QCD, Strings and Black holes

The large N limit of Field Theories

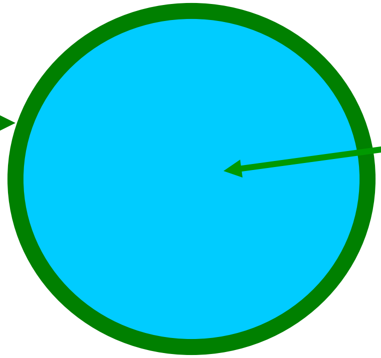
and

Gravity

Juan Maldacena

Field Theory = Gravity theory

Gauge Theories
QCD



Quantum Gravity
String theory

Plan

QCD, Strings, the large N limit

Supersymmetric QCD

↓ N large

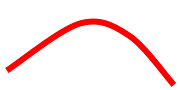
Gravitational theory in 10 dimensions

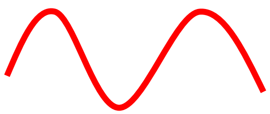
Calculations → Correlation functions
Quark-antiquark potential

Black holes

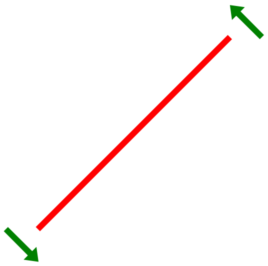
Strings and Strong Interactions

Before 60s → proton, neutron → elementary
During 60s → many new strongly interacting particles
Many had higher spins s = 2, 3, 4 ….
All these particles → different oscillation modes of a string.



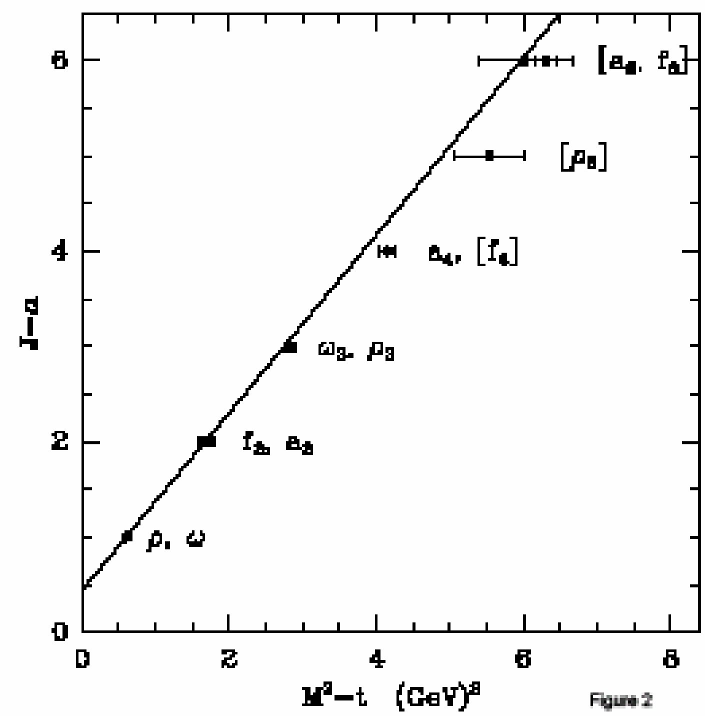


This model explained “Regge trajectories”



Rotating String model

$$m^2 \sim T J_{\max} + const$$



From E. Klempt **hep-ex/0101031**

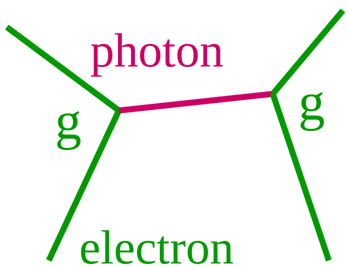
Strong Interactions from Quantum ChromoDynamics

Experiments at higher energies
revealed quarks and gluons

3 colors (charges)

They interact exchanging gluons

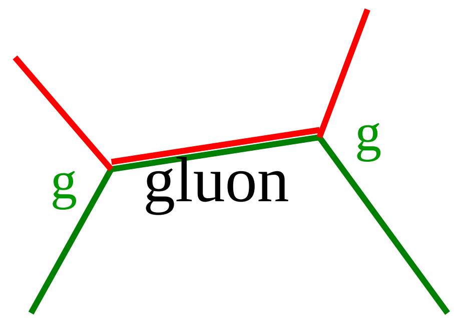
Electrodynamics



Gauge group

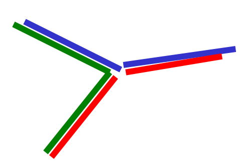
U(1)

Chromodynamics (QCD)



3 x 3 matrices

SU(3)



Gluons carry
color charge, so
they interact among
themselves

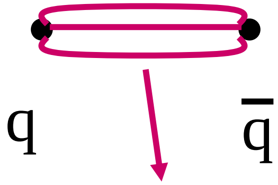
Coupling constant decreases at high energy

Gross, Politzer, Wilczek

$g \longrightarrow 0$ at high energies $\longrightarrow$ QCD is easier to study at high energies

Hard to study at low energies

Indeed, at low energies we expect to see confinement



Flux tubes of color field = glue

$V = T\ L$

At low energies we have something that looks like a string

Can we have an effective theory in terms of strings ?

