AST 513: Stellar Systems

Assignment 5

Due Wednesday, December 19, 2007

1. Dynamics of the solar neighborhood

By approximating the Galactic disk as an infinite slab, in which the density $\rho(z)$ depends only on the distance z from the Galactic midplane, we were able to simplify Poisson's equation in the solar neighborhood to the form

$$\rho(z) = \frac{1}{4\pi G}\frac{d^2\Phi}{dz^2}.$$

If we make the better approximation that the Galactic disk is axisymmetric, show that this equation is replaced by

$$\rho(z) = \frac{1}{4\pi G}\frac{d^2\Phi}{dz^2} + c,$$

and evaluate c both in terms of the Oort constants and $\mathrm{M}_\odot\,\mathrm{pc}^{-3}$. Is this correction significant?

2. Jeans instability I

Suppose that the gravitational potential due to a body of mass m is modified from the Newtonian form to the Yukawa potential, $\Phi(r) = -Gm\exp(-\alpha r)/r$. How does this modification affect the Jeans wavenumber for a homogeneous fluid?

3. Jeans instability II

An infinite homogeneous stellar system has density ρ_0 and equilibrium distribution function

$$f_0(\mathbf{v}) = \frac{\rho_0\theta}{\pi^2}\frac{1}{(v^2+\theta^2)^2}.$$

What is the Jeans wavenumber?

4. Jeans instability III

At typical sea-level conditions ($p = 1.01 \times 10^5\,\mathrm{N\,m}^{-2}$ and $T = 15^\circ\,\mathrm{C}$), the density of air is $1.22\,\mathrm{kg\,m}^{-3}$ and the speed of sound is $340\,\mathrm{m\,s}^{-1}$. Find (i) the fractional change in frequency due to the self-gravity of the air, for a sound wave with wavelength 1 meter; (ii) the Jeans length (this is Problem 5.4 from BT).

5. The homologous merger

Two identical galaxies are initially at rest, at a large distance from one another. They are spherical, composed solely of identical stars, and their light distributions obey the $R^{1/4}$ law with effective radius R_e. The galaxies fall together and merge. If the merger product also satisfies the $R^{1/4}$ law with the same index, what is its effective radius?

6. Relaxation in a disk

In a 3-dimensional spherical stellar system supported by the random velocities of its N stars, the relaxation time $t_{\rm relax}$ is related to the crossing time $t_{\rm cross}$ by

$$t_{\rm relax} \approx \frac{N}{\log N} t_{\rm cross}.$$

What would be the analogous formula for a hypothetical 2-dimensional stellar system (a razor-thin flat disk) supported by the random velocities of its stars (i.e., without rotation)?

8. Visits from nearby stars

(a) What is the closest approach that a star is likely to have made to the Sun during its lifetime (i.e., the impact parameter such that there is a 50% chance that no closer encounter has occurred), assuming that its environment has always been similar to the present solar neighborhood? Assume that the one-dimensional velocity dispersion in the solar neighborhood is $30\,\mathrm{km\,s^{-1}}$, that all stars have mass $0.5\,\mathrm{M_\odot}$, and that the stellar density is $\rho = 0.040\,\mathrm{M_\odot\,pc^{-3}}$.

(b) Approximately what eccentricity would such an encounter excite in Neptune's orbit?

9. Galactic globular clusters

A catalog of Galactic globular clusters compiled by W. E. Harris is available at http://phys-www.mcmaster.ca/%7Eharris/mwgc.dat.

(a) Use this catalog to estimate the following quantities:

(i) The median core radius, tidal radius, half-mass relaxation time, and luminosity (in the V band) of Galactic globular clusters.

(ii) The distance to the Galactic center (remember that clusters at low Galactic latitude may be obscured). If your result disagrees with the standard value, explain why.

(iii) The circular speed of the Galactic rotation curve, assuming it is flat (hint: you may wish to use the answer to question 3 of assignment 2). If your result disagrees with the standard value, explain why.

(b) Use the catalog to devise a simple interpolation formula to determine the half-mass radius r_h given the core radius r_c and the tidal radius r_t.