THEORY RELATING TO THE PVLAS EXPERIMENT

S. Adler IAS

1. PVLAS PARAMETERS

2. QED NONLINEAR EFFECTS

3. ELLIPTICALLY POLARIZED LIGHT

4. THEORY OF PVLAS AND RELEVANT(?) PHYSICAL EFFECTS

1. PVLAS PARAMETERS

BEAM MAKES 44,000 PASSES

THROUGH 1m MAGNETIC FIELD

$B \sim 5$ TESLA

$L = 1\,m \cdot 4.4 \times 10^4 = 4.4 \times 10^4\,m = 4.4 \times 10^6\,cm$

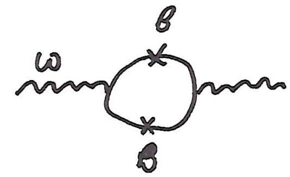
$B \sim 5\,T = 5 \times 10^4$ gauss (5.5T MAX)

$\lambda = 1064\,nm = 1.064 \times 10^{-4}\,cm$

$\Rightarrow \hbar\omega = 1.2\,eV$

2. QED NONLINEAR EFFECTS IN EXTERNAL B FIELD

• VACUUM BIREFRINGENCE



HEISENBERG-EULER — ALL POWERS OF B, SMALL ω

MINGUZZI, CORRECTED IN ADLER — ALL B, GENERAL $\omega < 2m_e$

FOR PVLAS, NEED ONLY B^2 TERM

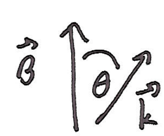
FROM ADLER et al PHOTON SPLITTING PRL

$$n_{\parallel,\perp} = 1 + \frac{\alpha}{\pi}\left(\frac{B}{2B_{cr}}\right)^2 N_{\parallel,\perp}(x)$$

$$B_{cr} \equiv \frac{m^2}{\sqrt{\alpha}} = \frac{m^2}{e} = 4.41 \times 10^9\ \mathrm{T}$$

$$x = \frac{\omega}{2m}\,\frac{B\sin\theta}{B_{cr}}$$

θ = PROPAGATION ANGLE RELATIVE TO $\vec{B}$



$\theta = \frac{\pi}{2}$ FOR PVLAS

$\parallel$ $\quad \vec{h}_{WAVE} \in \vec{B}\,\vec{k}$ PLANE $\qquad \vec{e}_{WAVE} \perp \vec{B}$

$\perp$ $\quad \vec{h}_{WAVE} \perp \vec{B} \qquad \vec{e}_{WAVE} \in \vec{B}\,\vec{k}$ PLANE

BAKALOV ET AL (PVLAS) USES
OPPOSITE CONVENTION

$N_{\perp}(0) - N_{\parallel}(0) = 0.13 \qquad \frac{B}{B_{cr}} = 1.25 \times 10^{-9}$

$\Rightarrow$

$\Delta n = 1.2 \times 10^{-22}$

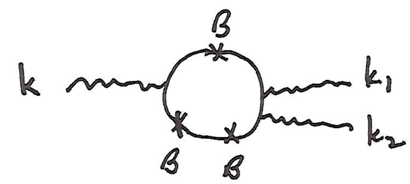
WILL SEE THAT PVLAS CUMULATIVE

EFFECT IS

$\pi \frac{L}{\lambda} \Delta n = 1.6 \times 10^{-11}$ rad $\quad$ VACUUM BIREFRINGENCE

$\ll 1.7 \times 10^{-7}$ rad $\quad$ OBSERVED

- PHOTON SPLITTING



KINEMATICS $\Rightarrow$

$k_1, k_2 \propto k$

ABSORPTION COEFFS PER UNIT PATH LENGTH $\alpha_\parallel \neq \alpha_\perp$

$$\alpha_{\left\{\substack{\parallel \\ \perp}\right\}} = \left\{\begin{matrix} 0.51 \\ 0.24 \end{matrix}\right\} \left(\frac{\omega}{m_e}\right)^5 \left(\frac{B \sin\theta}{B_{cr}}\right)^6 \ \mathrm{cm}^{-1}$$

