COLLAPSE MODELS WITH NON-WHITE NOISE

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- MEASUREMENT PROBLEM
- OBJECTIVE REDUCTION MODELS - WHITE NOISE
- PHENOMENOLOGY
- RELATIVISTIC REDUCTION MODELS ?
- OBJECTIVE REDUCTION MODELS - NON-WHITE NOISE
- QUANTUM THEORY AS THERMODYNAMICS OF PRE-QUANTUM "TRACE DYNAMICS"
- OPEN QUESTIONS

MEASUREMENT PROBLEM

SCHRODINGER DYNAMICS IS LINEAR

$$i\hbar \frac{\partial |\psi_1\rangle}{\partial t} = H |\psi_1\rangle$$

$$\rightarrow \quad i\hbar \frac{\partial}{\partial t} |\psi_1 + \psi_2\rangle = H |\psi_1 + \psi_2\rangle$$

$$i\hbar \frac{\partial |\psi_2\rangle}{\partial t} = H |\psi_2\rangle$$

EVEN STRONGER STATEMENT HOLDS:

DYNAMICS IS UNITARY

$$|\psi(t)\rangle = U(t) \, |\psi(0)\rangle \qquad U^\dagger U = 1$$

DYNAMICS IS DETERMINISTIC:

GIVEN $|\psi(0)\rangle$, IT PREDICTS $|\psi(t)\rangle$

MEASUREMENT INTRODUCES A STOCHASTIC ASSUMPTION:

BORN RULE

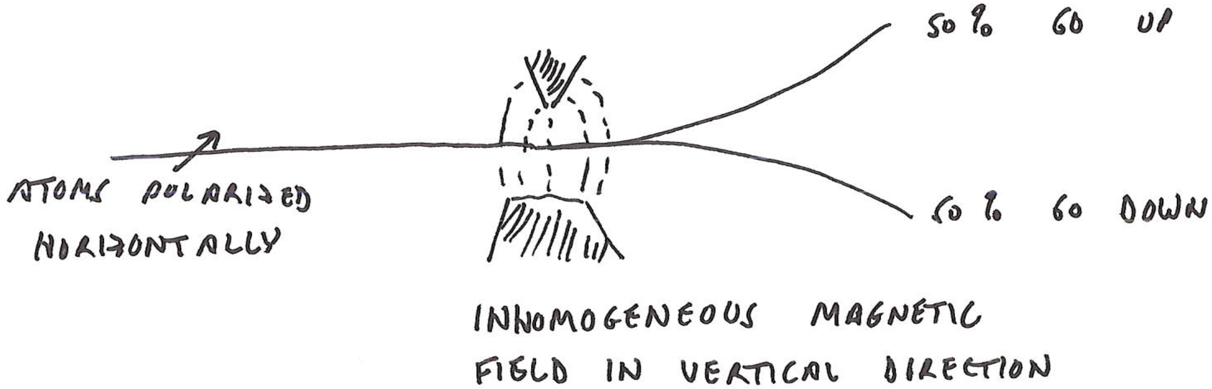
$$|\psi\rangle = \sum_\ell c_\ell \, |\psi_\ell\rangle$$

ℓ^{th} EIGENSTATE OF OBSERVABLE $\mathcal{O}$

P_ℓ = PROBABILITY OF MEASURING ℓ^{th} OUTCOME IS

$$P_\ell = |\langle\psi|\psi_\ell\rangle|^2 = |c_\ell|^2$$

STERN-GERLACH EXPERIMENT



INDIVIDUAL RUNS HAVE OUTCOMES, BUT CANNOT PREDICT WHETHER IT WILL BE ↑ OR ↓

MEASUREMENT PROBLEM

IF WE TRY TO APPLY QUANTUM MECHANICS VIA SCHRÖDINGER DYNAMICS TO INDIVIDUAL OUTCOMES OF SYSTEM AND APPARATUS - GET A CONTRADICTION

$$U|\rightarrow\rangle = |\uparrow\rangle \quad \text{RUN 1}$$

$$U|\rightarrow\rangle = |\downarrow\rangle \quad \text{RUN 2}$$

$$1 = \langle\rightarrow|\rightarrow\rangle = \langle\rightarrow|U^{\dagger}U|\rightarrow\rangle = \langle\downarrow|\uparrow\rangle = 0$$

EVEN THE WEAKER ASSUMPTION OF LINEAR EVOLUTION GIVES A CONTRADICTION

NON-SOLUTIONS:

