IS QUANTUM THEORY A FORM OF STATISTICAL MECHANICS?

[See: S. Adler
"Quantum Theory as an Emergent Phenomenon" Cambridge 2004

also hep-th/0510120]

QUANTUM MECHANICS IS OUR MOST SUCCESSFUL PHYSICAL THEORY. THERE ARE TWO POSSIBILITIES:

- IT IS EXACT, WITH (PERHAPS) A NEED FOR REINTERPRETATION AT A FOUNDATIONAL LEVEL: MANY WORLDS / HISTORIES
BOHMIAN / BAYESIAN ...

- IT IS <u>NOT</u> EXACT, BUT IS A VERY ACCURATE ASYMPTOTIC APPROXIMATION TO A DEEPER LEVEL THEORY - A PRE-QUANTUM MECHANICS

MOTIVATIONS FOR PURSUING SECOND POSSIBILITY:

- RIDDLE OF "CANONICAL QUANTIZATION"

STANDARD APPROACH -

WRITE DOWN A CLASSICAL THEORY

"QUANTIZE" IT BY: POISSON BRACKET

$\rightarrow -\frac{i}{\hbar}$ COMMUTATOR

Q's:

(1) IF QUANTUM THEORY IS MORE FUNDAMENTAL, WHY CAN'T IT BE OBTAINED DIRECTLY FROM AN OPERATOR THEORY?

(2) WHAT IS ORIGIN OF PLANCK CONSTANT $\hbar$?

CANONICAL QUANTIZATION LOOKS VERY MUCH LIKE AN ALGORITHM TO INVERT THE CLASSICAL LIMIT OF QUANTUM THEORY

- MEASUREMENT PROBLEM

Q MECH. A LINEAR THEORY - UNITARY TIME EVOLUTION

"MEASUREMENTS" INVOKE A NONLINEAR "STATE VECTOR REDUCTION" INDUCED BY A "CLASSICAL" APPARATUS

"PREFERRED BASIS" PROBLEM

HOW DO PROBABILITIES BECOME ACTUALITIES?

WHY SHOULD ONE HAVE PROBABILITIES
WITHOUT AN UNDERLYING SAMPLE SPACE?

- PECULARITIES OF QUANTUM THEORY

  NONLOCALITY - EPR

  INFINITIES IN QUANTUM FIELD THEORY

- UNIFICATION OF QUANTUM THEORY
  WITH GRAVITATION

  FOCUS HAS BEEN ON "PRE-GEOMETRIC"
  THEORIES LIKE STRING THEORY

  UNDERLYING THEORY COULD BE
  "PRE-QUANTUM MECHANICAL" AS WELL

- COSMOLOGICAL CONSTANT PROBLEM -
  HAS RESISTED ALL ATTEMPTS AT SOLUTION
  WITHIN STANDARD QUANTUM FRAMEWORK

- I HAVE A CONCRETE IDEA AS TO
  HOW ONE MIGHT PROCEED

CLASSICAL MECHANICS - - - - - - VARIABLES COMMUTATIVE

ALL $\propto$ UNIT MATRIX

CANONICAL QUANTIZATION

CLASSICAL LIMIT

$\hbar \to 0$

STOCHASTIC SCHRÖDINGER EQUATION $\leftarrow$ QUANTUM MECHANICS - - - - - VARIABLES SPECIAL (INFINITE) MATRICES

$[q_\ell, p_m] = i\hbar\,\delta_{\ell m}$

$[q_\ell, q_m] = 0$ etc

SPACELIKE COMMUTATIVE

BROWNIAN MOTION CORRECTIONS

THERMODYNAMICS

EQUILIBRIUM STATISTICAL MECHANICS

"TRACE DYNAMICS" - - - - VARIABLES <u>GENERAL</u> MATRICES

or

"GENERALIZED QUANTUM DYNAMICS"

NO A PRIORI COMMUTATIVITY PROPERTIES

DYNAMICS ASSUMED <u>GLOBAL</u> <u>UNITARY</u> <u>INVARIANT</u>

$[q_\ell, q_m] \neq 0$ etc

THIS IS BASIS FOR EMERGENT QUANTUM MECHANICS

- OVERVIEW OF TRACE DYNAMICS

"GENERALIZED QUANTUM DYNAMICS" OR "TRACE DYNAMICS" IS A NONCOMMUTATIVE GENERALIZATION OF CLASSICAL LAGRANGIAN AND HAMILTONIAN DYNAMICS

