DARK MATTER MODELING FOR FLYBY

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SOME FLYBYS GAIN VELOCITY (ANOMALOUS DRAG)

SOME FLYBYS LOSE VELOCITY (NORMAL DRAG)

MODEL ASSUMES <u>TWO</u> SPECIES OF DARK MATTER ORBITING EARTH

- INELASTIC, EXOTHERMIC SCATTER

  $$D(m_2) + N(m_1) \rightarrow D'(m_2') + N(m_1) \qquad m_2 > m_2'$$

  CAN GIVE VELOCITY GAIN IF $N(m_1)$ SCATTERS FORWARD

- ELASTIC SCATTER

  $$D''(m_2'') + N(m_1) \rightarrow D''(m_2'') + N(m_1)$$

  GIVES NORMAL DRAG

VELOCITY CHANGE FORMULAS

$\vec{u}_1$ = VELOCITY OF INITIAL NUCLEON (IN SPACECRAFT)

$\vec{u}_2$ = VELOCITY OF INCIDENT DARK MATTER

$\langle \delta \vec{v} \rangle$ = CROSS-SECTION AVERAGED NUCLEON VELOCITY CHANGE
(ASSUME NO AZIMUTHAL ANGLE DEPENDENCE)

INELASTIC

$$\langle \delta \vec{v} \rangle = \frac{\vec{u}_1 - \vec{u}_2}{|\vec{u}_1 - \vec{u}_2|} A_I \qquad A_I = \left[\frac{2 \, (m_2 - m_2') \, m_2'}{m_1 \, (m_1 + m_2')} \right]^{1/2} c \; \langle \cos\theta \rangle_I$$

ELASTIC

$$\langle \delta \vec{v} \rangle = -(\vec{u}_1 - \vec{u}_2) A_E \qquad A_E = \left(\frac{m_2''}{m_1 + m_2''} \right) \left[1 - \langle \cos\theta \rangle_E \right]$$

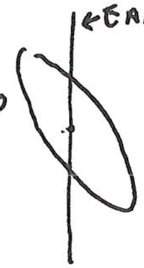
WILL LUMP CROSS SECTION x DENSITY INTO THE
PARAMETERS A_I , A_E

DARK MATTER ORBITAL GEOMETRY

ASSUME AXIAL SYMMETRY AROUND EARTH ROTATION AXIS

SIMPLEST MODEL - DISK IN EQUATORIAL PLANE - DOES NOT WORK - "NEAR" GETS SMALLEST VELOCITY CHANGE

NEXT SIMPLEST MODEL -



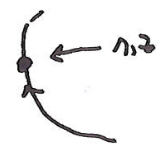
E, L_z CONSERVED, BUT ORBIT WILL PRECESS BECAUSE OF EARTH QUADRUPOLE - WILL FILL SHELL OBTAINED BY AVERAGING ORBIT OVER AZIMUTHAL ANGLE AROUND EARTH AXIS

GET THIS

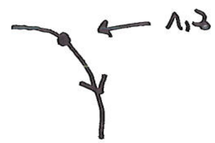
DENSITY $\propto \dfrac{1}{\sqrt{(r \sin\gamma)^2 - z^2}}$

DARK MATTER VELOCITY $\vec{u}_2$

FOR GIVEN r, ψ, z, THERE ARE TWO $\vec{u}_2$ VALUES, ONE FOR AN UP-GOING SEGMENT



AND ONE FOR A DOWN-GOING SEGMENT



$$\vec{u}_2 = \sqrt{\frac{GM_\oplus}{r}} \left(c_\pm \, \hat{n}_\parallel + d_\pm \, \hat{n}_\perp \right)$$

$\hat{n}_\parallel$ PARALLEL TO EQUATORIAL PLANE

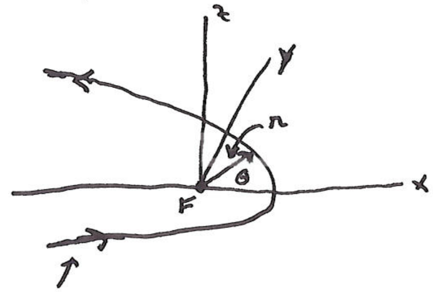
$\hat{n}_\perp$ PERPENDICULAR TO $\hat{r}$ AND $\hat{n}_\parallel$

$$c_\pm = \frac{r \cos\psi}{\sqrt{r^2 - z^2}} \qquad d_\pm = \pm \frac{\sqrt{r^2 \sin^2\psi - z^2}}{\sqrt{r^2 - z^2}}$$

$$c_\pm^2 + d_\pm^2 = 1$$

FLYBY ORBITAL GEOMETRY

WORK IN FLYBY ORBIT PLANE

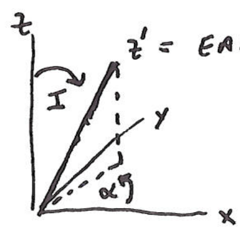


FLYBY HYPERBOLIC ORBIT

$$r = \frac{p}{1 + e\cos\theta}$$

$$\vec{r} = (x, y, 0) \qquad x = r\cos\theta \qquad y = r\sin\theta$$

EARTH AXIS AS SEEN FROM FLYBY ORBIT PLANE



z' = EARTH ROTATION AXIS (CALLED THIS z BEFORE)