DECOHERENCE - IMPLIES RAPID DIAGONALIZATION OF DENSITY MATRIX BUT DOES NOT EXPLAIN INDIVIDUAL OUTCOMES

STATISTICAL INTERPRETATION - NOT ENOUGH WHEN WE CAN TRAP INDIVIDUAL ATOMS FOR HOURS AND FOLLOW THEIR TRANSITIONS

MANY WORLDS - NO SATISFACTORY DERIVATION OF BORN RULE

OBJECTIVE REDUCTION MODELS - WHITE NOISE

GRWP

GHIRARDI, RIMINI, WEBER (GRW) PROGRAM - RANDOM PROCESSES
"HITS" ALTER SCHRODINGER EQUATION

PEARLE " GAMBLER'S RUIN" IDEA -
A RANDOM PROCESS INCORPORATING THE IDEA OF
A "FAIR GAME" CAN GIVE THE BORN RULE

MERGER OF THE TWO PROGRAMS - GHIRARIDI, PEARLE, RIMINI

CSL MODEL - "CONTINUOUS SPONTANEOUS LOCALIZATION"

SUGGESTS A MODIFIED SCHRÖDINGER EQUATION AS AN "EFFECTIVE THEORY" SOLUTION TO THE MEASUREMENT PROBLEM - IF THERE IS A PRE-QUANTUM DYNAMICS THAT EXPLAINS MEASUREMENT, IT SHOULD IMPLY A PHENOMENOLOGICAL EFFECTIVE THEORY EXPRESSED IN TERMS OF OUR LABORATORY MEASUREMENT DEGREES OF FREEDOM AND VARIABLES: WAVE FUNCTIONS
OPERATORS - q, p
TIME

STOCHASTIC MODIFIED SCHRÖDINGER EQUATION - SIMPLE CASE FOR POINTER WITH CENTER OF MASS VARIABLE q (LEADING SMALL DISPLACEMENT APPROXIMATION TO THE GRW AND CSL MODELS)

$$d|\psi\rangle = -\frac{i}{\hbar} H|\psi\rangle\, dt - \frac{\eta}{2}(q - \langle q\rangle)^2 |\psi\rangle\, dt + \sqrt{\eta}\,(q - \langle q\rangle)|\psi\rangle\, dW_t$$

HERE: dt = TIME STEP

dW_t = FLUCTUATION $\sim (dt)^{1/2}$

$$\left.\begin{aligned} (dW_t)^2 &= dt \\ dW_t\, dt &= (dt)^2 = 0 \end{aligned}\right\} \text{ITÔ CALCULUS RULES}$$

$\langle q\rangle = \langle\psi|q|\psi\rangle$ EXPECTATION VALUE OF q IN STATE $|\psi\rangle$

BECAUSE $|\psi\rangle$ APPEARS IN $\langle q\rangle$, THIS IS A <u>NONLINEAR</u> STOCHASTIC DIFFERENTIAL EQUATION

MULTIPLYING THROUGH BY $i\hbar$,

$$i\hbar\, d|\psi\rangle = H|\psi\rangle\, dt + \cdots + \underbrace{i\hbar\sqrt{\eta}\,(q - \langle q\rangle)\,|\psi\rangle\, dW_t}_{\text{ANTI-SELF-ADJOINT}}$$

THE DETAILED STRUCTURE OF THIS EQUATION IS FIXED BY 2 REQUIREMENTS:

(1) STATE VECTOR NORMALIZATION

$$d\langle\psi|\psi\rangle = 0$$

(2) NO SUPERLUMINAL COMMUNICATION

DENSITY MATRIX $\hat{\rho} = |\psi\rangle\langle\psi|$ OBEYS

$$d\hat{\rho} = \frac{-i}{\hbar}[H, \hat{\rho}]\, dt - \frac{1}{2}\eta\,[q, [q, \hat{\rho}]]\, dt + \sqrt{\eta}\,[\hat{\rho}, [\hat{\rho}, q]]\, dW_t$$

ONLY NONLINEARITY IS STOCHASTIC TERM

$\Rightarrow$ STOCHASTIC EXPECTATION $\rho = E[\hat{\rho}]$ OBEYS A LINEAR EQUATION, SO SEQUENCES OF MEASUREMENTS CANNOT SEND / RECEIVE SUPERLUMINAL SIGNALS

BORN RULE

CAN NOW PROVE: WHEN THE $-\frac{i}{\hbar}[H,\rho]\,dt$ $\left(-\frac{i}{\hbar}H|\psi\rangle\,dt\right)$ TERM IS UNIMPORTANT (WHEN REDUCTION IS VERY FAST, OR WHEN THE DIFFERENT POINTER STATES ARE DEGENERATE IN ENERGY) THIS EQUATION IMPLIES THE BORN RULE!