HOW IT WORKS (BOSONIC CASE)

$$\{q_r\} \qquad \{\dot{q}_r\} \qquad \cdot = \frac{\partial}{\partial t}$$

NONCOMMUTING COORDINATES – OPERATORS ON UNDERLYING COMPLEX HILBERT SPACE

$$L = L[\{q_r\}, \{\dot{q}_r\}]$$

ORDERING IMPORTANT, SO $\frac{\delta L}{\delta q_r}$ NOT DEFINED

NOW USE CYCLIC INVARIANCE OF TRACE : DEFINE

TRACE LAGRANGIAN $\mathbf{L}$

$$\mathbf{L} = \mathrm{Tr}\, L$$

FORM $\delta \mathbf{L}$ AND REORDER CYCLICALLY SO THAT ALL δq_r, $\delta \dot{q}_r$ STAND ON THE RIGHT

<u>DEFINITION</u>

$$\delta \mathbf{L} = \mathrm{Tr} \sum_r \left(\frac{\delta \mathbf{L}}{\delta q_r} \delta q_r + \frac{\delta \mathbf{L}}{\delta \dot{q}_r} \delta \dot{q}_r \right)$$

CAN NOW SHOW

① $$0 = \delta \mathbf{S} = \delta \int_{-\infty}^{\infty} dt\, \mathbf{L}$$

$$\Rightarrow \frac{\delta \mathbf{L}}{\delta q_r} - \frac{d}{dt} \frac{\delta \mathbf{L}}{\delta \dot{q}_r} = 0$$

DIRECTLY GIVES <u>OPERATOR</u> EULER-LAGRANGE EQUATIONS

HAVE A DYNAMICS THAT IS MORE GENERAL THAN QUANTUM MECHANICS

TO RECOVER QUANTUM MECHANICS, WE CONSIDER THE EQUILIBRIUM STATISTICAL MECHANICS OF THIS DYNAMICS

HAVE THREE GENERIC CONSERVED QUANTITIES:

① TRACE HAMILTONIAN

DEFINE $\quad p_r = \dfrac{\delta \underset{\sim}{L}}{\delta \dot{q}_r}$

$$\underset{\sim}{H} = \mathrm{Tr}\left(\sum_r p_r \dot{q}_r\right) - \underset{\sim}{L}$$

THEN

$$\underbrace{\frac{\delta \underset{\sim}{H}}{\delta q_r} = -\dot{p}_r \qquad \frac{\delta \underset{\sim}{H}}{\delta p_r} = \dot{q}_r}_{\text{OPERATOR HAMILTON EQUATIONS}}$$

$$\frac{d\underset{\sim}{H}}{dt} = \mathrm{Tr}\sum_r\left(\frac{\delta \underset{\sim}{H}}{\delta q_r}\dot{q}_r + \frac{\delta \underset{\sim}{H}}{\delta p_r}\dot{p}_r\right)$$

$$= \mathrm{Tr}\sum_r\left(-\dot{p}_r \dot{q}_r + \dot{q}_r \dot{p}_r\right) = 0$$

ANDREW MILLARD

② OPERATOR $\tilde{C} = \sum_{r} [q_r, p_r] = -\tilde{C}^{\dagger}$

(WITH FERMIONS, $\tilde{C} = \sum_{r,B} [q_r, p_r] - \sum_{r,F} \{q_r, p_r\}$)

SUPPOSE $\underset{\sim}{H}$ IS GLOBAL UNITARY INVARIANT

(i.e. IT INVOLVES NO NONCOMMUTATIVE CONSTANTS)

$\tilde{C}$ IS CORRESPONDING NOETHER CHARGE OPERATOR

$$\frac{d\tilde{C}}{dt} = 0$$

③ PHASE SPACE MEASURE

LET

$$d\mu = \prod_{r,m,n,A} d\langle m|q_r|n\rangle^A \; d\langle m|p_r|n\rangle^A$$