POLAR ANGLES I, α

- FIRST, EXPRESS DARK MATTER ORBIT ON EARTH-FIXED AXES
- THEN, REWRITE ON FLYBY PLANE AXES

FLYBY PARAMETERS ON FLYBY PLANE AXES
(AND INTRINSIC PARAMETERS p, e)

	GLL-I	GLL-II	NEAR	CASSINI	ROSETTA	MESSENGER
I (°)	142.9	138.7	108.0	25.4	144.9	133.1
α (°)	−45.1	−147.4	−55.1	−158.4	−53.1	0.0
p (km)	25,473	22,155	19,448	51,689	19,262	20,563
e	2.473	2.319	1.814	5.851	1.312	1.360

$$e = 1 + \frac{2V_\infty^2}{V_f^2 - V_\infty^2} \qquad p = \frac{4GM_\oplus}{V_\infty^2}\left[\left(\frac{V_\infty^2}{V_f^2 - V_\infty^2}\right)^2 + \frac{V_\infty^2}{V_f^2 - V_\infty^2}\right]$$

α REQUIRES ATTENTION TO $\pm$ IN ARC COSINE; ABOVE VALUES REPRODUCE THE δ_i, δ_0, $\alpha_i - \alpha_0$ VALUES IN ANDERSON ET AL PRL

WHERE THINGS ARE NOW

PROGRAM INTEGRATES OVER ORBIT OF FLYBY TO CALCULATE $\vec{V}_\infty \cdot \delta \vec{V}_\infty$

HAVE TAKEN A QUICK LOOK AT AN 8 PARAMATER MODEL (NO SYSTEMATIC OPTIMIZATION OVER PARAMETERS)

INELASTIC SCATTERERS	A_I	χ_I	$a_I \le r \le b_I$
ELASTIC SCATTERERS	A_E	χ_E	$a_E \le r \le b_E$

EXAMPLE:

$A_I = 0.99$	$\chi_I = 1.69$ rad	$\frac{1}{2}(a_I + b_I) = 39,400$ km
		$b_I - a_I = 10,000$ km
$A_E = 0.076$	$\chi_E = 1.29$ rad	$\frac{1}{2}(a_E + b_E) = 30,000$ km
		$b_E - a_E = 10,000$ km

ASSUME JUST $\frac{1}{r}$ FROM JACOBIAN AS DENSITY PROFILE $a \le r \le b$

NO AVERAGING OVER DIFFERENT χ VALUES

NO DEPARTURES FROM CIRCULAR ORBIT VELOCITY

GET FROM THESE PARAMETERS

	GLL-I	GLL-II	NEAR	CASSINI	ROSETTA	MESSENGER
δV_∞ exp (mm/s)	3.92	−4.6	13.5	−2	1.8	.02
δV_∞ model	3.0	−18	12	−33	0.7	3.4

CAN GET QUALITATIVE SIGNS RIGHT

MAGNITUDES NOT GOOD!

$$\text{CHI-SQUARE} = \sum_{k=1}^{6} \left[\frac{\delta V_\infty \text{exp}(k) - \delta V_\infty \text{model}(k)}{\sigma(k)}\right]^2 = 1.3 \times 10^5 \quad \text{LOUSY!}$$

- DON'T KNOW WHETHER A SYSTEMATIC SEARCH OF PARAMETER SPACE WILL GIVE A BETTER FIT
- CAN MAKE THE MODEL MORE ELABORATE

KEY THINGS NEEDED FROM FLYBY REANALYSIS

- REAL EFFECT (NEW PHYSICS) OR ARTIFACT ?
- δV_∞ VALUES
- σ VALUES

$$\sigma = \begin{cases} .01 & \text{MESSENGER} \\ .03 & \text{ROSETTA} \end{cases}$$

GIVES THEM A VERY BIG WEIGHT IN CHI - SQUARE FOR FIT

ALSO TO LOOK FOR

- GLASST, PAMELA RESULTS ON DARK MATTER SEARCHES
- LHC SEARCH FOR SUPERSYMMETRY

HEAVY, SELF-ANNIHILATING SUSY DARK MATTER DOESN'T OBEY FLYBY MODEL CONSTRAINTS