THAT IS, DIFFERENT EVOLUTIONS OF THE STOCHASTIC PROCESS GIVE OUTCOMES $|q_\ell\rangle$ WITH PROBABILITY $|\langle\psi(0)|q_\ell\rangle|^2$

SO THE STOCHASTIC SCHRÖDINGER EQUATION GIVES A CONSISTENT EFFECTIVE THEORY FOR STATE VECTOR REDUCTION - THIS IS A REMARKABLE RESULT: SUCH AN EFFECTIVE THEORY DID NOT HAVE TO EXIST!

PHENOMENOLOGY

GIVEN A MODEL FOR MODIFIED SCHRÖDINGER EQUATION, ONE CAN TRY TO BOUND ITS PARAMETERS. IN FULL CSL MODEL, PARAMETERS AS FOLLOWS:

$$d|\psi(t)\rangle = \left[-\frac{i}{\hbar} H\,dt + \int d^3x \left(M(x) - \langle M(x)\rangle \right) dB(x) - \frac{\gamma}{2} \int d^3x \left(M(x) - \langle M(x)\rangle \right)^2 dt \right] |\psi(t)\rangle$$

- $dB(x)$ BROWNIAN MOTION OBEYING

$$dt\, dB(x) = 0 \qquad dB(x)\, dB(y) = \gamma\, \delta^3(x-y)\, dt$$

- $\langle M \rangle$ IS EXPECTATION OF M IN STATE $|\psi(t)\rangle$

- $M(x)$ IS SMEARED MASS DENSITY OPERATOR

$$M(x) = m_N^{-1} \int d^3y\; g(x-y) \sum_s \underset{\substack{\uparrow \\ \text{MASS}}}{m_s}\; \underset{\substack{\uparrow \\ \text{NUMBER DENSITY OPERATOR}}}{N_s(y)}$$

$$g(x) = \left(\frac{\alpha}{2\pi}\right)^{3/2} e^{-(\alpha/2)x^2} \qquad \int d^3x\; g(x) = 1$$

TWO PARAMETERS

CONVENTIONAL VALUES
BASSI & GHIRARDI, PHYS REP 379, 257 (2003)

NOISE STRENGTH γ — $\gamma \simeq 10^{-30}\ cm^3\ s^{-1}$

CORRELATION LENGTH r_c (OR a) — $r_c \simeq 10^{-5}\ cm$

OFTEN CONVENIENT TO REPLACE γ BY RATE PARAMETER λ

$$\lambda = \gamma \left(\frac{\alpha}{4\pi}\right)^{3/2} = \frac{\gamma}{8\pi^{3/2} r_c^3}$$

CONVENTIONAL VALUE
$\lambda_c \simeq 2.2 \times 10^{-17}\ s^{-1}$

LOWER BOUND - REDUCTION MUST OCCUR IN MEASUREMENT

$$\Gamma_R \simeq \lambda n^2 N$$

n = NUMBER OF NUCLEONS WITHIN RADIUS r_c THAT MOVE BY MORE THAN r_c

N = NUMBER OF SUCH GROUPS OF n NUCLEONS

EXAMPLE: POINTER WITH 10^{15} ATOMS

$$(10^{-5}\ cm)^3 \cdot 10^{24}/cm^3 = 10^9 = n$$

$$10^{15}/10^9 = 10^6 = N$$

$$\Gamma_R \sim 2.2 \cdot 10^{-17}\ s^{-1} \times 10^{18} \times 10^6 = 2.2 \cdot 10^7\ s^{-1}$$

$\Rightarrow$ REDUCTION IN 10^{-7} s

UPPER BOUNDS

- SECULAR ENERGY GAIN OF BODY OF MASS M IGM HEATING

$$\frac{dE}{dt} = \frac{3}{4}\lambda \frac{\hbar^2}{r_c^2} \frac{M}{m_N^2}$$

- FULLERENE DIFFRACTION
- DECAY OF SUPERCURRENTS
- EXCITATION OF BOUND ATOMIC AND NUCLEAR SYSTEMS
- RADIATION BY FREE OR ATOMIC ELECTRONS

ENHANCED LOWER BOUND ?   ADLER J PHYS A **40**, 2935 (2007)
(E) A **40**, 13501 (2007)

LATENT IMAGE FORMATION IN
PHOTOGRAPHIC EMULSIONS
ETCHED TRACK DETECTORS

FEW ATOMS MOVE ON CONVENTIONAL MODELS OF THESE PROCESSES

IF LATENT IMAGE FORMATION (AND NOT SUBSEQUENT DEVELOPMENT, ETCHING, ETC) CONSTITUTES MEASUREMENT, NEED $\lambda \sim 10^{9\pm2}$ TIMES "CONVENTIONAL" VALUE λ_c. IGM HEATING REQUIRES $\lambda \lesssim 10^{8\pm1} \lambda_c$

