AST 513: Stellar Systems

Assignment 3

Due Monday, November 12, 2007

1. The stellar slab, part 1

A simple model for a galactic disk consists of N infinite, parallel sheets, each having surface density σ. Let z be the coordinate perpendicular to the sheets and label the position of the i^{th} sheet by z_i.

(a) What is the equation of motion of the i^{th} sheet? Hint: your answer should involve the quantities N_{+i} and N_{-i}, the number of sheets with $z > z_i$ and $z < z_i$.

(b) The kinetic and potential energy per unit area can be written

$$K = \tfrac{1}{2}\sigma \sum_{i=1}^{N} v_i^2, \qquad W = \pi G \sigma^2 \sum_{\substack{i,j=1 \\ i \neq j}}^{N} |z_i - z_j|,$$

where $v_i = \dot{z}_i$. What is the relation between K and W in a steady state, when $N \gg 1$?

2. The stellar slab, part 2

Consider an infinite slab with density $\rho(z)$ and potential $\Phi(z)$. All properties of the slab are independent of the horizontal coordinates x and y. According to Jeans's theorem the distribution function of the slab can depend only on the energy,

$$F(z, v_z) \equiv \int f(\mathbf{x}, \mathbf{v}) dv_x dv_y = F(E_z), \qquad \text{where} \qquad E_z = \tfrac{1}{2}v_z^2 + \Phi(z).$$

We assume that the slab is self-gravitating, so that

$$\frac{d^2\Phi}{dz^2} = 4\pi G \rho(z);$$

in addition we assume that the slab is symmetric about $z = 0$ and the distribution function is isothermal, $F(E_z) \propto \exp(-E_z/\sigma_z^2)$, where σ_z is the velocity dispersion. Find the density distribution $\rho(z)$ in terms of σ_z and the central density $\rho_0 \equiv \rho(0)$.

3. Galaxy classification

Find images of the following galaxies and classify them using Hubble's classification scheme (E0,. . . ,E7, S0, SB0, Sa,. . . , Sc, SBc, Irr).

NGC 300	NGC 1232
NGC 1371	NGC 1433
NGC 1566	NGC 3344
NGC 3379	NGC 3783
NGC 3992	NGC 4111
NGC 4261	NGC 4350
NGC 4478	NGC 4501
NGC 4596	NGC 4660
NGC 4698	NGC 5085
NGC 7741	NGC 7793

4. The Sersic law

The Sersic law for the surface-brightness distribution is spherical galaxies is

$$I(R) = I_0 \exp(-bR^{1/m}),$$

and includes the de Vaucouleurs profile ($m = 4$) and the exponential profile ($m = 1$) as special cases. The total luminosity can be written in the form

$$L = f(m) I_e R_e^2,$$

where $I_e = I(R_e)$ and R_e is the effective radius (the projected radius containing half of the total luminosity associated with $I(R)$). Find $f(m)$ for $m = 4$ and $m = 1$.

5. Stars in a logarithmic potential

(a) A spherical system of test particles orbits in the logarithmic potential $\Phi(r) = v_c^2 \ln r$. The system is in a steady state, with luminosity density $j(r) \propto r^{-k}$ and anisotropy parameter $\beta = 1 - \sigma_\theta^2/\sigma_r^2$ independent of radius. What is $\sigma_r^2(r)$?

(b) In Assignment 2 it was shown that the mean-square velocity of an equilibrium distribution of test particles in the logarithmic potential is $\langle v^2 \rangle = v_c^2$. Is this result consistent with the result of part (a)? If not, why not?