$L\,(\alpha_\parallel - \alpha_\perp) \sim 3 \cdot 10^{-76}$ PVLAS CUMULATIVE EFFECT

TOO SMALL TO BE OF INTEREST IN PVLAS

(OF ASTROPHYSICAL INTEREST)

REFS: LEADING ORDER ADLER, BAHCALL, CALLAN, ROSENBLUTH PRL **25**, 1061 (1970)

GENERAL ADLER ANN PHYS **67**, 599 (1971)

3. ELLIPTICALLY POLARIZED LIGHT

REF: BORN + WOLF "PRINCIPLES OF OPTICS" MACMILLAN 1964

HECHT "OPTICS" ADDISON-WESLEY 1998

WAVE IN z DIRECTION

$$\tau = \omega t - \vec{k} \cdot \vec{r} = \omega t - kz$$

$$E_x = a_1 \cos(\tau + \delta_1)$$

$$E_y = a_2 \cos(\tau + \delta_2)$$

- $\delta_1 - \delta_2 = 2n\pi$ / $(2n+1)\pi$ $\qquad \dfrac{E_x}{E_y} = \pm \dfrac{a_1}{a_2}$ LINEARLY POLARIZED

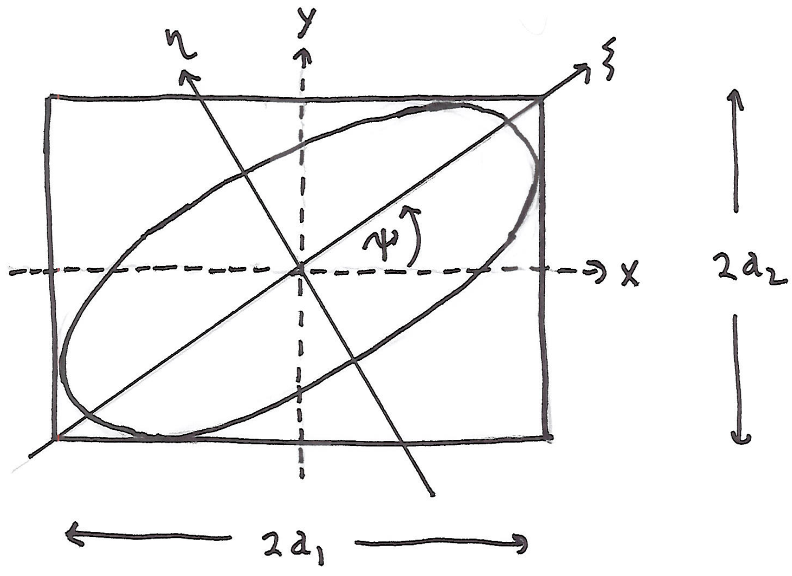
- $\delta_1 - \delta_2 = \pm \frac{\pi}{2} + 2n\pi$ CIRCULARLY POLARIZED

  AND $a_1 = a_2$

- GENERAL CASE $a_1 \neq a_2$

  GENERAL δ_1, δ_2

ELLIPTICALLY POLARIZED: $\vec{E}$ TRAVERSES AN ELLIPSE IN PLANE $\perp \vec{k}$



BORN + WOLF FIG 1.6

HECHT FIG 8.6

ξ, η PRINCIPAL AXES

$\delta = \delta_2 - \delta_1 \quad a_1 \quad a_2$ GIVEN

AFTER SEVERAL PAGES, BORN + WOLF GET:

α = AUXILIARY ANGLE $\quad 0 \le \alpha \le \frac{\pi}{2}$

χ = AUXILIARY ANGLE $\quad -\frac{\pi}{4} \le \chi \le \frac{\pi}{4}$

$$\tan \alpha = \frac{a_2}{a_1} \qquad a^2 + b^2 = a_1^2 + a_2^2$$

$$\tan 2\psi = \tan 2\alpha \cos \delta$$

$$\sin 2\chi = \sin 2\alpha \sin \delta$$

$$\tan \chi = \pm b/a$$

- QUARTER WAVE PLATE

REFERRED TO PRINCIPAL AXES ξ, η

$$E_\xi = a \cos(\tau + \delta_0)$$

$$E_\eta = \pm b \sin(\tau + \delta_0)$$

INSERT A PLATE OF BIREFRINGENT MATERIAL (QUARTER WAVE PLATE) ALIGNED TO GIVE $\frac{\pi}{2}$ PHASE DIFFERENCE ON ξ, η AXES

$$E_\xi \rightarrow a \cos(\tau + \delta_1)$$

$$E_\eta \rightarrow \pm b \sin\left(\tau + \delta_1 + \frac{\pi}{2}\right) = \pm b \cos(\tau + \delta_1)$$

EMERGING WAVE IS LINEARLY POLARIZED

ANGLE OF LINEAR POLARIZATION GIVES $\frac{b}{a}$

ANGLE OF QUARTER WAVE PLATE THAT PRODUCES LINEAR POLARIZATION GIVES ψ

SO CAN MEASURE PARAMETERS OF ELLIPSE

CONVERSELY, LINEAR POL $\rightarrow$ QWP $\rightarrow$ ELLIPTIC POL

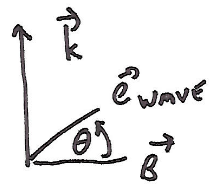
- APPLICATION TO PVLAS

<u>VACUUM BIREFRINGENCE</u> (BAKALOV CONVENTIONS FOR $\parallel, \perp$)

$$n_{\parallel} \neq n_{\perp}$$

$\vec{k}_{\text{WAVE}} \perp \vec{B}$ ($\theta = \pi/2$ IN QED DISCUSSION)

NOW USE BAKALOV ET AL DEFINITION OF θ:



$\vec{e}_{\text{WAVE}}$ MAKES ANGLE θ WITH $\vec{B}$

$$\vec{e} = \vec{B}\cos\theta + \hat{k}\times\vec{B}\sin\theta \qquad k = n\frac{\omega}{c}$$

AFTER DISTANCE L

$$\vec{e} = \vec{B}\cos\theta\cos\left(n_{\parallel}\frac{\omega}{c}L - \omega t\right) + \hat{k}\times\vec{B}\sin\theta\cos\left(n_{\perp}\frac{\omega}{c}L - \omega t\right)$$

$$= \vec{B}\cos\theta\cos\left(\Delta n\frac{\omega}{c}L + \tau\right) + \hat{k}\times\vec{B}\sin\theta\cos(\tau)$$

ELLIPTICALLY POL

$$a_1 = \cos\theta$$
$$a_2 = \sin\theta$$
$$\delta = \Delta n\frac{\omega}{c}L$$

$$\tau = n_{\perp}\frac{\omega}{c}L - \omega t$$

SOLVING BORN-WOLF EQS

$$\tan \alpha = \frac{d_2}{d_1} = \tan\theta \qquad \alpha = \theta$$

$$\sin 2\chi \approx 2\chi = \sin 2\theta \sin\delta \approx \sin 2\theta\, \delta$$

$$\text{ELLIPTICITY} = \Psi_{\text{BAKALOV}} = \frac{b}{a} = \tan\chi$$

$$\approx \chi = \frac{1}{2}\sin 2\theta\, \delta = \frac{1}{2}\Delta n \underbrace{\frac{\omega}{c}}_{\frac{2\pi}{\lambda}} L \sin 2\theta$$

$$= \frac{\pi L}{\lambda}\Delta n \sin 2\theta$$

← BAKALOV ET AL QUANT SEMICLASS OPT 10, 239 (1998)