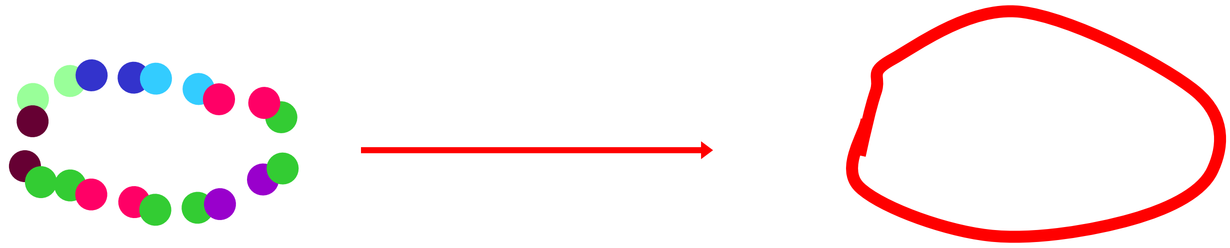
Large N limit

t' Hooft '74

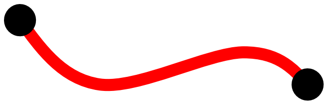
Take N colors instead of 3, SU(N)

Large N and strings

Gluon: color and anti-color



Open strings → mesons
Closed strings → glueballs



Looks like a string theory, but…

1. Simplest action = Area

Not consistent in D=4 (D=26 ?)

↓ generate

At least one more dimension (thickness)

Polyakov

2. Strings theories always contain a state with m=0, spin =2: a Graviton.

Scherk-Schwarz
Yoneya

For this reason strings are commonly used to study quantum gravity

We combine these two problems into a solution. We will look for a 5 dimensional theory that contains gravity. We have to find an appropriate 5 dimensional curved spacetime.

Most supersymmetric QCD

Supersymmetry

Ramond
Wess, Zumino

Bosons ⟷ Fermions

Gluon ⟷ Gluino

Many supersymmetries

B1 F1
B2 F2

Maximum 4 supersymmetries, $N = 4$ Super Yang Mills

A_μ	Vector boson	spin = 1
Ψ_α	4 fermions (gluinos)	spin = 1/2
Φ^I	6 scalars	spin = 0

SO(6) symmetry

All NxN matrices

Susy might be present in the real world but spontaneously broken at low energies.

We study this case because it is simpler.

Similar in spirit to QCD

Difference: most SUSY QCD is scale invariant

Classical electromagnetism is scale invariant

$$V = 1/r$$

QCD is scale invariant classically but not quantum mechanically, g(E)

Most susy QCD is scale invariant even quantum mechanically

Symmetry group

Lorentz + translations + scale transformations + other

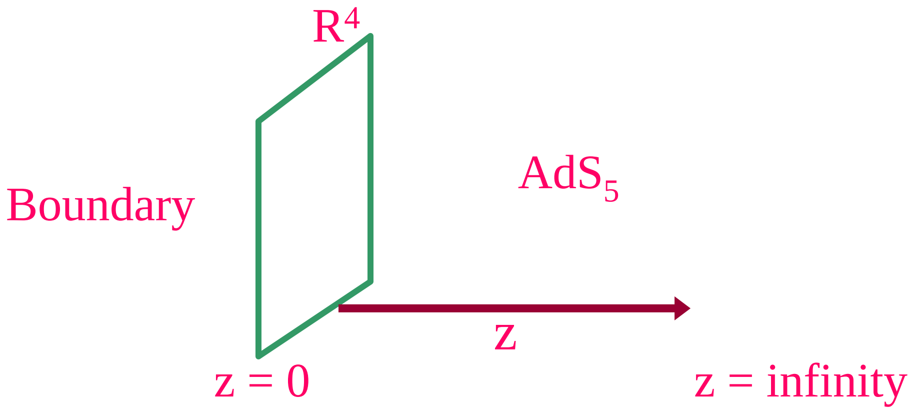
The string should move in a space of the form

$$ds^2 = R^2 \; w^2(z) \; (dx^2_{3+1} + dz^2)$$

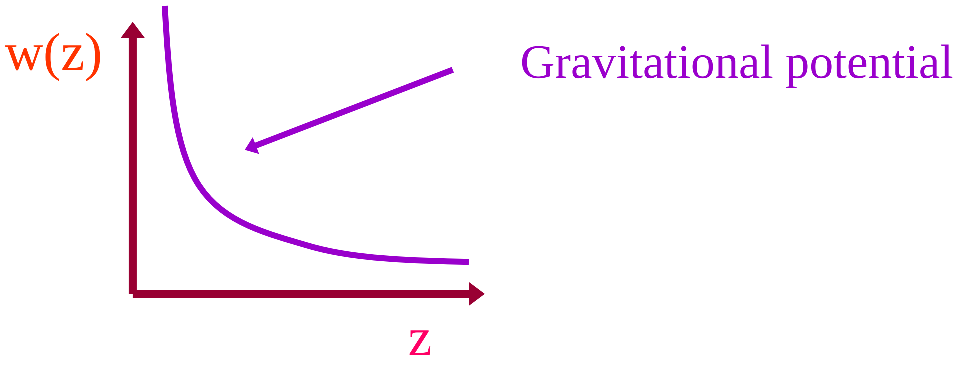
$\downarrow$ redshift factor = warp factor ~ gravitational potential