HOWEVER, 11 keV RADIATION BOUND FROM GERMANIUM (FU PHYS REV A 56, 1806)

$\Rightarrow \lambda < 3 \times 10^{6} \lambda_c$ FOR WHITE NOISE

STILL TRUE FOR BOUND ATOMIC SYSTEMS (ADLER + RAMAZANOĞLU J PHYS A 40, 13395 (2007))

SO ENHANCED λ CONFLICTS WITH THIS BOUND FOR WHITE NOISE

NO PROBLEM IF NOISE IS NON-WHITE, WITH FREQUENCY CUTOFF BELOW $10^{18}\ s^{-1}$ (FREQUENCY OF 11 keV γ)

BASSI + GHIRARDI REVIEW SUGGESTS POSSIBLE CUTOFF AT $c/r_c \sim 10^{15}\ s^{-1}$

EVEN CUTOFF AT $10^{11}\ s^{-1}$ WOULD ALLOW NANOSECOND MEASUREMENTS

$\Updownarrow$

ENERGY CUTOFF 10^{-4} eV

$\Updownarrow$

NOISE TEMPERATURE 1° K (COSMOLOGICAL?)

SO POSSIBLE ENHANCED NOISE PARAMETER, SUGGESTED BY LATENT IMAGE FORMATION, MOTIVATES STUDY OF NON-WHITE NOISE

PROSPECTS FOR DETECTING CSL EFFECTS

LARGE MOLECULE DIFFRACTION

GRATING 10^{-5} cm $\sim r_C$

$\Gamma_R \sim 10^2\ s^{-1}$

$\Gamma_R = \lambda n^2$ $\quad$ $\lambda = \lambda_c$ NEED $n \sim 2 \times 10^9$

$\lambda = 2 \times 10^7 \lambda_c$ NEED $n \sim 5 \times 10^5$ = 500 × MASS OF FULLERENE

LATENT IMAGE LOWER BOUND

MARSHALL ET AL MIRROR DEFLECTION EXPERIMENT

(PHYS REV LETT **91**, 130401 (2003))

CUBICAL MIRROR SIDE S, DENSITY D

CSL REDUCES TO ONE DIMENSIONAL MODEL $\eta = 8\pi r_C^2 \lambda S^2 D^2$

ANALYZED IN ADLER, BASSI + IPPOLITI PHYS REV LETT **94**, 030401 (2005)

$\lambda = \lambda_c$ MIRROR EXPERIMENT 6 ORDERS OF MAGNITUDE AWAY FROM A TEST OF CSL

$\lambda = 2 \times 10^7 \lambda_c$ $\quad$ $r_C = 10^{-5}$ cm $\quad$ EFFECT SWAMPED BY THERMAL DECOHERENCE BACKGROUND

$\lambda = 1.4 \times 10^9 \lambda_c$ $\quad$ $r_C = 10^{-4}$ cm $\quad$ FACTOR 600 EFFECT ABOVE BACKGROUND IS POSSIBLE (IF FULL SENSITIVITY ATTAINED IN MIRROR EXPERIMENT)

RELATIVISTIC REDUCTION MODELS ?

EVERYTHING SO FAR HAS BEEN NON- RELATIVISTIC

IS THERE A RELATIVISTIC EXTENSION ?

MY BELIEF IS THAT THE ANSWER IS: NO

REASONS

(1) ATTEMPTS AT A COVARIANT EXTENSION OF CSL FAIL

EXAMPLE (ONE OF SEVERAL) ADLER + BRUN J PHYS A 34, 1 (2001)

GO TO INTERACTION PICTURE AND USE THE TOMONAGA - SCHWINGER FORMALISM TO GENERALIZE SCHRÖDINGER EQUATION TO A LOCAL EQUATION, WITH WAVE FUNCTION $|\psi(\sigma)\rangle$ DEFINED ON GENERAL SPACELIKE SURFACE σ. LET $dt(\vec{x})$ INCREMENT THE SURFACE FORWARD IN VICINITY OF POINT $\vec{x}$

$$d_{\vec{x}}|\psi\rangle = -i\, H_{int}(\vec{x})\,|\psi\rangle\, dt(\vec{x})$$

FOR ENTIRE SURFACE

$$d|\psi\rangle = \int d^3x\; d_{\vec{x}}|\psi\rangle$$

CAN NOW INTRODUCE A LOCAL NOISE $dB(\vec{x})$ OBEYING

$$dB(\vec{x})\, dB(\vec{y}) = \delta^3(\vec{x}-\vec{y})\, dt(\vec{x})$$

[A+B PAPER USES COMPLEX NOISE]

AND WRITE A FORMALLY COVARIANT CSL EQUATION

PROBLEM (FIRST NOTED BY PEARLE) IN GENERIC FIELD THEORY MODELS, THE ENERGY NON-CONSERVATION IS NOW PROPORTIONAL TO $\delta^3(0) = \infty$!