AT $2\theta = \pi/2$,

IS $\sim 1.6 \times 10^{-11}$ rad

VACUUM BIREFRINGENCE EFFECT FOR PVLAS SETUP

VACUUM DICHROISM

$$a_{\parallel} \neq a_{\perp}$$

$$\vec{e} = \left[(1 - a_{\parallel} L)\, \hat{B} \cos\theta + (1 - a_{\perp} L)\, \hat{k} \times \hat{B} \sin\theta\right] \cos\tau$$

$$a_1 = \cos\theta\,(1 - a_{\parallel} L)$$

$$a_2 = \sin\theta\,(1 - a_{\perp} L)$$

$$\delta = 0 \quad \Rightarrow \quad \psi = \alpha$$

$$\chi = 0 \Rightarrow b = 0$$

$$\tan\alpha = \frac{a_2}{a_1} = \tan\theta\left[1 + (a_{\parallel} - a_{\perp})L\right]$$

WRITE $\alpha = \theta + \Delta$ $\quad \tan(\theta + \Delta) = \tan\theta + \Delta(1 + \tan^2\theta)$

$\Rightarrow \Delta$ (WHICH PVLAS CALL α !)

$$= \frac{\tan\theta\,(a_{\parallel} - a_{\perp})L}{1 + \tan^2\theta} = \frac{1}{2}\sin 2\theta\,(a_{\parallel} - a_{\perp})L$$

ZAVATTINI ET AL

PRL **96**, 110406 (2006)

MEASURED VALUE OF $\frac{1}{2}(d_{\parallel} - d_{\perp}) L$

IN PVLAS IS

$$3.9 \cdot 10^{-12} \frac{\text{rad}}{\text{pass}} \times 4.4 \times 10^{4} \text{ passes}$$

$$= 1.7 \times 10^{-7} \text{ rad}$$

$\gg$ BIREFRINGENCE EFFECT

$$\sim 1.6 \times 10^{-11} \text{ rad}$$

ALTERNITIVE WAY TO GET PVLAS FORMULA:

$$\vec{e} = (1 - d_{\parallel} L)\, \hat{B} \cos\theta + (1 - d_{\perp} L)\, \hat{k} \times \hat{B} \sin\theta$$

$$= \eta \left[\hat{B} \cos(\theta + \Delta) + \hat{k} \times \hat{B} \sin(\theta + \Delta) \right]$$

$$\Rightarrow \quad (1 - d_{\parallel} L) \cos\theta = \eta \cos(\theta + \Delta)$$

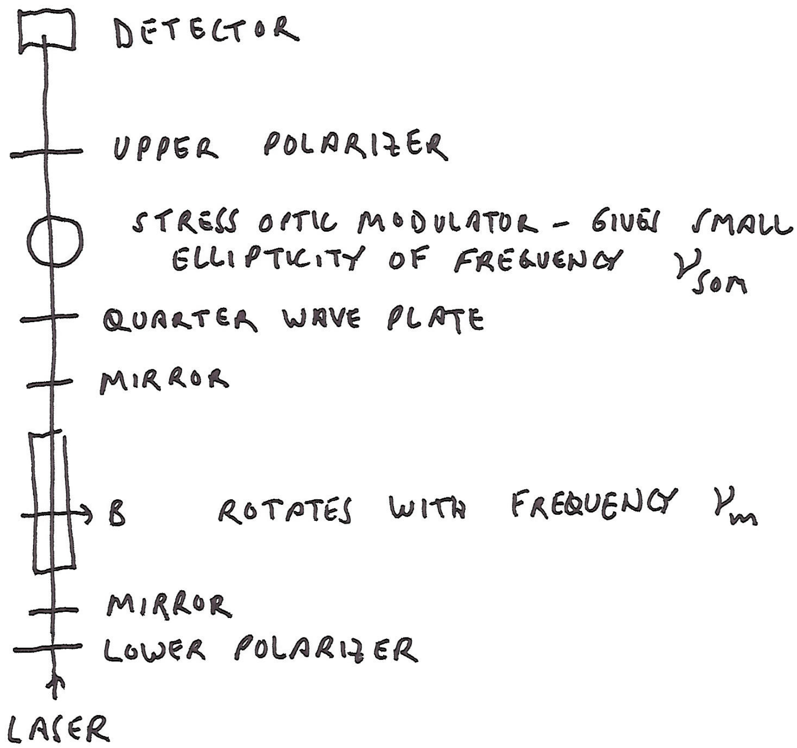
$$(1 - d_{\perp} L) \sin\theta = \eta \sin(\theta + \Delta)$$

$$\Rightarrow \tan(\theta + \Delta) = \tan\theta \left[1 + (d_{\parallel} - d_{\perp}) L \right]$$

$$\Rightarrow \Delta = \frac{1}{2} \sin 2\theta \, (d_{\parallel} - d_{\perp}) L$$