Demanding that the metric is symmetric under scale transformations $x \rightarrow \lambda x$, we find that $w(z) = 1/z$

$$ds^2 = R^2 \frac{(dx^2_{3+1} + dz^2)}{z^2}$$



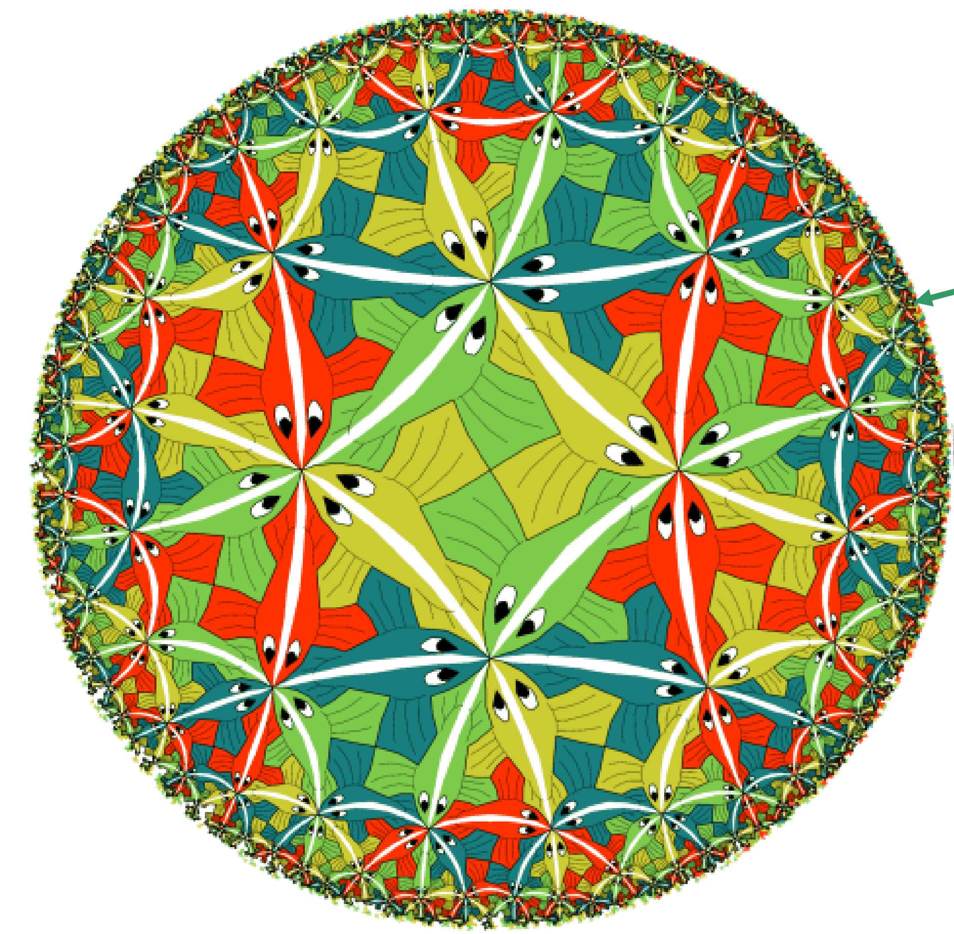
This metric is called anti-de-sitter space. It has constant negative curvature, with a radius of curvature given by R.



Anti de Sitter space

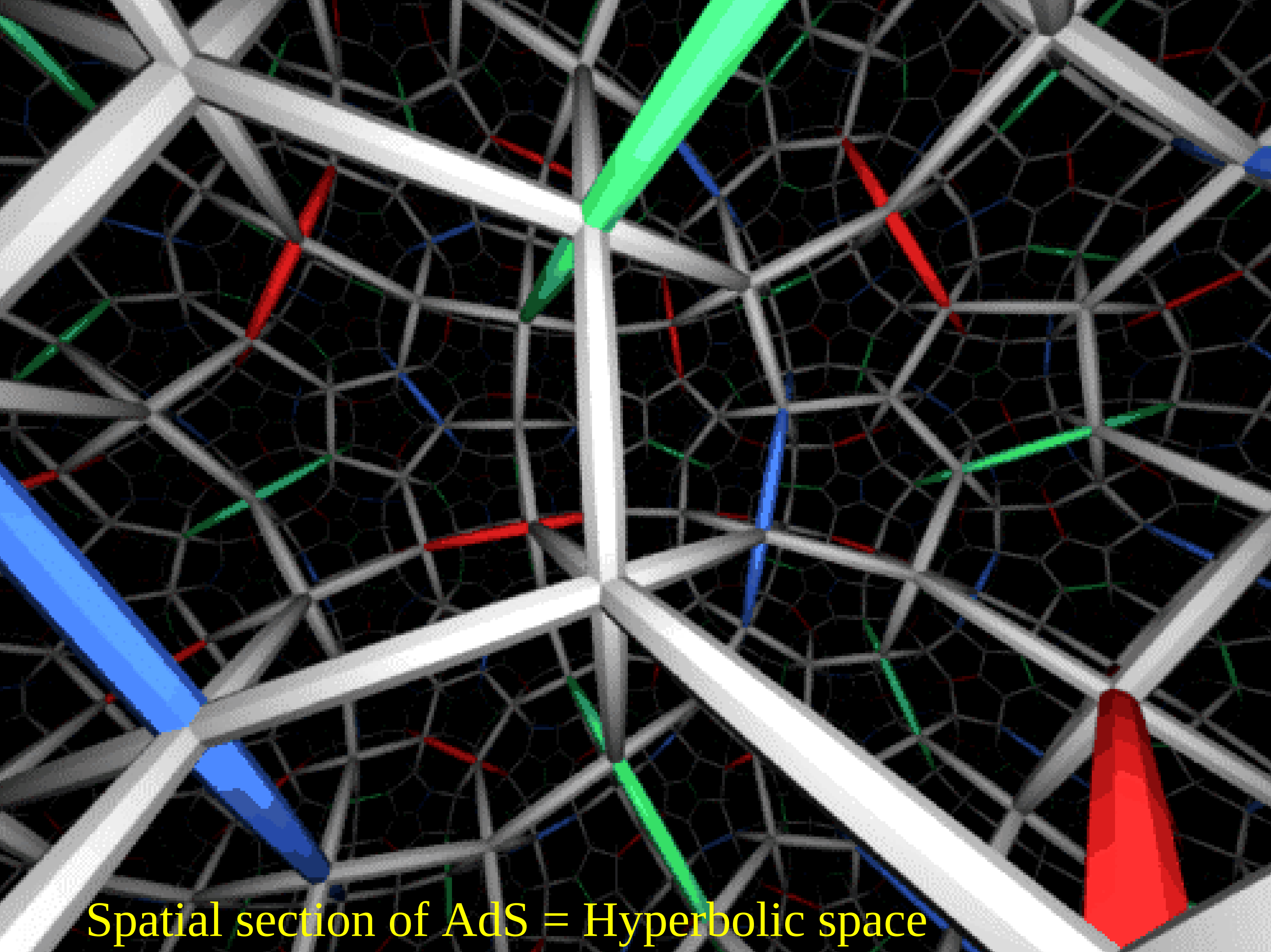
Solution of Einstein's equations with negative cosmological constant.

