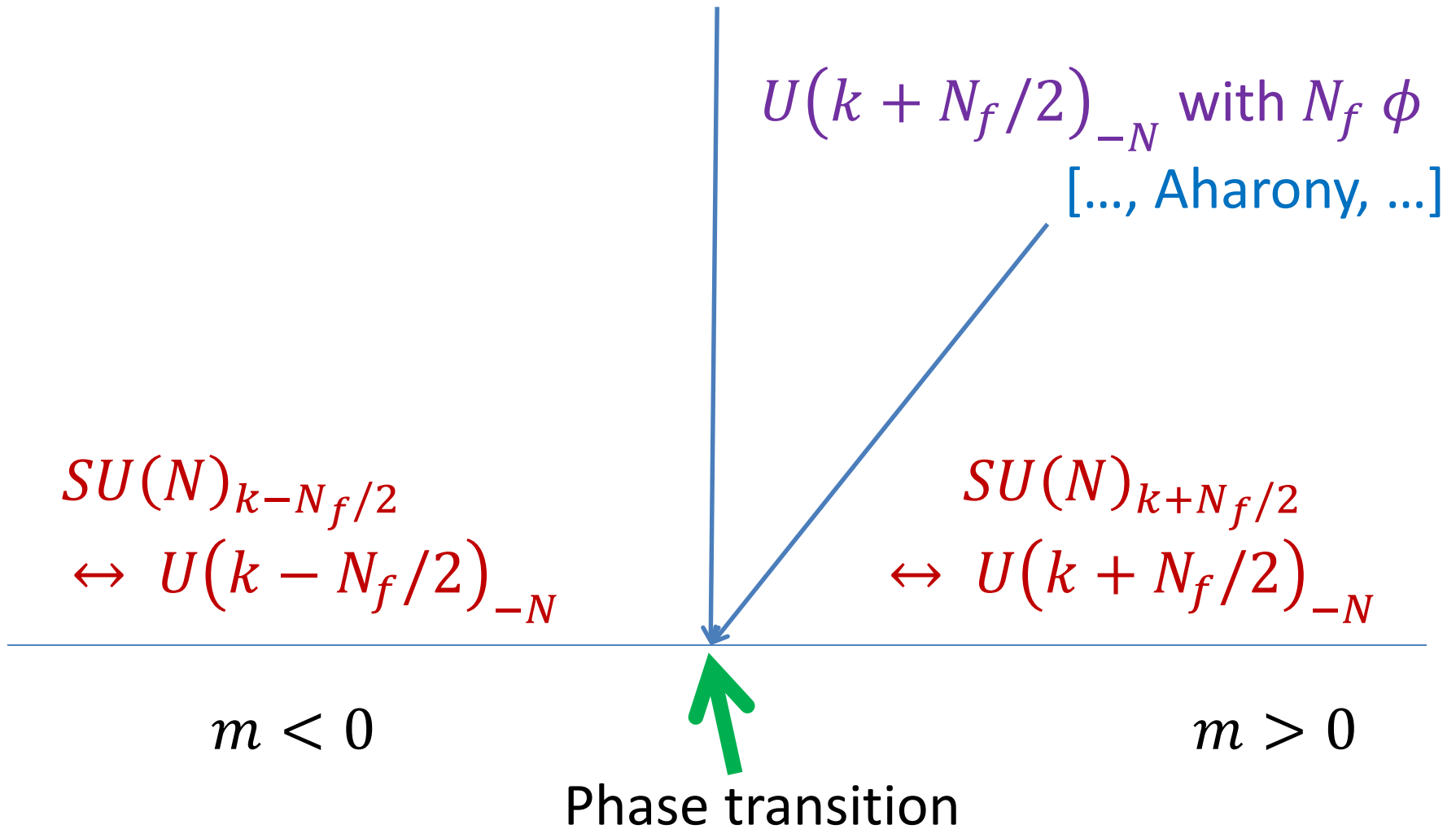
$U(1)_k$ with N_f electrons

- In the scalar picture there is a $|\phi|^4$ interaction and the duality is valid in the IR (gauged Wilson Fisher fixed point).
- In the scalar picture the gauge group is Higgsed and the level does not change.
- Monopoles \leftrightarrow baryons = $\phi^{k+N_f/2}$.
- The dual description reproduces the same gapped phases using level-rank duality