INDEXES REAL COMPONENTS

$d\mu$ IS INVARIANT UNDER CANONICAL TRANSFORMATIONS $\delta p_r = -\frac{\delta \underset{\sim}{G}}{\delta q_r}$ $\delta q_r = \frac{\delta \underset{\sim}{G}}{\delta p_r}$

⇒ GENERALIZED LIOUVILLE THEOREM

CAN USE STATISTICAL MECHANICS

CANONICAL ENSEMBLE:

$$\rho = Z^{-1} e^{-\tau \underset{\sim}{H} - \mathrm{Tr}(\tilde{\lambda}\tilde{C})}$$

$$\int d\mu \, \rho = 1 \quad \Rightarrow$$

$$Z = \int d\mu \, e^{-\tau \underset{\sim}{H} - \mathrm{Tr}(\tilde{\lambda}\tilde{C})}$$

$\tilde{\lambda}, \tau$ ARE ENSEMBLE PARAMETERS

$$\langle \tilde{C} \rangle_{AV} \equiv i_{eff} \, D$$

$$i_{eff}^2 = -1$$

$$i_{eff}^{+} = -i_{eff}$$

$$[i_{eff}, D] = 0$$

SIMPLEST CASE IS D = A MULTIPLE OF UNIT MATRIX

$$D = \hbar \, 1$$

$\langle \tilde{\tau} \rangle_{AV}$ A FUNCTION OF $\tilde{\lambda}, \tau$

$\Rightarrow \tilde{\lambda} = i_{eff}\, \lambda$

SO THE CANONICAL ENSEMBLE ONLY PARTIALLY BREAKS THE GLOBAL UNITARY INVARIANCE : ρ INVARIANT UNDER U_{eff} FOR WHICH $U_{eff}^{\dagger}\, i_{eff}\, U_{eff} = i_{eff}$

SINCE $\mathrm{Tr}\, \tilde{\lambda}\, \tilde{\xi} \to \mathrm{Tr}\, \tilde{\lambda}\, U_{eff}^{\dagger}\, \tilde{\xi}\, U_{eff}$

$$= \mathrm{Tr}\, U_{eff}\, \tilde{\lambda}\, U_{eff}^{\dagger}\, \tilde{\xi} = \mathrm{Tr}\, \tilde{\lambda}\, \tilde{\xi}$$

SO WE WILL AVERAGE TOO MUCH IF WE USE $\int d\mu\, \rho$

LET $d\mu' = d\mu \Big|_{\substack{\text{OVERALL } U_{eff} \\ \text{FROZEN}}}$

USE THIS RESTRICTED MEASURE TO FORM THERMODYNAMIC AVERAGES

LET X BE ANY q_r OR p_r

$X_{eff} \equiv \frac{1}{2}\left(X - i_{eff}\, X\, i_{eff}\right)$ = PART OF X THAT COMMUTES WITH i_{eff}

$$i_{eff}\, X_{eff} = \frac{1}{2}\left(i_{eff}\, X + X\, i_{eff}\right) = X_{eff}\, i_{eff}$$

- HOW QUANTUM FIELD THEORY EMERGES

$$\frac{\int d\mu' \rho \; x_{1\,eff} \cdots x_{n\,eff}}{\int d\mu' \rho} \Leftrightarrow X_{1\,eff} \cdots X_{n\,eff}$$

↑ OPERATORS IN A QUANTUM THEORY WITH ROLE OF i PLAYED BY i_{eff}

CAN DERIVE EQUIPARTITION THEOREMS (WARD IDENTITIES) FOR THESE AVERAGES

WHEN (i) $\tau \sim \frac{1}{\text{MASS}}$ VERY SMALL

↗ PLANCK MASS ?