(De Sitter → solution with positive cosmological constant, accelerated expanding universe)



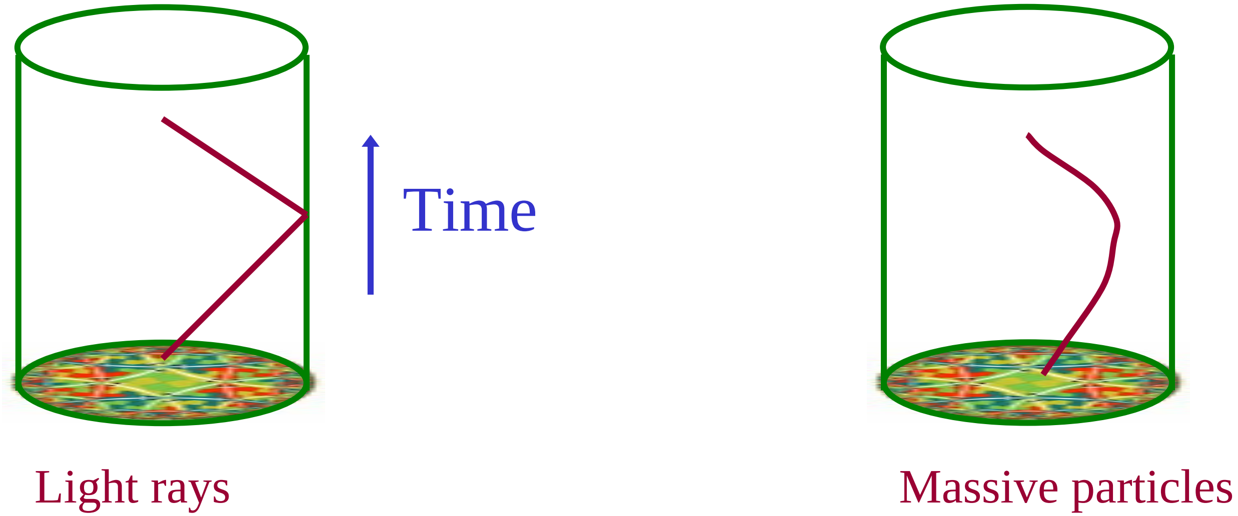
Boundary is S^3 x time

Spatial cross section of AdS = hyperbolic space



Spatial section of AdS = Hyperbolic space

R = radius of curvature



Energies of particles in AdS are quantized, particles feel as if they were in a gravitational potential well, they cannot escape to infinity.

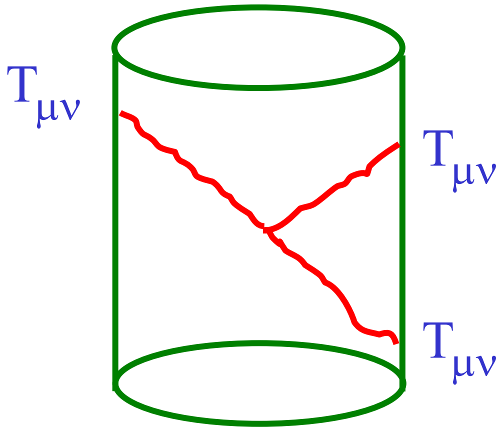
Qualitatively AdS is like a box of size R.

Boundary is S^3 x Time

The Field theory is defined on the boundary of AdS.

Building up the Dictionary

Graviton → stress tensor



Gubser, Klebanov,
Polyakov - Witten

$< T_{\mu\nu}(x)\ T_{\mu\nu}(y)\ \ T_{\mu\nu}(z) >_{\text{Field theory}}$ = Probability amplitude that gravitons go between given points on the boundary

Other operators

→ Other fields (particles) propagating in AdS.

Mass of the particle → scaling dimension of the operator

$$\Delta = 2 + \sqrt{4 + (mR)^2}$$

Most supersymmetric QCD

We expected to have string theory on AdS.

Supersymmetry ⟶ D=10 superstring theory on AdS^5 x $(\text{something})^5$

SO(6) symmetry ↓

S^5

Type IIB superstrings on AdS^5 x S^5

(J. Schwarz)

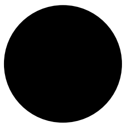
5-form field strength F = generalized magnetic field → quantized

$$\int_{S^5} F = N$$

Similar solution in 4 dimensional gravity + electromagnetism:

AdS^2 x S^2, with a flux of magnetic field on S^2:

Start with a charged, extremal black hole → near horizon geometry



String Theory

Veneziano
Scherk
Schwarz
Green
…..