$U(1)_{k=N_f/2}$ with N_f electrons

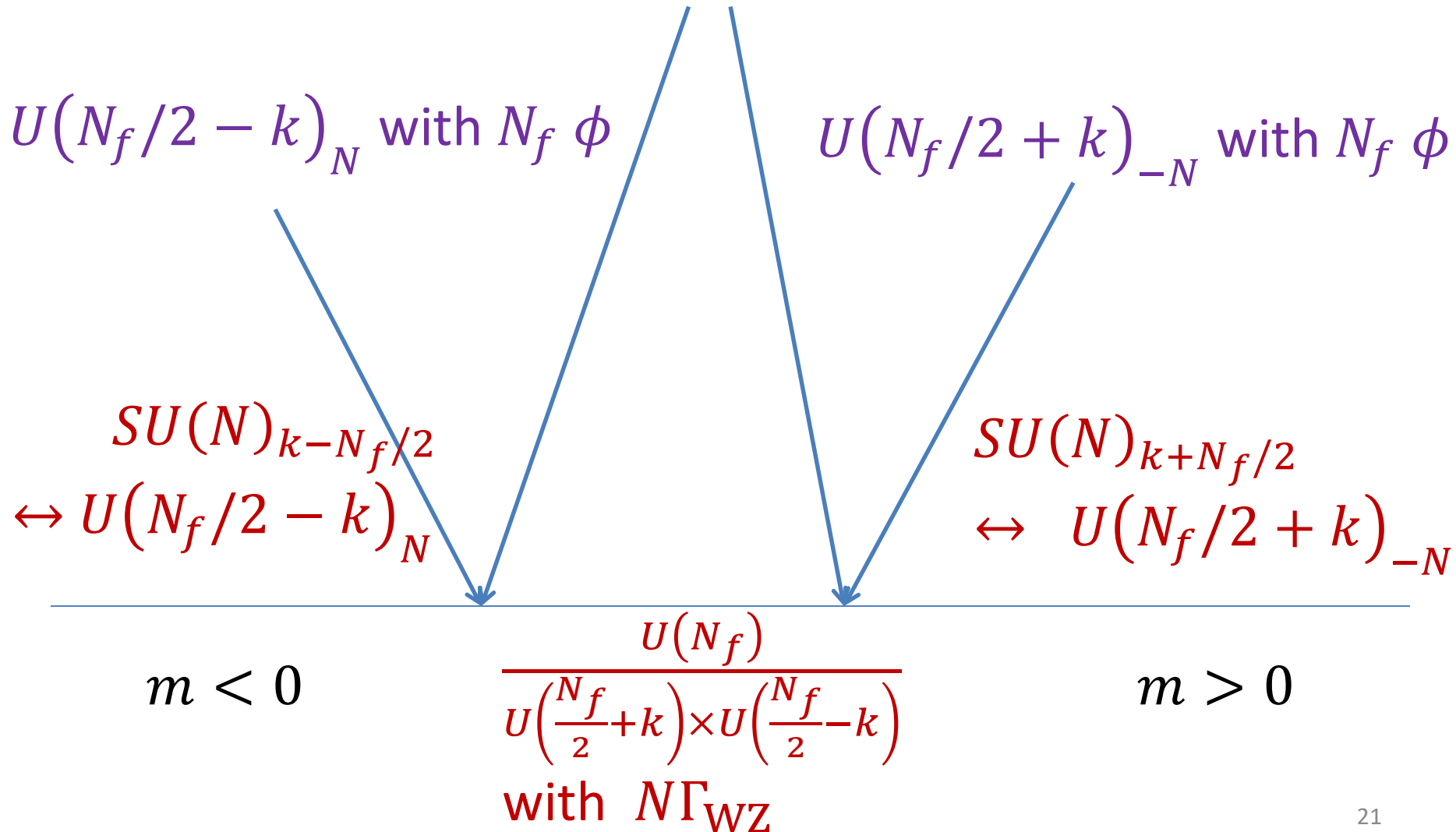


$SU(N)_k$ with N_f quarks $N_f/2 \leq k$



$SU(N)_k$ with N_f quarks

$2k < N_f < N^*(N, k)$ [Komargodski, NS]



$SU(N)_k$ with N_f quarks

$$2k < N_f < N^*(N, k)$$

- Three phases

- For large $|m|$ semiclassical physics – gapped, topological.

- For small $|m|$ a new quantum phase with global symmetry

breaking $U(N_f)/U\left(\frac{N_f}{2} + k\right) \times U\left(\frac{N_f}{2} - k\right)$

- Each phase transition has a weakly coupled bosonic dual description

- The intermediate phase

- Wess-Zumino term from the Chern-Simons term

- For $k = 0$: $U(N_f) \rightarrow U(N_f/2) \times U(N_f/2)$ [Vafa, Witten]

with a WZ term

- Assuming this, we can derive for other values of k

- Skyrmions in the nonlinear model are monopoles in the bosonic theory and are the baryons in the fermionic theory ²²

More (will not discuss here)

- $SO(N)_k$, $Spin(N)_k$, $O(N)^r_{k,L}$ with fermions in the vector representation
- $Sp(N)_k$ with fermions in the fundamental representation
- Relation to theories on domain walls and interfaces in four dimensions