= INDUCED ROTATION IN $\vec{e}$

4. THEORY OF PVLAS AND RELEVANT(?) PHYSICAL EFFECTS



LOWER POLARIZER AXIS $\quad \hat{E}_0 + \hat{k}\times\hat{E}_0\,\xi_L$

UPPER POLARIZER AXIS $\quad -\hat{E}_0\,\xi_U + \hat{k}\times\hat{E}_0$

QUARTER WAVE PLATE:

SLOW AXIS $\quad \hat{E}_0 + \hat{k}\times\hat{E}_0\,\xi_{QW}$

FAST AXIS $\quad -\hat{E}_0\,\xi_{QW} + \hat{k}\times\hat{E}_0$

SMALL ROTATION ANGLES:

$$\Delta_{SOM} = \eta_0 \cos(2\pi \nu_{SOM} t + \theta_{SOM})$$

Δ_F = POSSIBLE FARADAY ROTATION

$$\Delta = \alpha_0 \cos(4\pi \nu_m t + \theta_m) \qquad \alpha_0 = \tfrac{1}{2}(a_\parallel - a_\perp) L$$

4 BECAUSE EFFECT $\propto B^2$

AFTER $\vec{B}$ FIELD

$$\vec{E} = \vec{E}_{WAVE} = \eta \left[\hat{E}_0 + (\Delta + \Delta_F + \xi_L)\, \hat{k} \times \hat{E}_0 \right] \cos\tau$$

$$= \eta \left[(\hat{E}_0 + \hat{k} \times \hat{E}_0\, \xi_{QW}) \cos\tau \right.$$

$$\left. + (\xi_L - \xi_{QW} + \Delta + \Delta_F)(-\hat{E}_0\, \xi_{QW} + \hat{k} \times \hat{E}_0) \cos\tau \right]$$

AFTER QWP

$$\rightarrow \eta \left[(\hat{E}_0 + \hat{k} \times \hat{E}_0\, \xi_{QW}) \cos\tau \right.$$

$$\left. + (\xi_L - \xi_{QW} + \Delta + \Delta_F)(-\hat{E}_0\, \xi_{QW} + \hat{k} \times \hat{E}_0) \sin\tau \right]$$

AFTER SOM

$$\rightarrow \eta \left[(\hat{E}_0 + \hat{k} \times \hat{E}_0\, \xi_{QW}) \cos\tau \right.$$

$$\left. + (\xi_L - \xi_{QW} + \Delta + \Delta_F + \Delta_{SOM})\, \hat{k} \times \hat{E}_0 \sin\tau + \text{higher order} \right]$$

DOT PRODUCT WITH UPPER POLARIZER TO GET

$$A_{detector} = \eta \left[(\xi_{QW} - \xi_U) \cos \tau \right.$$

$$\left. + (\xi_L - \xi_{QW} + \Delta + \Delta_F + \Delta_{SOM}) \sin \tau \right]$$

$$I_{detector} = \langle A^2_{detector} \rangle_{\tau \text{ AVERAGE}}$$

$$= I_0 \left[\sigma^2 + (\xi_{QW} - \xi_U)^2 + (\xi_L - \xi_{QW} + \Delta + \Delta_F + \Delta_{SOM})^2 \right]$$

$\uparrow$ EXTINCTION FACTOR

WHEN $\xi_{QW} - \xi_U = 0$ THIS IS THE PVLAS FORMULA

$$I = I_0 \left[\sigma^2 + (\alpha(t) + \eta(t) + \Gamma(t))^2 \right]$$

$$\alpha(t) = \Delta = \alpha_0 \cos(4\pi \nu_m t + \theta_m)$$

$$\eta(t) = \Delta_{SOM} = \eta_0 \cos(2\pi \nu_{SOM} t + \theta_{SOM})$$

$$\Gamma(t) = \xi_L - \xi_{QW} + \Delta_F$$

PULAS SIGNAL IS THE $\alpha_0 \eta_0$ CROSS TERM

FREQUENCY $\nu_{SOM} \pm 2\nu_m$

POSSIBLE COMPETING EFFECTS:

① EARTH ROTATION

CONTRIBUTES TO $\xi_L - \xi_{QW} \sim 2\pi \frac{\nu_e}{\sqrt{2}} \frac{D}{c}$ (D = traversed distance between polarizers)

45° LATITUDE

DOES NOT CONTRIBUTE TO $\nu_{SOM} \pm 2\nu_m$ SIDEBAND

(DITTO FOR MUCH SMALLER LENS-THIRRING EFFECT)

② FARADAY EFFECT

IF FRINGING FIELD HAS A COMPONENT ALONG AXIS B_{fr} HAVE ROTATION

$$\Delta_F = \mathcal{V} B_{fr} d$$

$\mathcal{V}$ = VERDET CONSTANT

d ~ TRAVERSED THICKNESS

BAKALOV ET AL QUOTE $V_d \sim 9 \times 10^{-6}$ rad / T reflection
(QUANT SEMICLASS OPT.) op. cit. p. 247

PVLAS EFFECT $\sim 9 \times 10^{-12}$ rad/pass @ 5T

WRITE

$$B_{Fr} = B_0 + \underbrace{B_1 \cos(2\pi \nu_m t + \Delta_1)}_{} + B_2 \cos(4\pi \nu_m t + \Delta_2)$$

POSSIBLE EFFECTS OF NON-AXIAL SYMMETRY OF APPARATUS + SURROUNDINGS

$$B_{Fr} V_d \sim 2 \times 10^{-5} \text{ rad} \left[\frac{B_0}{5.5T} + \frac{B_1}{5.5T} \cos\begin{pmatrix} 2\pi\nu_m t \\ +\Delta_1 \end{pmatrix} + \frac{B_2}{5.5T} \cos\begin{pmatrix} 4\pi\nu_m t \\ +\Delta_2 \end{pmatrix} \right]$$

$\frac{B_1}{5.5T} \sim 2 \times 10^{-7}$ GIVES PEAK AT $\nu_{som} \pm \nu_m$ COMPARABLE TO SIGNAL

$\frac{B_2}{5.5T} \sim 2 \times 10^{-7}$ GIVES PEAK AT $\nu_{som} \pm 2\nu_m$ " " " (SIMULATES SIGNAL)

CAN DISTINGUISH THROUGH $\vec{B}$ DEPENDENCE:

Δ SCALES AS B^2

Δ_F SCALES AS B

③ STRAINS IN APPARATUS PRODUCED BY MAGNETIC STRESSES IN MAGNET COILS:

THESE SCALE AS B^2

SUPPOSE $\quad \xi_L - \xi_{QW} = \delta_0 + \delta_1 \cos(2\pi \nu_m t + \Delta_3)$

$$+ \delta_2 \cos(4\pi \nu_m t + \Delta_4)$$

NEED $\quad \delta_2 \ll 4 \times 10^{-12} \frac{\text{rad}}{\text{pass}} \times 4.4 \times 10^{9} \text{ passes}$

$$\sim 2 \times 10^{-7} \text{ rad}$$

AT A 3 cm LEVER ARM

$$2 \times 10^{-7} \text{ rad} \times 3 \text{ cm} = 6 \times 10^{-7} \text{ cm}$$

$= 6$ nanometers

60 atomic diameters

VERY SEVERE REQUIREMENTS ON APPARATUS RIGIDITY!

CAN DISTINGUISH BY SPLITTING LASER BEAM:

PART 1 GOES THROUGH $\vec{B}$ $\quad \Delta + \xi_L - \xi_{QW}$ CONTRIBUTES

PART 2 BYPASSES $\vec{B}$ $\quad \xi_L - \xi_{QW}$ CONTRIBUTES