AST 513: Stellar Systems

Assignment 2

Due Monday, October 22, 2007

Short questions

1. A typical disk galaxy has central surface brightness I_0 equal to 21 mag $(\mathrm{sec})^{-2}$ when viewed face on, and scale length $R_d = 3h^{-1}\,\mathrm{kpc}$ where the surface brightness at radius R is $I(R) = I_0 \exp(-R/R_d)$. What is the absolute magnitude of such a galaxy?

2. The typical size of the seeing disk (the angular size of the image of a star after blurring by the atmosphere) is ~ 1 arcsec, far smaller than the angular resolution of the eye (about 1 arcmin). Describe an experiment that could measure the size of the seeing disk of a star (or at least set an upper limit of a few arcsec) without the use of any optical instruments.

3. A particle moves in a bound orbit in a spherical potential $\Phi(r)$. Use a derivation similar to that of the virial theorem to find an expression for its mean-square velocity in terms of some average of $\Phi(r)$ over the orbit. What does this relation imply for the mean-square velocity of an equilibrium distribution of test particles in a logarithmic potential, $\Phi(r) = v_c^2 \log r$?

4. Prove that the density of a spherical, ergodic, self-consistent stellar system must decrease outward.

5. The figures at the end of the assignment show the distribution of three populations of celestial objects in Galactic coordinates. The figures are equal-area projections in which the north and south Galactic poles are at top and bottom, and the Galactic center is in the middle of each figure. Estimate the typical distance range for the objects in each population, from the following choices (in some cases more than one choice may be consistent with the data): 100 pc or less, 1 kpc, 10 kpc, 100 kpc, 1 Mpc, 10 Mpc, 100 Mpc or greater. You should justify your answers.

6. Gravitational focusing

Consider an infinite homogeneous system of collisionless zero-mass test particles in D-dimensional space. The particles have an isotropic velocity distribution $f(v)$. Initially the particles are subject to no forces. At $t = 0$ a gravitational potential well suddenly appears in a finite region of the space. As $t \to \infty$, is the density of unbound particles traveling through the well larger or smaller than the asymptotic density at large distance? Answer for $D = 1$, 2, and 3.

7. The Plummer model

(a) What is the surface brightness $I(R)$ of a transparent spherical galaxy with luminosity density $j(r) = j_0(1 + r^2/b^2)^{-5/2}$ (the Plummer model)?

(b) What is the core radius r_c in units of the scale length b?

(c) If the distribution function is ergodic and the mass-to-light ratio Υ is independent of position, what is the velocity dispersion at radius r?

(d) What is the line-of-sight velocity dispersion at the center of the galaxy, $\sigma_{\|}(0)$?

(e) The core-fitting method for determining the mass-to-light ratio near the center of a stellar system states that

$$\Upsilon = \eta \frac{9\sigma_{\|}^2(0)}{2\pi G I(0) r_c},$$

where η is nearly unity. What is η for the Plummer model with an ergodic distribution function?

(f) Verify that the virial theorem holds for this system.

8. Central cusps

The goal of this problem is to explore the behavior of the velocity dispersion near the center of a spherical galaxy. At radii $r < r_0$ assume that the density has the power-law form

$$\rho(r) = \rho_0 \frac{r_0^\gamma}{r^\gamma}, \qquad 0 \leq \gamma < 3$$

(why is the constraint $\gamma < 3$ necessary?). Assume that the velocity dispersion is isotropic at all radii and equal to some given constant σ_0^2 at r_0.

(a) What is the dispersion $\sigma^2(r)$ for $r < r_0$?

(b) For what value(s) of γ is $\sigma^2(r)$ independent of r as $r \to 0$? For what range of γ does $\sigma^2(r) \to 0$ as $r \to 0$? For what range of γ does $\sigma^2(r)$ diverge as $r \to 0$?

9. Stars around a black hole

Within about 1 pc of the center of the Milky Way, the gravitational potential is dominated by the central black hole. If the distribution function of the stars in this region is ergodic, what is the distribution of their eccentricities, that is, what is the probability $p(e)de$ that a star has eccentricity in the range $e \to e + de$?