$SU(N)_k$ with λ in adjoint for $N \leq 2k$

SUSY

$$SU(N)_{k-\frac{N}{2}} \leftrightarrow U(k - N/2)_{-N}$$

$$SU(N)_{k+\frac{N}{2}} \leftrightarrow U(k + N/2)_{-N}$$

$m < 0$

$m > 0$

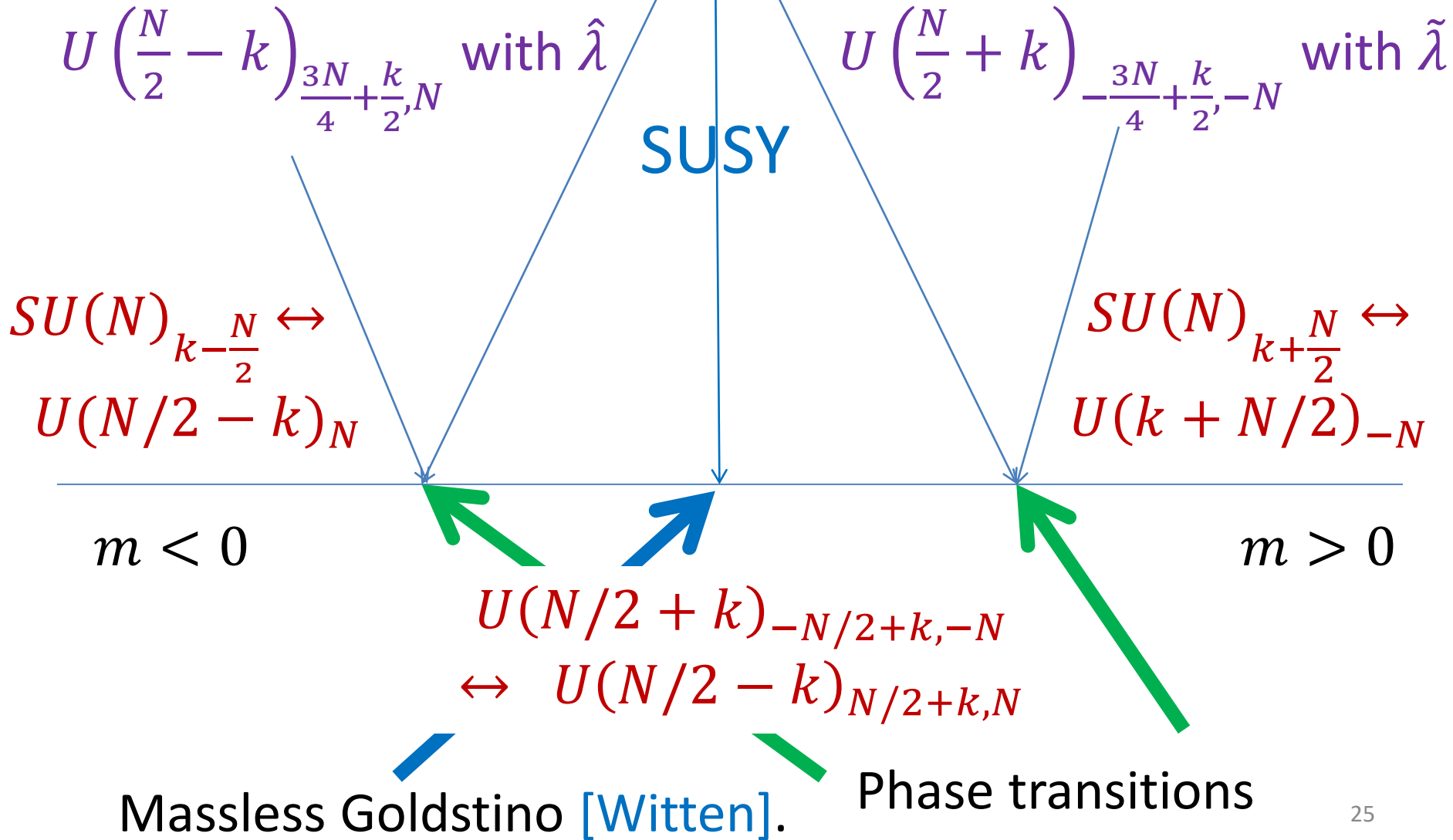
Phase transition

No transition. Supersymmetry is unbroken [Witten].

Holographic duals [Maldacena, Nastase] with $k - \frac{N}{2}$ branes.

$SU(N)_k$ with λ in adjoint for $0 \leq 2k < N$

[Gomis, Komargodski, NS]



$SU(N)_k$ with λ in adjoint for $0 \leq 2k < N$

- Three phases

- For large $|m|$ semiclassical physics – gapped, topological.

- For small $|m|$ a new quantum phase

$$U\left(\frac{N}{2} + k\right)_{-\frac{N}{2}+k, -N} \leftrightarrow U\left(\frac{N}{2} - k\right)_{\frac{N}{2}+k, N}$$