(ii) $\tilde{C} \to \langle \tilde{C} \rangle_{AV} = i_{eff} \hbar$

THESE WARD IDENTITIES HAVE THE STRUCTURE OF QUANTUM MECHANICS

$$[q_{r\,eff}, p_{s\,eff}] = i_{eff} \hbar \, \delta_{rs} + O(\tau)$$

$$[q_{r\,eff}, q_{s\,eff}] = 0 + O(\tau)$$

LOCALITY IS <u>EMERGENT</u>

$$\dot{x}_{eff} = \frac{i_{eff}}{\hbar} [H_{eff}, x_{eff}] + O(\tau)$$

INSIDE AVERAGES

SKETCH OF GENERAL WARD IDENTITY DERIVATION:

$$\frac{\int d\mu' \rho\, \mathcal{O}}{\int d\mu' \rho} \equiv \langle \mathcal{O} \rangle_{AV}$$

TAKE $\mathcal{O} = \{\tilde{C}, i_{eff}\} W$

- USE $\int d\mu' \, \delta[\rho\, \mathcal{O}] = 0$

  (TRANSLATION INVARIANCE OF MEASURE $d\mu'$)

- NEGLECT τ TERM COMING FROM $\delta\rho$

- REPLACE $\tilde{C} \to \langle \tilde{C} \rangle_{AV}$ IN INTEGRANDS

- MAKE VARIOUS CHOICES FOR W

  $W \propto$ CANONICAL q OR p

  GIVES CANONICAL ALGEBRA INSIDE $\langle\rangle_{AV}$

  $W \propto H$ (OPERATOR HAMILTONIAN)

  GIVES HEISENBERG EQ. OF MOTION INSIDE $\langle\rangle_{AV}$

  $W \propto G = G^{\dagger}$ GIVES UNITARY CANONICAL TRANSFORMATION INSIDE $\langle\rangle_{AV}$

CAN INCLUDE SOURCES IN ρ SO THAT $\langle\,\rangle_{AV}$ HAS SOURCE TERMS THAT CAN BE VARIED

REMARKS:

① $O(\tilde{\lambda})$ TERM IN WARD IDENTITIES VANISHES BECAUSE

$$\{ i_{eff}, [\tilde{\lambda}, x] \} \propto [\tilde{\lambda}, x_{eff}] = 0$$

② CONDITIONS FOR NEGLECT OF $O(\tau)$ TERMS IN WARD IDENTITIES

NEED BOSON - FERMION BALANCE

SUFFICES FOR $\dot{X}_{eff}$ AND $\tilde{\zeta}_{eff}$ TO HAVE DISJOINT SUPPORT ON OPERATOR PHASE SPACE

WHY BOSON - FERMION BALANCE IS NEEDED

$$\langle [q_s, p_r] \rangle_{AV} = i_{eff}\, \hbar\, \delta_{rs}$$

$r = s$ AND SUM

$$\langle \sum_r [q_r, p_r] \rangle_{AV} = \langle \tilde{C} \rangle_{AV} = i_{eff}\, \hbar$$

$$= i_{eff}\, \hbar \underbrace{\sum_r 1}_{= N_B} \quad \text{CONTRADICTION}$$

WHEN <u>BOTH</u> BOSONS + FERMIONS PRESENT

BOSON $\langle [q_s, p_r] \rangle_{AV} = i_{eff} \hbar \, \delta_{rs} + \varepsilon \ldots$

FERMION $\langle \{q_s, p_r\} \rangle_{AV} = i_{eff} \hbar \, \delta_{rs} + \varepsilon \ldots$

$r = s$ AND SUM

$$\left\langle \sum_{r, B} [q_r, p_r] - \sum_{r, F} \{q_r, p_r\} \right\rangle_{AV}$$

$$= \langle \tilde{C} \rangle_{AV} = i_{eff} \hbar$$

$$= i_{eff} \hbar \, (N_B - N_F) + \sum \varepsilon \ldots$$

NO CONTRADICTION IF $N_B = N_F$

- WHY THERE ARE BROWNIAN MOTION CORRECTIONS

$$\tilde{C} = \underset{i_{eff}\hbar}{\underset{\parallel}{\langle C\rangle_{AV}}} + \Delta C$$

$\uparrow$ RAPIDLY FLUCTUATING

$$\frac{1}{2}\{\tilde{C}, i_{eff}\} = -\hbar + \underbrace{\frac{1}{2}\{\Delta\tilde{C}, i_{eff}\}}_{-\hbar(K+N)}$$