(2) NON-WHITE NOISE IMPLIES A PREFERRED FRAME - CAN'T WRITE A LORENTZ-INVARIANT CUTOFF TO NOISE SPECTRUM

(3) THE IDEAS FOR A PRE-QUANTUM THEORY, WITH CSL AS A PHENOMENOLOGY, DEVELOPED IN MY 2004 BOOK "QUANTUM THEORY AS AN EMERGENT PHENOMENON" (CAMBRIDGE U PRESS) REQUIRE A PREFERRED FRAME - MORE ON THIS BELOW

THERE IS A NATURAL PREFERRED LORENTZ FRAME SELECTED BY THE COSMIC MICROWAVE BACKGROUND RADIATION (CMB) - THE CMB REST FRAME IN WHICH THE DIPOLE ANISOTROPY VANISHES. OUR LOCAL GROUP MOVES WITH VELOCITY $627 \pm 22\ \frac{km}{s} \ll c = 3 \times 10^5\ \frac{km}{s}$ RELATIVE TO THIS FRAME

(4) <u>EMPIRICALLY</u>, THERE IS NO EVIDENCE FOR RELATIVISTIC MEASUREMENTS - ALL MEASUREMENTS INVOLVE APPARATUS WITH $V_{\text{CMB REST FRAME}} \ll c$

(5) <u>CONWAY - KOCHEN</u> "STRONG FREE WILL THEOREM" (ARXIV: 0807.3286) ARGUES THAT THERE CAN BE NO RELATIVISTIC GRW - A PREFERRED FRAME IS NEEDED

THERE HAS BEEN A LIVELY ONGOING CONTROVERSY ABOUT THIS, BUT I BELIEVE CONWAY & KOCHEN HAVE PREVAILED

[EARLIER WORK USING "TWINNED" KOCHEN - SPECKER :

STAIRS PHIL SCI 50, 578 (1983)

ELBY FOUND PHYS 20, 1389 (1990)

BROWN + SVETLICHNY FOUND PHYS 20, 1379 (1990)]

OBJECTIVE REDUCTION MODELS - NON-WHITE NOISE

I WILL NOW SURVEY SOME RESULTS OF A STUDY OF NON-WHITE NOISE MODELS THAT I MADE WITH A. BASSI

ADLER & BASSI J. PHYS. A **40**, 15083 (2007)
" " " II " " " **41**, 395308 (2008)

FIRST, A FURTHER PROPERTY OF THE WHITE NOISE CASE

WITH CHANGES IN NOTATION ($x \to i$, $B \to W$) EQUATION WE HAD BEFORE IS:

$$d|\psi_t\rangle = \left[-\frac{i}{\hbar} H\,dt + \sqrt{\gamma} \sum_{i=1}^{N} (A_i - \langle A_i \rangle_t)\,dW_{i,t} - \frac{\gamma}{2} \sum_{i=1}^{N} (A_i - \langle A_i \rangle_t)^2\,dt \right] |\psi_t\rangle$$

$$\langle A_i \rangle_t \equiv \langle \psi_t | A_i | \psi_t \rangle$$

NOW GENERALIZE THIS BY INCLUDING A COMPLEX FACTOR $\xi = \xi_R + i\,\xi_I$ IN THE NOISE. THE CORRESPONDING NORM-PRESERVING EQUATION IS

$$d|\psi_t\rangle = \Big[-\frac{i}{\hbar} H\, dt + \sqrt{\gamma} \sum_{i=1}^{N} \left[\xi A_i - \xi_R \langle A_i \rangle_t \right] dW_{i,t}$$

$$- \frac{\gamma}{2} \sum_{i=1}^{N} \left(|\xi|^2 A_i^2 - 2 \xi \xi_R A_i \langle A_i \rangle_t + \xi_R^2 \langle A_i \rangle_t^2 \right) dt \Big] |\psi_t\rangle$$

USING THE ITÔ CALCULUS, THE DENSITY MATRIX $\rho(t) = E[\, |\psi_t\rangle \langle \psi_t| \,]$ OBEYS

$$\frac{d}{dt} \rho(t) = -\frac{i}{\hbar} [H, \rho(t)] + \frac{\gamma}{2} |\xi|^2 \sum_{i=1}^{N} \left(2 A_i \rho(t) A_i - \{ A_i^2, \rho(t) \} \right)$$