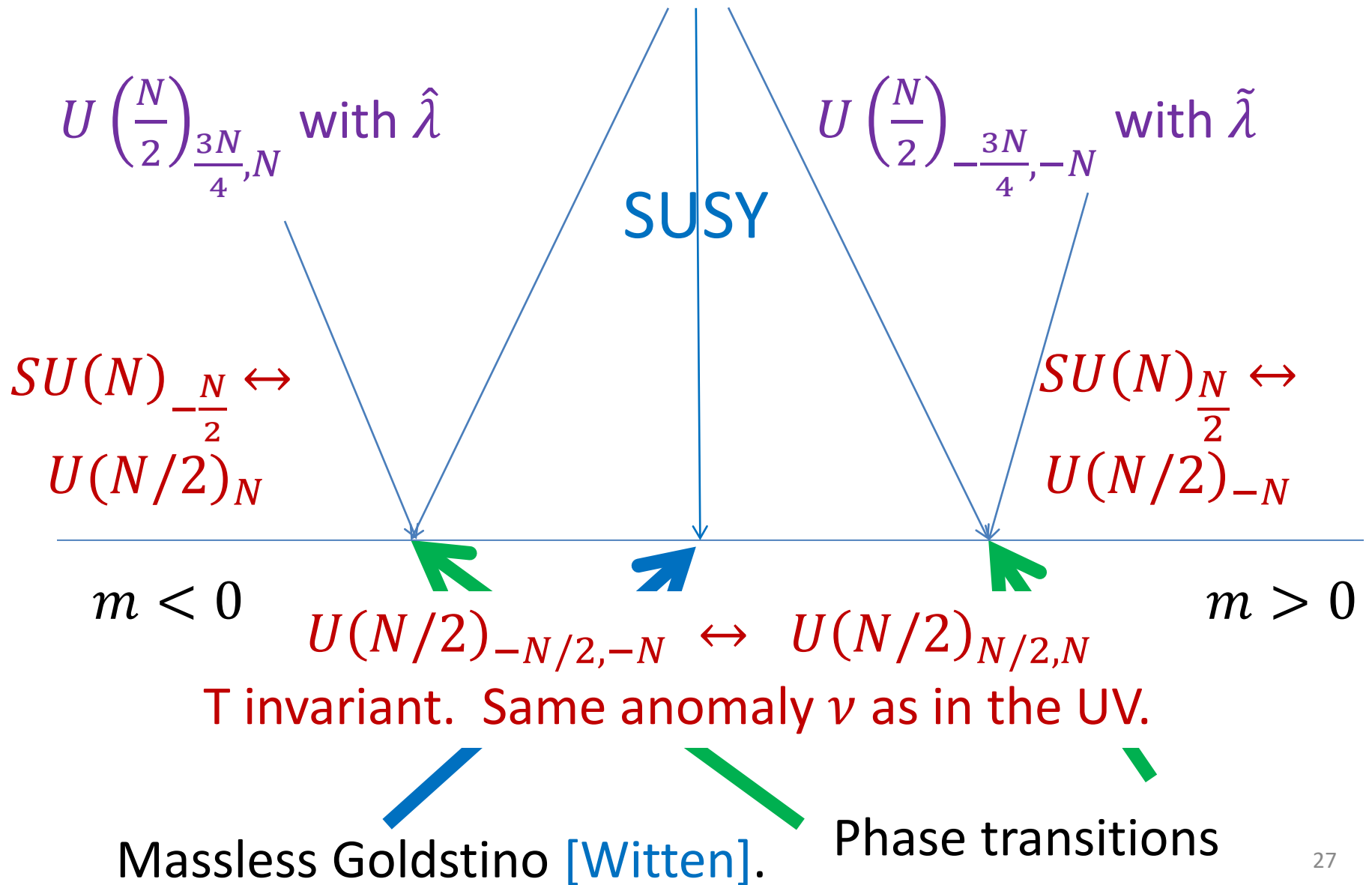
- Each phase transition has a fermionic dual description.

- $m_\lambda = 0$ supersymmetry. Spontaneously broken with a massless Goldstino [Witten] in addition to the TQFT.

- For special N and k the picture simplifies.

- Many consistency checks

$SU(N)_0$ with λ in adjoint [Gomis, Komargodski, NS]



$SU(2)_0$ with λ in adjoint [Gomis, Komargodski, NS]

SUSY

$SU(2)_{-1} \leftrightarrow$
 $U(1)_2$

$SU(2)_1 \leftrightarrow$
 $U(1)_{-2}$

$m < 0$

$m > 0$

Massless Goldstino [Witten].

More (will not discuss here)

- $SO(N)_k, Spin(N)_k, O(N)_{k,L}^r$ with adjoint fermions. (Highly nontrivial consistency tests of our picture.)
- $Sp(N)_k$ with adjoint fermions
- $SO(N)_k$ with fermions in a traceless symmetric tensor
- $Sp(N)_k$ with fermions in a traceless anti-symmetric tensor

They exhibit new phenomena.