K = FLUCTUATING C-NUMBER

N = FLUCTUATING MATRIX, WITH OPERATOR ANALOG

see hep-th/0510120 for update to book Chapter 6

FROM N TERM: APPROPRIATE ANSATZ CORRESPONDS TO MASS-PROPORTIONAL FORM OF "CONTINUOUS SPONTANEOUS LOCALIZATION" (CSL) STOCHASTIC SCHRÖDINGER EQUATION

STOCHASTIC MODIFIED SCHRÖDINGER EQN. - SIMPLE CASE FOR POINTER WITH CENTER OF MASS VARIABLE q

(THIS IS THE LEADING SMALL DISPLACEMENT APPROXIMATION TO THE GRW AND CSL MODELS)

$$d|\psi\rangle = \frac{-i}{\hbar} H|\psi\rangle dt \quad - \frac{\eta}{2}\left(q - \langle q\rangle\right)^2 |\psi\rangle dt$$

$$+ \sqrt{\eta}\left(q - \langle q\rangle\right)|\psi\rangle dW_t$$

$$\langle q\rangle = \langle\psi|q|\psi\rangle$$

EXPECTATION VALUE OF q IN STATE $|\psi\rangle$

BECAUSE $|\psi\rangle$ APPEARS IN $\langle q\rangle$, THIS IS A <u>NONLINEAR</u> STOCHASTIC DIFFERENTIAL EQUATION

- THIS EQUATION CAN BE PROVED TO GIVE STATE VECTOR REDUCTION ON POSITION EIGENSTATES WITH BORN RULE PROBABILITIES

- FOR DETAILED DISCUSSION OF RATES + ALLOWED PARAMETER VALUES FOR CSL REDUCTION SEE:

  S. Adler quant-ph/0605072

SUMMARY

TRACE DYNAMICS AS PRE-QUANTUM MECHANICS

- THERMODYNAMICS - VIA EQUIPARTITION THEOREMS (WARD IDENTITIES)

  $\Leftrightarrow$ UNITARY EVOLUTION

- BROWNIAN MOTION CORRECTIONS - VIA CSL PHENOMENOLOGY

  $\Leftrightarrow$ BORN RULE, PROBABILITY INTERPRETATION

I WILL NOW GIVE SOME DETAILS OF DERIVATIONS :

- CONSERVATION OF $\tilde{\mathcal{E}}$

- WARD IDENTITY DERIVATION (SIMPLIFIED EXAMPLE)

- STOCHASTIC SCHRÖDINGER EQN
  $\Rightarrow$ REDUCTION WITH BORN RULE PROBABILITIES

- CONSERVATION OF $\tilde{C}$

ASSUME GLOBAL UNITARY INVARIANCE

$$\underset{\sim}{H}\left[\{U^\dagger q_n U\},\{U^\dagger p_n U\}\right] = \underset{\sim}{H}\left[\{q_n\},\{p_n\}\right]$$

$$U = e^{\Lambda},\quad U^\dagger = e^{-\Lambda},\quad \Lambda = -\Lambda^\dagger$$

$\Rightarrow$

$$\underset{\sim}{H}\left[\{q_n - [\Lambda, q_n]\},\{p_n - [\Lambda, p_n]\}\right] = \underset{\sim}{H}\left[\{q_n, p_n\}\right]$$

$\Rightarrow$

$$\mathrm{Tr}\,\underset{\sim}{\rho}\left(-\frac{\delta \underset{\sim}{H}}{\delta q_n}[\Lambda, q_n] - \frac{\delta \underset{\sim}{H}}{\delta p_n}[\Lambda, p_n]\right) = 0$$

$\Rightarrow$

$$\mathrm{Tr}\,\Lambda\left(\underset{\sim}{\rho}\left[\frac{\delta \underset{\sim}{H}}{\delta q_n}, q_n\right] + \left[\frac{\delta \underset{\sim}{H}}{\delta p_n}, p_n\right]\right) = 0$$

Λ ARBITRARY $\Rightarrow$ OPERATOR RELATION

$$\underset{\sim}{\rho}\left[\underbrace{\frac{\delta \underset{\sim}{H}}{\delta q_n}}_{-\dot{p}_n}, q_n\right] + \left[\underbrace{\frac{\delta \underset{\sim}{H}}{\delta p_n}}_{\dot{q}_n}, p_n\right] = 0$$