Free strings

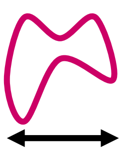
String

$$\text{Tension} = T = \frac{1}{l_s^2} \quad , \qquad l_s = \text{string length}$$

Relativistic, so T = (mass)/(unit length)

Excitations along a stretched string travel at the speed of light

Closed strings



l_s

Can oscillate ⟶ Normal modes ⟶ Quantized energy levels

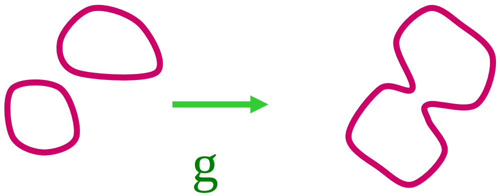
Mass of the object = total energy

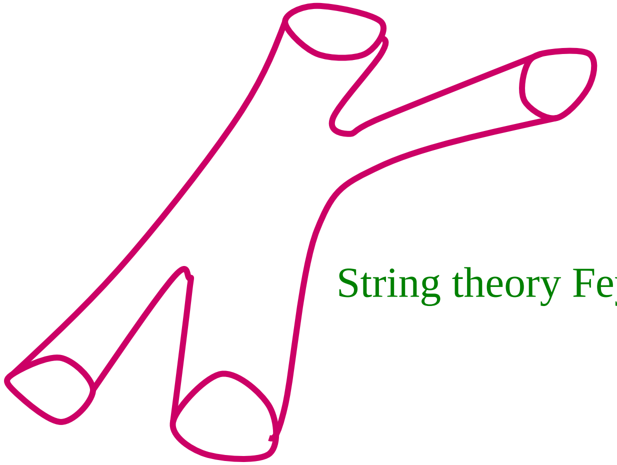
M=0 states include a graviton (a spin 2 particle)

First massive state has $M^2 \sim T$

String Interactions

Splitting and joining





String theory Feynman diagram

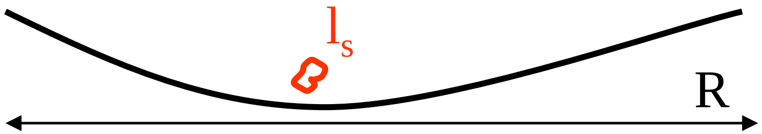
Simplest case: Flat 10 dimensions and supersymmetric

Precise rules, finite results, constrained mathematical structure

At low energies, energies smaller than the mass of the first massive string state

↓

Gravity theory



Radius of curvature >> string length → gravity is a good approximation

(Incorporates gauge interactions → Unification)

Particle theory = gravity theory

Most supersymmetry QCD theory	=	String theory on $AdS_5 \times S^5$

(J.M.)

N colors

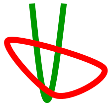
N = magnetic flux through S^5

Radius of curvature

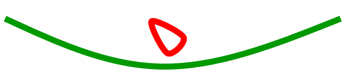
$$R_{S^5} = R_{AdS_5} = \left(g_{YM}^2 \; N\right)^{1/4} l_s$$

Duality:

$g^2 N$ is small → perturbation theory is easy – gravity is bad



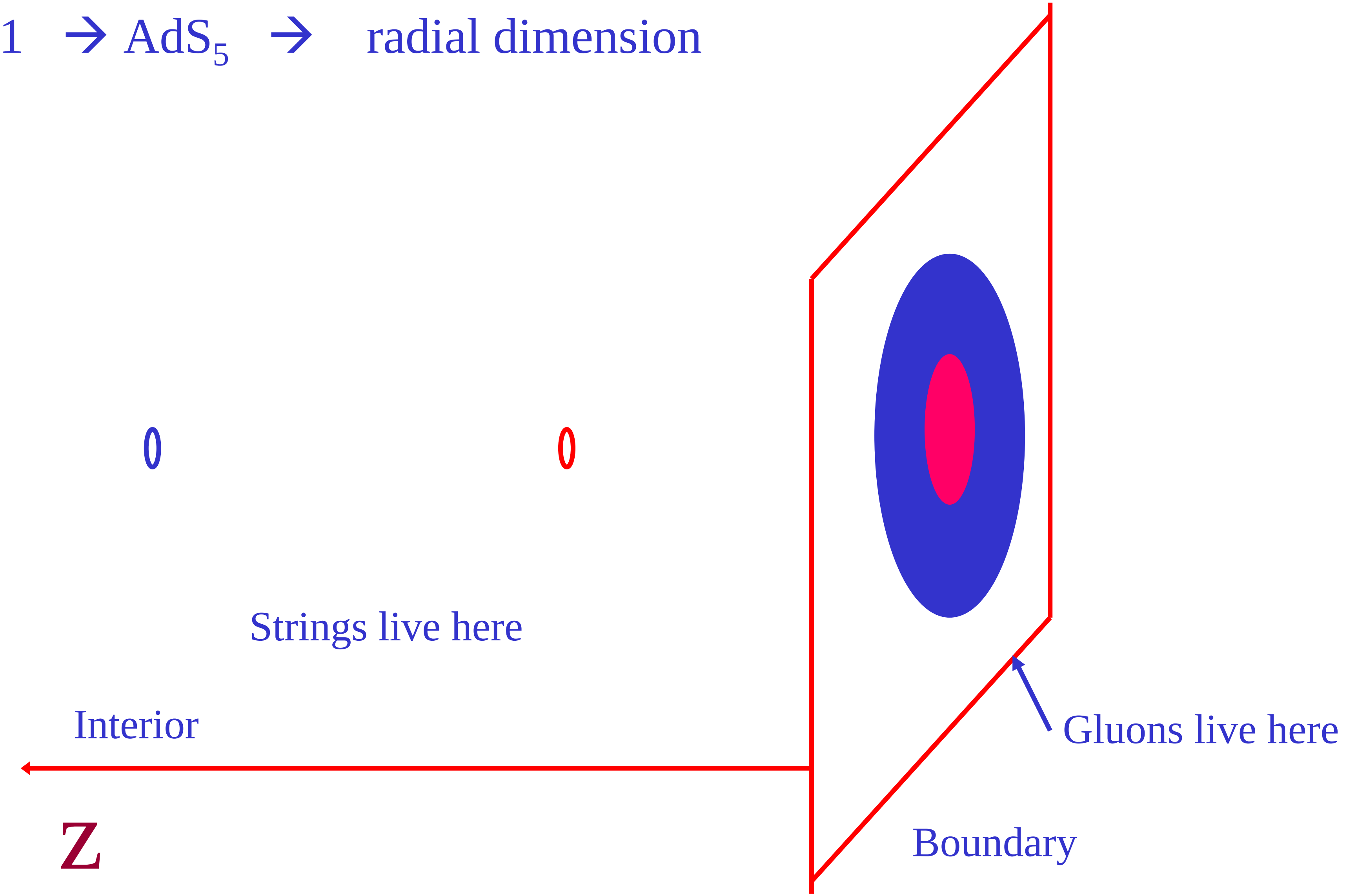
$g^2 N$ is large → gravity is good – perturbation theory is hard