Fermion-Fermion Dualities

[NS, T. Senthil, Wang, Witten]

Starting with the boson-fermion dualities (with the necessary background fields), we can derive a fermion-fermion duality (earlier version by [Son; Wang, Senthil; Metlitski, Vishwanath])

$$i\bar{\Psi} \not{D}_A \Psi \quad \leftrightarrow \quad i\bar{\chi} \not{D}_a \chi - \frac{2}{4\pi} bdb + \frac{1}{2\pi} adb + \frac{1}{2\pi} Adb - \frac{1}{4\pi} AdA$$

A is a classical $U(1)$ background field. (Note its counterterm.)

The dynamics in the RHS is not that of QED (different critical exponents). It leads in the IR to the free LHS.

Fermion-Fermion Dualities [Cordova, Hsin, NS]

Using this duality we can derive

$$U(1)_0 + \psi \text{ with charge } 2 \leftrightarrow \chi + U(1)_2$$

- Both sides are time-reversal invariant at all scales
 - unlike $U(1)_{1/2}$ with a single fermion of charge one
- The RHS includes a free Dirac fermion and a decoupled TQFT. Can view the LHS as UV and the RHS as IR.
- The monopole operator in the UV is mapped to the free fermion χ in the IR.
- The Wilson line in the LHS is the Wilson line of $U(1)_2$.

Fermion-Fermion Dualities [Cordova, Hsin, NS]

$$U(1)_0 + \psi \text{ with charge } 2 \leftrightarrow \chi + U(1)_2$$

- The magnetic global $U(1)_M$ acts in the RHS both on χ and on the anyons of the TQFT.
- All local operators satisfy $(-1)^M = (-1)^F$.
- As is common in dualities (using standard definitions),

$$T_{UV} \leftrightarrow (CT)_{IR}$$

$$(CT)_{UV} \leftrightarrow T_{IR}$$

- The anomalies in T and CT match

$$- \nu_{T_{UV}} = 0, \quad \nu_{(CT)_{UV}} = 2$$

$$- \nu_{T_{IR}} = 2, \quad \nu_{(CT)_{IR}} = 0$$

Fermion-Fermion Dualities [Cordova, Hsin, NS]

$$U(1)_0 + \psi \text{ with charge } 2 \leftrightarrow \chi + U(1)_2$$

- Add to the UV Lagrangian a charge 2 monopole operator.
 - Cannot add a charge 1 monopole operator because it is a fermion.
- In the IR it is $\chi_1^2 - \chi_2^2$ with $\chi = \chi_1 + i\chi_2$. It splits the IR fixed point to two points with massless $\chi_{1,2}$.
- It breaks the magnetic $U(1)_M$ to \mathbf{Z}_2 : $(-1)^M = (-1)^F$.
- It preserves C and breaks T_{UV} .
- But...

Fermion-Fermion Dualities [Cordova, Hsin, NS]

$$U(1)_0 + \psi \text{ with charge } 2 \leftrightarrow \chi + U(1)_2$$

- It preserves a subgroup – modified time-reversal

$$T' = T_{UV} e^{\frac{i\pi}{2}M} = (CT)_{IR} e^{\frac{i\pi}{2}M}$$

- Its algebra is not the standard one

$$\begin{aligned} T' C &= CT' (-1)^M \\ (CT')^2 &= (-1)^M = (-1)^F \\ (T')^2 &= (-1)^F (-1)^M = 1 \end{aligned}$$

T' is not a conventional time-reversal symmetry. CT' is.

Such a deformation of the naïve global symmetry group is ubiquitous.

More

- Many other cases have been analyzed.
 - They exhibit many new phenomena.
 - New insights about confinement
- Many more generalizations have not yet been analyzed.

QFT is fun