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$$\Rightarrow \sum_n [q_n, \dot{p}_n) + [\dot{q}_n, p_n) = 0$$

$$\Rightarrow \dot{\mathcal{C}} = 0$$

$$\mathcal{C} = \sum_n [q_n, p_n) \qquad \text{BOSONS}$$

WHEN FERMIONS (GRASSMANN ODD q_n, p_n) INCLUDED

$$\mathcal{C} = \sum_{n,\,\text{BOSONS}} [q_n, p_n) - \sum_{n,\,\text{FERMIONS}} \{q_n, p_n\}$$

$$\dot{\tilde{\mathcal{C}}} = 0$$

- EXACT WARD IDENTITY
  (BOSON CASE)

$$Z \langle \mathrm{Tr}\, \tilde{C} p_a \rangle_{AV} = \int d\mu \; e^{-\tau \underset{\sim}{H} - \mathrm{Tr}\, \tilde{\lambda} \tilde{C}} \; \mathrm{Tr}\, \tilde{C} p_a$$

MAKE A CONSTANT SHIFT OF p_s

$$\delta_s (d\mu) = 0$$

$\Rightarrow$

$$0 = \int d\mu \; \delta_s \left[e^{-\tau \underset{\sim}{H} - \mathrm{Tr}\, \tilde{\lambda} \tilde{C}} \; \mathrm{Tr}\, \tilde{C} p_a \right]$$

$$= \int d\mu \; e^{-\tau \underset{\sim}{H} - \mathrm{Tr}\, \tilde{\lambda} \tilde{C}}$$

$$\times \Big[\big(-\tau \, \delta_s \underset{\sim}{H} - \mathrm{Tr}\, \tilde{\lambda} \, \delta_s \tilde{C} \big) \mathrm{Tr}\, \tilde{C} p_a$$

$$+ \mathrm{Tr} (\delta_s \tilde{C}) p_a + \mathrm{Tr}\, \tilde{C} \, \delta_{as} \, \delta p_s \Big]$$

FOR

$$\underset{\sim}{H} = \mathrm{Tr} \Big[\sum_a \tfrac{1}{2} p_a^2 + V(\{q_a\}) \Big] ,$$

$$\delta_s \underset{\sim}{H} = \mathrm{Tr}\, p_s \, \delta p_s$$

19 ALSO, $\delta_s \tilde{C} = [q_s, \delta p_s]$

$\Rightarrow$

$$0 = \Big\langle \Big(-\tau \, \mathrm{Tr}\, p_s \delta p_s - \mathrm{Tr}\, [\tilde{\lambda}, q_s] \delta p_s \Big) \mathrm{Tr}\, \tilde{C} p_r$$

$$+ \mathrm{Tr}\, [p_r, q_s] \delta p_s + \mathrm{Tr}\, \tilde{C} \delta_{rs} \delta p_s \Big\rangle_{AV}$$

δp_s ARBITRARY $\Rightarrow$ OPERATOR RELATION

$$0 = \Big\langle \big(-\tau p_s - [\tilde{\lambda}, q_s] \big) \mathrm{Tr}\, \tilde{C} p_r$$

$$+ [p_r, q_s] + \tilde{C} \delta_{rs} \Big\rangle_{AV}$$

$$= -\Big[\tilde{\lambda}, \langle q_s \, \mathrm{Tr}\, \tilde{C} p_r \rangle_{AV}\Big] = 0$$

FUNCTION OF $\tau, \tilde{\lambda}$

SUBSTITUTING $p_r = p_r^c + p_r'$

CLASSICAL PART

TRACELESS PART

$$\langle \tilde{C} \rangle_{AV} = i_{eff} \hbar$$

REMAINING THREE TERMS GIVE

$$\langle [q_s, p_n] \rangle_{AV} = i_{eff} \hbar \, \delta_{ns} - \tau \langle p_s' T_n \tilde{C} p_n' \rangle_{AV}$$