SAME FOR $\xi = 1$ REDUCTION

$\xi = i$ UNITARY EVOLUTION

SINCE STATISTICS OF OUTCOMES OF EXPERIMENTS MEASURING O ARE GIVEN BY $E[\langle \psi_t | O | \psi_t \rangle] \equiv Tr(\rho(t)\, O)$

WE CAN USE ANY STOCHASTIC EQUATION WITH $|\xi| = 1$ TO GET OUTCOME AVERAGES FOR REDUCTION EQUATION WITH $\xi = 1$

IMAGINARY NOISE TRICK CAN CALCULATE AVERAGES USING THE LINEAR, UNITARY EQUATION WITH $\xi = i$

NON-WHITE NOISE

INTRODUCE GAUSSIAN RANDOM PROCESS $W_i(t)$ OBEYING

$$E_Q[W_i(t)] = 0 \qquad E_Q[W_i(t_1)W_j(t_2)] = D_{ij}(t_1,t_2)$$

START FROM LINEAR EQUATION

ALL A's COMMUTE $[A_i, A_j] = 0$

$$\frac{d|\phi(t)\rangle}{dt} = \left[-\frac{i}{\hbar}H + \sqrt{\gamma}\,\xi \sum_{i=1}^{N} A_i W_i(t) + \mathcal{O}\right]|\phi(t)\rangle$$

$\mathcal{O}$ A LINEAR OPERATOR TO BE DETERMINED BY

$$E_Q[\langle\phi(t)|\phi(t)\rangle] = 1 \quad , \quad \frac{d}{dt}E_Q[\langle\phi(t)|\phi(t)\rangle] = 0$$

USING THE FURUTSU-NOVIKOV FORMULA FOR GAUSSIAN NOISE AVERAGES

$$E_Q\left[F[\{W(t)\}]W_i(t)\right] = \sum_{j=1}^{N}\int_0^t ds\, D_{ij}(t,s)\, E_Q\left[\frac{\delta F[\{W(t)\}]}{\delta W_j(s)}\right]$$

FIND

$$\mathcal{O} = -2\sqrt{\gamma}\,\xi_R \sum_{i,j=1}^{N} A_i \int_0^t ds\, D_{ij}(t,s)\frac{\delta}{\delta W_j(s)}$$

ALTHOUGH $|\phi(t)\rangle$ IS NORMED IN AN <u>AVERAGED</u> SENSE, $\langle\phi(t)|\phi(t)\rangle \neq 1$. THE PHYSICAL NORMED WAVE FUNCTION IS

$$|\psi(t)\rangle = \frac{|\phi(t)\rangle}{\|\phi(t)\|}$$

AND THE PHYSICAL AVERAGE E (E_P IN THE PAPERS) IS

$$E[F] = E_Q[F\langle\phi(t)|\phi(t)\rangle]$$

$$\Rightarrow \quad E[|\psi(t)\rangle\langle\psi(t)|] = E_Q[|\phi(t)\rangle\langle\phi(t)|]$$

EVIDENTLY, THE FORMALISM IS TECHNICAL, AND GENERAL RESULTS ARE IMPLICIT! HOWEVER, CAN GET EXPLICIT ANSWERS IN CERTAIN CASES:

① WHEN $H = 0$, CAN CALCULATE $\mathcal{O}$ EXPLICITLY TO GET

$$\frac{d|\phi(t)\rangle}{dt} = \left[\sqrt{\gamma}\,\xi\sum_{i=1}^{N} A_i W_i(t) - 2\gamma\,\xi\,\xi_R\sum_{i,j=1}^{N} A_i A_j F_{ij}(t)\right]|\phi(t)\rangle$$

$$F_{ij}(t) = \int_0^t ds\; D_{ij}(t,s)$$

CORRESPONDING TO THIS, $|\psi(t)\rangle$ OBEYS

$$\frac{d|\psi(t)\rangle}{dt} = \left[\sqrt{\gamma} \sum_{i=1}^{N} \left(\xi A_i - \xi_R \langle A_i \rangle_t \right) W_i(t) - 2\gamma \xi_R \sum_{i,j=1}^{N} \left(\xi A_i A_j - \xi_R \langle A_i A_j \rangle_t \right) F_{ij}(t) \right] |\psi(t)\rangle$$

BY A LONG CALCULATION, ONE CAN NOW SHOW:

DEFINE VARIANCE $V_A(t) = \langle A^2 \rangle_t - \langle A \rangle_t^2$

THEN

$$\underbrace{E[V_A(t)]}_{\geq 0} = V_A(0) - 8\xi_R^2 \gamma \sum_{i,j} \int_0^t ds \, \underbrace{E\left[\langle (A_i - \langle A_i \rangle_s) A \rangle_s \, \langle (A_j - \langle A_j \rangle_s) A \rangle_s \right] F_{ij}(s)}$$