Strings made with gluons become fundamental strings.

Where Do the Extra Dimensions Come From?

3+1 → AdS_5 → radial dimension

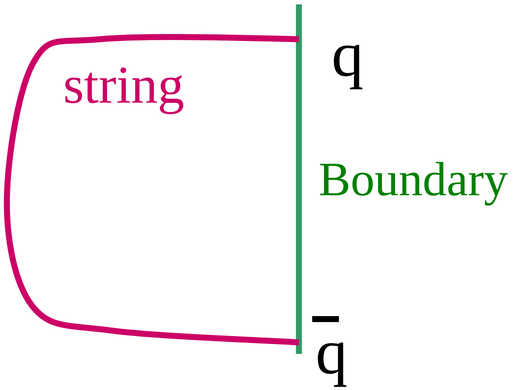


What about the S^5 ?

- Related to the 6 scalars
- $S^5 \rightarrow$ other manifolds = Most susy QCD $\rightarrow$ less susy QCD.
- Large number of examples

Klebanov, Witten,
Gauntlett, Martelli, Sparks,
Hannany, Franco, Benvenutti,
Tachikawa, Yau …..

Quark anti quark potential



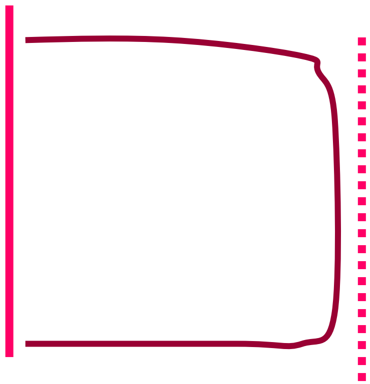
V = potential = proper length of the string in AdS

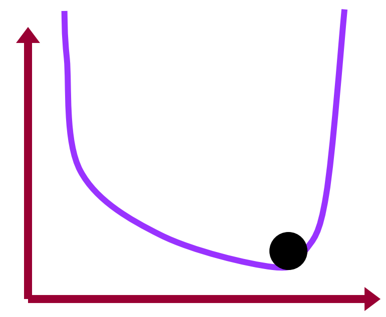
$$V \approx -\frac{\sqrt{g^2 N}}{L}$$

Weak coupling result: $$V \approx -\frac{g^2 N}{L}$$

Confining Theories

Add masses to scalars and fermions → pure Yang Mills at low energies → confining theory. There are many concrete examples.
At strong coupling → gravity solution is a good description.



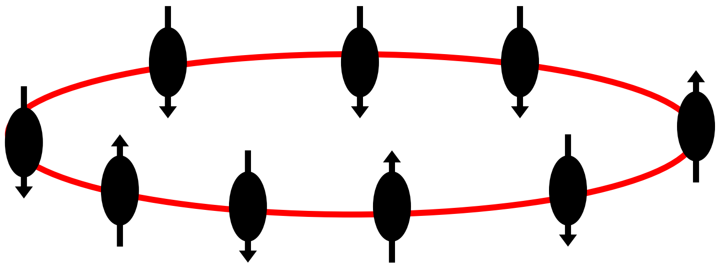


String at z_0 has finite tension from the point of view of the boundary theory.
Graviton in the interior → massive spin=2 particle in the boundary theory = glueball.

Baryons → D-branes

Checking the conjecture

- Energies of string states in AdS = Dimensions of operators in gauge theory
- Direct computation in the gauge theory
- Planar diagrams → "spin" chains
- Connections to "integrable" systems that also appear in condensed matter.
- Works nicely in a special, large charge, limit
- Not yet done in general, active research area…



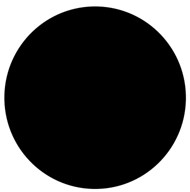
Berenstein, J.M, Nastase, Minahan, Zarembo, Staudacher, Beisert,….

What can we learn about gravity from the field theory ?

- Useful for understanding quantum aspects of black holes

Black holes

Gravitational collapse leads to black holes



Classically nothing can escape once it crosses the event horizon

Quantum mechanics implies that black holes emit thermal radiation.