<u>IF</u> τ TERM CAN BE NEGLECTED

$\Rightarrow$

$$\langle [q_s, p_n] \rangle_{AV} = i_{eff} \hbar \, \delta_{ns}$$

- REDUCTION WITH BORN RULE PROBABILITIES

STRUCTURE OF THE CSL EQUATION IS FIXED BY 2 REQUIREMENTS:

(1) STATE VECTOR NORMALIZATION

$$d\,\langle\psi|\psi\rangle = 0$$

(2) NO SUPERLUMINAL COMMUNICATION

DENSITY MATRIX $\hat{\rho} = |\psi\rangle\langle\psi|$ OBEYS

$$d\hat{\rho} = -\frac{i}{\hbar}[H,\hat{\rho}]\,dt - \frac{1}{2}\eta\,[q,[q,\hat{\rho}]]\,dt + \sqrt{\eta}\,[\hat{\rho},[\hat{\rho},q]]\,dW_t$$

ONLY NONLINEARITY IS STOCHASTIC TERM

→ STOCHASTIC EXPECTATION $\rho = E[\hat{\rho}]$ OBEYS A <u>LINEAR</u> EQN., SO SEQUENCES OF MEASUREMENTS CANNOT SEND / RECEIVE SUPERLUMINAL SIGNALS

IGNORE HAMILTONIAN TERM

(e.g. POINTER EIGENSTATES $|q_1\rangle$, $|q_2\rangle$ ARE ENERGY-DEGENERATE)

(i) WHEN $\hat{\rho} = \Pi_q = |q\rangle\langle q|$

$$[q, \hat{\rho}] = 0 \Rightarrow d\hat{\rho} = 0$$

SO ANY Π_q CAN BE ENDPOINT OF STOCHASTIC EVOLUTION

(ii) IF $[G, q] = 0$, THEN

$$d\,E[\langle G\rangle] = d\,E[\mathrm{Tr}\, G\hat{\rho}]$$

$$= \mathrm{Tr}\, G\, dE[\hat{\rho}]$$

$$= -\tfrac{1}{2}\eta\, \mathrm{Tr}\, G\, [q, [q, \hat{\rho}]]\, dt$$

$$= -\tfrac{1}{2}\eta\, \mathrm{Tr}\, [G, q]\, [q, \hat{\rho}]\, dt = 0$$

(iii) FOR $V = (\Delta q)^2 = \mathrm{Tr}\,\hat{\rho}\, q^2 - (\mathrm{Tr}\,\hat{\rho}\, q)^2$,

$$E\left[\frac{dV}{dt}\right] = -4\eta\, E\left[V^2\right]$$

[PROOF USES (ii) AND $(dW_t)^2 = dt$]

(iv) INTEGRATING

$$E[V(t)] = E[V(0)] - 4\eta \int_0^t ds\, E\left[V(s)^2\right]$$

$$\leq E[V(0)] - 4\eta \int_0^t ds\, E[V(s)]^2$$

BUT $V \geq 0 \Rightarrow E[V(\infty)] = 0$

$\Rightarrow$ $V(s)$ VANISHES AS $s \to \infty$

(APART FROM A SET OF OUTCOMES OF MEASURE ZERO)

$\Rightarrow$ AS $t \to \infty$, SYSTEM IS DRIVEN TO A COORDINATE EIGENSTATE

(v) NOW APPLY (iii) TO

$$G = \Pi_q$$

$$[\Pi_q, q] = 0 \Rightarrow dE[\langle \Pi_q \rangle] = 0$$

AT TIME ZERO,

$$E[\langle \Pi_q \rangle] = \langle \Pi_q \rangle = \mathrm{Tr}\, \hat{\rho}(0)\, \Pi_q$$

$$= |\langle \psi(0) | q \rangle|^2$$

AT TIME INFINITY,

$$E[\langle \Pi_q \rangle] = \sum_f \rho_f \langle f | \Pi_q | f \rangle = \rho_q$$

PROBABILITY FOR OUTCOME q

HENCE $\rho_q = |\langle \psi(0) | q \rangle|^2$ BORN RULE

USUAL MIXED STATE DENSITY MATRIX

IS $\rho = E[\hat{\rho}] = \sum_f \rho_f \, |f\rangle \langle f|$