IF $F_{ij}(\infty)$ is POSITIVE DEFINITE, MUST HAVE

$$\langle (A_i - \langle A_i \rangle_s) A \rangle_s \to 0 \qquad s \to \infty$$

TAKING $A = A_i \Rightarrow V_{A_i}(t) \to 0$

THIS GIVES A <u>NON-WHITE NOISE VERSION OF</u> <u>STANDARD REDUCTION PROOF</u>

② PERTURBATION EXPANSION IN $\sqrt{\gamma}$

BY MAKING A PERTURBATION EXPANSION, THE DERIVATIVE $\frac{\delta}{\delta W_j(s)}$ IN O CAN BE EVALUATED ORDER BY ORDER, GIVING EXPLICIT EQUATIONS FOR $|\phi(t)\rangle$ AND $|\psi(t)\rangle$

WORKING THROUGH ORDER γ (DROPPING $\gamma^{3/2}$) GET

$$\frac{d}{dt}\rho(t) = -\frac{i}{\hbar}[H, \rho(t)] + |\xi|^2 \gamma \int_0^t ds\, D_{ij}(t,s) \Big[A_i\, \rho(t)\, A_j(s-t) + A_j(s-t)\, \rho(t)\, A_i - A_i\, A_j(s-t)\, \rho(t) - \rho(t)\, A_j(s-t)\, A_i \Big]$$

$|\xi|^2 = 1$ FOR $\xi = 1$, $\xi = i$

$$A_j(s) = e^{\frac{i}{\hbar}Hs} A_j\, e^{-\frac{i}{\hbar}Hs}$$

INTERACTION PICTURE

GIVES NON-WHITE NOISE GENERALIZATION OF IMAGINARY NOISE TRICK

(3) PARTICLE DENSITY COUPLED NOISE (PAPER II)

$i \rightarrow$ SPATIAL COORDINATE $\vec{x}$

$A_i \rightarrow$ PARTICLE DENSITY $M(\vec{x}) = \sum_i m_i \, \delta^3(\vec{x} - \vec{q}_i)$

$$\int d^3x \, M(\vec{x}) = \sum_i m_i = \text{C-NUMBER}$$

$W_i \rightarrow$ CLASSICAL NOISE FIELD $\phi(\vec{x}, t)$

MEAN $E[\phi(\vec{x},t)] = \phi_0$

AUTOCORRELATION $E[(\phi(\vec{x},t_1) - \phi_0)(\phi(\vec{y},t_2) - \phi_0)] = D(\vec{x}-\vec{y}, t_1 - t_2)$

NOISE COUPLING IS THROUGH $\int d^3x \, [M(\vec{x}) - \langle M(\vec{x})\rangle_t] \, \phi(\vec{x},t)$,

SO ϕ_0 CONTRIBUTION DROPS OUT:

$$\int d^3x \, [M(\vec{x}) - \langle M(\vec{x})\rangle_t] \, \phi_0 = \phi_0 \left[\sum_i m_i - \langle \sum_i m_i \rangle_t \right] = 0$$

DEFINE $F(\vec{x}-\vec{y},t) = \int_0^t ds\, D(\vec{x}-\vec{y},\, t-s)$

EVOLUTION EQUATION FOR THE NORMALIZED STATE VECTOR BECOMES

$$\frac{d|\psi(t)\rangle}{dt} = \left[-iH + \sqrt{\gamma} \int d^3x \left[M(\vec{x}) - \langle M(\vec{x})\rangle_t \right] \phi(\vec{x},t) + \gamma\left(B - \langle B\rangle_t\right) \right] |\psi(t)\rangle$$

$$B = -2 \int d^3x \int d^3y\; F(\vec{x}-\vec{y},t) \left[M(\vec{x}) - \langle M(\vec{x})\rangle_t \right] \left[M(\vec{y}) - \langle M(\vec{y})\rangle_t \right]$$

SINCE $\int d^3x\, M(\vec{x})$ IS A C-NUMBER, B IS INVARIANT UNDER

$$F(\vec{x}-\vec{y},t) \rightarrow F(\vec{x}-\vec{y},t) - F(\vec{0},t)$$

IMPROVES INFRARED BEHAVIOR OF SPATIAL FOURIER TRANSFORM

A COUPLING $\int d^3x\, M(\vec{x})\, \phi(\vec{x},t)\, |\psi(t)\rangle$ CAN ARISE AS THE NON-RELATIVISTIC LIMIT OF A RELATIVISTICALLY INVARIANT ACTION; THE FRAME DEPENDENCE COMES FROM THE ASSUMED AUTOCORRELATOR OF THE NOISE FIELD $\phi(\vec{x},t)$