(Hawking)

$$T \approx \frac{1}{r_s} \approx \frac{1}{G_N M} \qquad T \approx 10^{-8} K \left(\frac{M_{sun}}{M} \right)$$

Black holes evaporate

Evaporation time $$\tau = \tau_{\text{universe}} \left(\frac{M}{10^{12}\, Kg} \right)^3$$

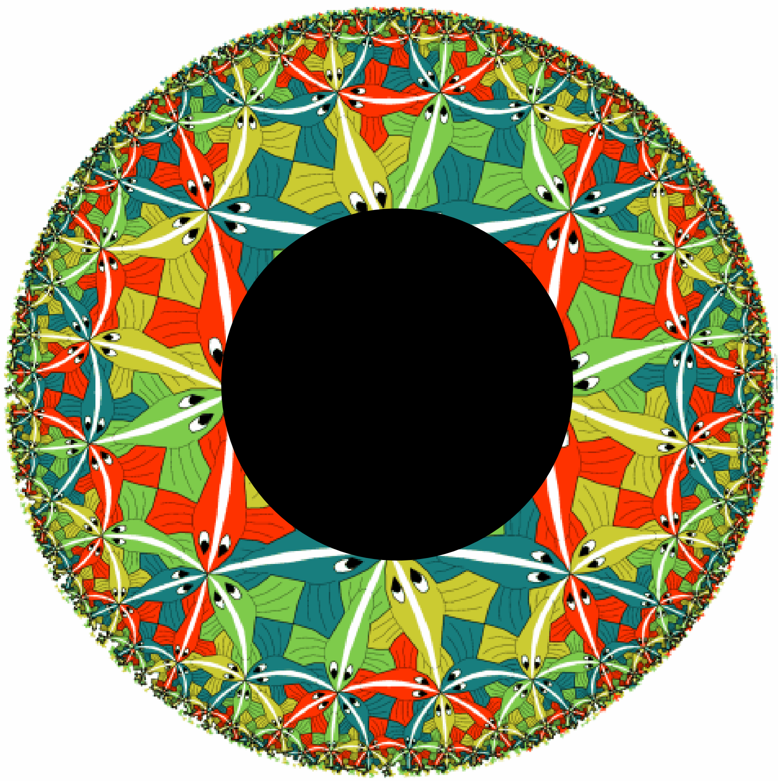
Temperature is related to entropy

dM = T dS $$S = \frac{\text{Area of the horizon}}{4\, L^2_{\text{Planck}}}$$ (Hawking-Bekenstein)

What is the statistical interpretation of this entropy?

Black holes in AdS

Thermal configurations in AdS.



Entropy:

$S_{GRAVITY}$ = Area of the horizon =
$S_{FIELD\ THEORY}$ =
Log[Number of states]

Evolution: Unitary

(these calculations are easier in the AdS_3 case)

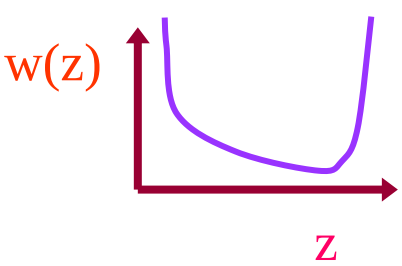
Confining Theories and Black Holes

Low temperatures

boundary

$z=0$ $\quad z=z_0$, $w(z_0) > 0$

Confinement



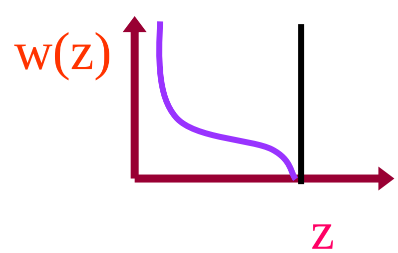
Gravitational potential

High temperatures

Horizon

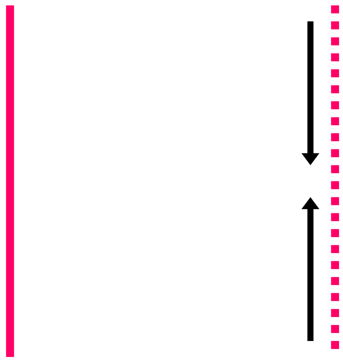
$z=0$ $\quad z=z_0$

Deconfinement=
black hole (black brane)



Gravitational potential

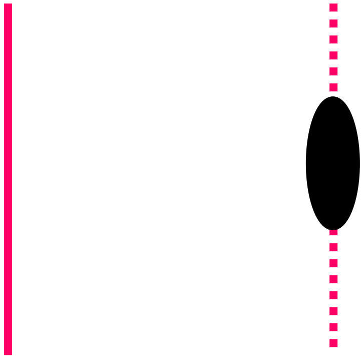
Black holes in the Laboratory



$z=0$ $z=z_0$,

QCD → 5d string theory

High energy collision→ produces a black hole =
droplet of deconfined phase ~
quark gluon plasma .



