Comments on Alfven propulsion (Steve Adler, IAS)

Conversion of the Drell, Foley, Ruderman formulas to SI notation:

Their equations (6), (8), (9), (10) for V, P, I, and Z become respectively, with μ_0 the vacuum permeability:

$$V = v_c B_0 M \tag{6}$$

$$P = \frac{ML}{\mu_0} \frac{v_c^2}{v_a} B_0^2 \tag{8}$$

$$I = \frac{L}{\mu_0} \frac{v_c}{v_a} B_0 \tag{9}$$

$$Z = \frac{M}{L} v_a \mu_0 \tag{10}$$

I have corrected an apparent factor of 2 error in their Eqs. (8-10), since I think that the energy density should be $h^2/8\pi$ rather than $h^2/4\pi$, so I replaced the $h^2/4\pi$ in their (8) by $h^2/2\mu_0$. The SI formula of Eq. (10) above reproduces their $Z = .23\Omega$ in the example on page 174 of their article, since with $M \sim L$, $v_a = 2 \times 10^5$m/s, and $\mu_0 = 1.256 \times 10^{-6}$mkg/(s^{2}A^2), one gets $Z = 0.25$m^{2}kg/(s^{3}A^2)=0.25Ω (with Ω=ohm).

For the NEAR satellite solar panels, the ratio $\frac{M}{L}$ will be smaller than 1 if M is the panel width, and much smaller than 1 if M is the panel thickness. Since the altitude is higher than 1600m, and since the Alfven velocity v_a increases with decreasing ionic density (see the discussion following their Eq. (4)), one will have $v_a > 10^7$m/s; scaling from their example, one has

$$Z = \frac{M}{L} \frac{v_a}{(2 \times 10^5 \text{m/s})} \times 0.25\Omega \quad ;$$

since $v_a < c = 3 \times 10^8$m/s, this gives

$$12.5 \frac{M}{L} \Omega < Z < 375 \frac{M}{L} \Omega \tag{I}$$

For a solar cell of voltage V, internal series resistance R_S and insulation resistance R_I between the cell elements and the plasma, the power that can be developed in Alfven propusion when N cells in a square array are wired in parallel will be

$$P = \frac{V^2}{(Z/N^{1/2} + (R_S + R_I)/N)} \tag{II}$$

Here Z is the impedance from Eq. (10) evaluated with the dimension L of the individual cell; since a square array will have side length $LN^{1/2}$, the Z value for the array will be $Z/N^{1/2}$. Similarly, for N cells wired in parallel, the resistances R_S and R_I of an individual cell are divided by N in an array. Note that Eq. (II) has the correct invariance property if each cell is imagined to be cut into four sub-cells wired in parallel: the cell side is decreased by a factor of 2, and so Z increases by a factor of 2, but since N is increased by a factor of 4, this increase in Z is compensated by the factor $1/N^{1/2}$. Similarly, the resistances of each subcell are increased by a factor of 4 (one expects the series resistance and the insulation resistance, per unit area, to be constant), but this is compensated by the factor $1/N$.

To make a rough estimate, from advertisements on the internet I find typical polycrystalline solar cell parameters $V \sim 7$volt, $R_S \sim 150\Omega$, $L \sim 5$cm, and from pictures it looks like $M/L \sim 1/12.5$ as a rough order of magnitude. Then Eq. (I) gives $1\Omega < Z < 30\Omega$. Taking the lowest Z value in this range, Eq. (II) gives an upper bound on P. Assuming that $N \sim 10^4$, this becomes

$$P < \frac{49}{(2.5 \times 10^{-2} + 10^{-4} R_I)} \text{watt} \quad , \tag{III}$$

with the insulation resistance R_I in ohms. So the possible power going into Alfven propulsion depends crucially on the quality of the solar cell insulation resistance R_I. For $R_I \sim 10^5$ ohms, one could get 5 watts of propulsion power, of the order needed for the flyby anomalies. But good glass insulation should be much better; for a R_I of 10 megohms, one gets a bound on the Alfven propulsion power of 0.05 watt, too small to be of interest.