ADLER + BASSI II ANALYZES THIS MODEL IN DETAIL:

DENSITY MATRIX DIAGONALIZATION
STATE VECTOR REDUCTION RATES

$\Big\}$ BOTH $e^{-\Gamma(t)}$

FOKKER-PLANCK EQUATION

ENERGY PRODUCTION BY THE NOISE

SPECIFIC MODELS FOR CORRELATOR, INCLUDING THERMAL CORRELATION FUNCTION MODELS

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SIMPLIFIED MODEL WITH A SINGLE QUANTUM PARTICLE: BASSI + FERIALDI arXiv: 0901.1254
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QUANTUM THEORY AS THERMODYNAMICS OF PRE-QUANTUM "TRACE DYNAMICS"

TAKE DYNAMICAL VARIABLES AS GENERAL, NON-COMMUTING OPERATORS:

$$[q_r, q_s] \neq 0 \qquad [p_r, p_s] \neq 0 \qquad [q_r, p_s] \neq i\hbar\, \delta_{rs}$$

NO A PRIORI COMMUTATIVITY PROPERTIES

DYNAMICS GENERATED BY

$$\delta \int dt \; \mathrm{Tr}\, L(q, \dot{q}) = 0$$

USING CYCLIC PERMUTATION UNDER TRACE

GIVES OPERATOR EQUATIONS OF MOTION DIRECTLY, WITHOUT "QUANTIZATION" OF A CLASSICAL THEORY

RECOVER QUANTUM MECHANICS BY USING STATISTICAL MECHANICS OF UNDERLYING TRACE DYNAMICS

- CAN PROVE A LIOUVILLE THEOREM FOR OPERATOR PHASE SPACE

- GENERIC CONSERVED QUANTITIES

$$\underset{\sim}{H} = \mathrm{Tr}\left(\sum_{r} p_r \dot{q}_r - \mathcal{L} \right) \qquad \text{CONSERVED}$$

IF $\underset{\sim}{H}$ INVOLVES NO FIXED OPERATOR COEFFICIENTS, THERE IS A GLOBAL UNITARY INVARIANCE

CORRESPONDING CONSERVED NOETHER CHARGE IS

$$\tilde{C} = \sum_{r,\,\text{BOSON}} [q_r, p_r] - \sum_{r,\,\text{FERMION}} \{q_r, p_r\}$$

ALREADY SEE A RESEMBLANCE TO CANONICAL ALGEBRA OF QUANTUM THEORY, WHICH WOULD FOLLOW FROM "EQUIPARTITIONING" OF $\tilde{C}$

IF THERE IS A LARGE HIERARCHY OF SCALE BETWEEN UNDERLYING AND OBSERVED DYNAMICS, SO THAT CERTAIN TERMS COMING FROM $\underset{\sim}{H}$ DECOUPLE FROM THE WARD IDENTITIES,

STATISTICAL THERMODYNAMICS OF TRACE DYNAMICS $\Leftrightarrow$ QUANTUM FIELD THEORY

EQUIPARTITION OF $\tilde{C}$ $\Rightarrow$ $[q_{r\,eff}, p_{s\,eff}] = i_{eff}\,\hbar\,\delta_{rs}$

WITH $\langle \tilde{C} \rangle_{\text{ENSEMBLE AVERAGE}} = i_{eff}\,\hbar$ LORENTZ INVARIANT

FOR FLUCTUATIONS AROUND AVERAGES TO BE FINITE, NEED $e^{-\tau \underset{\sim}{H}}$ FOR CONVERGENCE, SO FRAME-DEPENDENCE ENTERS HERE. WITH PLAUSIBLE ASSUMPTIONS, BROWNIAN MOTION CORRECTIONS $\Leftrightarrow$ STOCASTIC MODIFICATIONS TO SCHRÖDINGER EQUATION

NECESSARILY FRAME DEPENDENT

OPEN QUESTIONS (A FEW OF MANY)

- DOES LATENT IMAGE FORMATION CONSTITUTE MEASUREMENT?
- EXPERIMENTAL TESTS OF CSL?
- JUSTIFICATION FOR REAL-VALUED (IE ANTI-SELF-ADJOINT) NOISE TERM? FOR NORMALIZATION ASSUMPTION?
- COSMOLOGICAL CONNECTION?
- BETTER UNDERSTANDING OF TRACE DYNAMICS AS UNDERPINNING TO QUANTUM THEORY AND OBJECTIVE REDUCTION MODELS?