$z=0$ $z=z_0$

Black hole→ Very low shear viscosity→
similar to what is observed at RHIC:
" the most perfect fluid"

Kovtun, Son, Starinets, Policastro

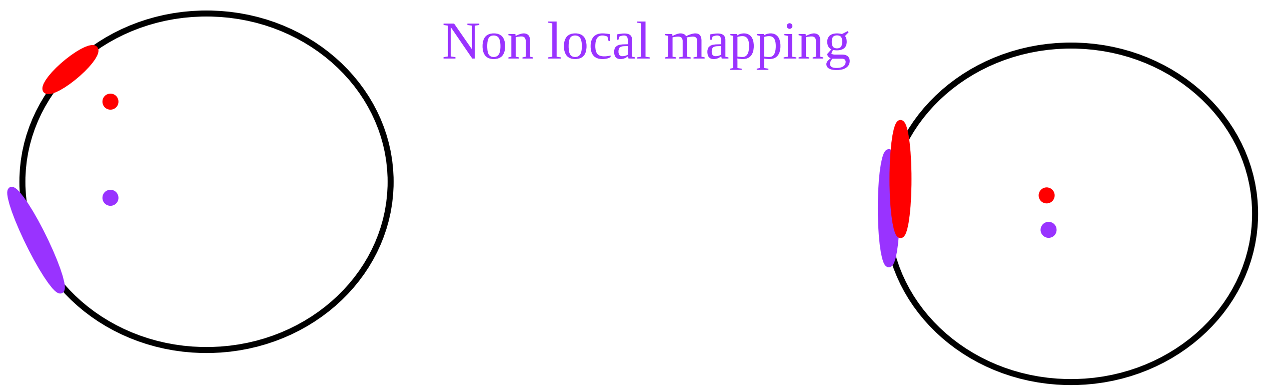
Very rough model, we do not yet know the precise string theory

Holography

Quantum theories of gravity should be **holographic.**

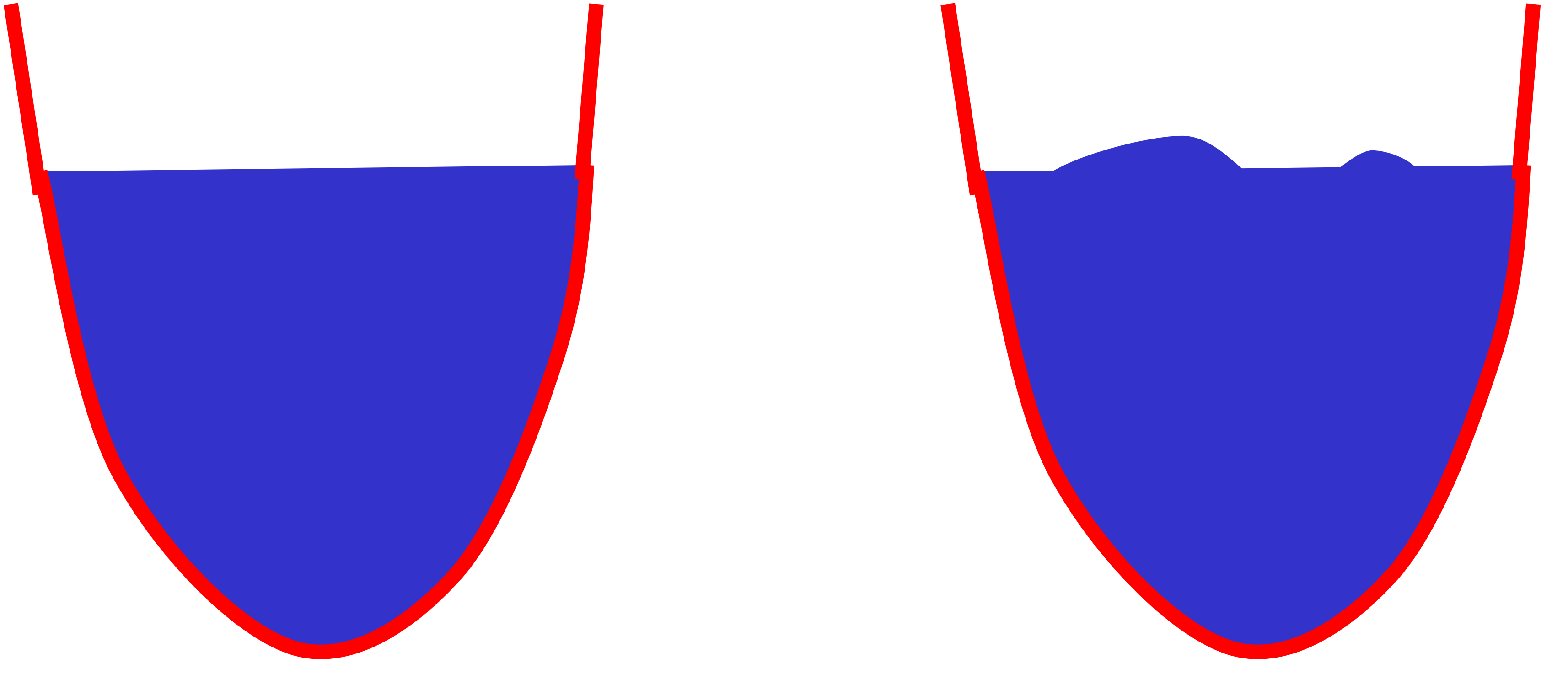
t’ Hooft
Susskind

All physics in a region → described in terms of a number of q-bits which is smaller that the area (not volume) of the region (in Planck units) .



The gauge theory gravity duality → concrete realization → gives a non-perturbative definition of quantum gravity in AdS.

Emergent space time



Spacetime: like the fermi surface,
only defined in the classical limit

Lin, Lunin, J.M.

Conclusions

- Gravity and particle physics are "unified"

  Usual: Quantum gravity → particle physics.

  New: Particle physics → quantum gravity.

  Most elementary particles → quantum gravity → particles physics of the real world

- Black holes and confinement are related
- Emergent space-time
- Tool to do computations in gauge theories.
- Tool to do computations in gravity.

Future

Field theory:

→ Theories closer to the theory of strong interactions

→ Solve large N QCD

Gravity:

→ Quantum gravity in other spacetimes

→ Understand cosmological singularities