AST 513: Stellar Systems

Assignment 1

Due October 8, 2007

1. Perspective acceleration

A nearby star has proper motion μ (arcsec/yr), parallax π (arcsec), and radial velocity v_r ($\mathrm{km\,s^{-1}}$, with $v_r > 0$ for stars travelling away from the Sun). If the star travels at constant velocity relative to the Sun, show that its proper motion slowly changes with time, and derive a formula for the range of change of proper motion ("perspective acceleration") $\dot{\mu}$ (arcsec/(yr)^2).

The star with the largest proper motion is Barnard's star. Using the Hipparcos proper motion and parallax for Barnard's star (these are available on the Hipparcos web site), and its heliocentric radial velocity $v_r = -106.8\,\mathrm{km\,s^{-1}}$, evaluate the perspective acceleration.

2. Malmquist bias

Consider a population of stars whose luminosity function is

$$\Phi(M) = \begin{cases} \Phi_0, & M > M_0, \\ 0, & M < M_0. \end{cases}$$

The density of stars is independent of position and there is no absorption.

We construct a magnitude-limited catalog of these stars. Some of the stars in the catalog will be in binary systems, which can be recognized because the two stars are so close together on the sky that the chance of accidental superposition is negligible. You may assume that the distribution of luminosities of stars in binary systems is the same as for single stars, and the luminosity of each component of the binary is independent of the other (this is a good assumption, at least for widely separated binary systems).

(a) Find the mean absolute magnitude of the stars in the catalog.

(b) Find the mean absolute magnitude of the stars in the catalog that are in detected binary systems.

3. Surface brightness and inclination

(this is problem 1.4 of BT) An axisymmetric transparent galaxy has luminosity density that is constant on spheroids $R^2 + z^2/q^2$ having axis ratio q. A distant observer located on the symmetry axis of the galaxy sees an image with circular isophotes and central surface brightness I_n. A second distant observer, observing the galaxy from a line of sight that is inclined by an angle i to the symmetry axis, sees an image with elliptical isophotes with axis ratio $Q < 1$ and central surface brightness I_0.

(a) What is the relation between I_0, I_n, and Q? Hint: the answers are different for oblate ($q < 1$) and prolate ($q > 1$) galaxies.

(b) What is the relation between q, Q, and i?

(c) Assuming that galaxies are oriented randomly, what fraction are seen from a line of sight that lies within 10° of the symmetry axis? From within 10° of the equatorial plane?

Short problems

4. The central surface brightness of a globular cluster is 17 mag/arcsec^2. What is the covering fraction of stars (i.e. the probability that a randomly selected straight line will interesect a star)? You may assume that most of the light comes from stars similar to the Sun.

5. If two disk galaxies similar to the Milky Way collide face-on at a relative speed of $1000\,\mathrm{km\,s^{-1}}$, roughly how many stars will collide?

6. For this problem, idealize the Galactic disk as a rigid body, somewhat like a rotating plate. Suppose that the disk is tumbling, that is, there is a component of angular velocity along an axis that lies in the disk. Hierarchical models of galaxy formation suggest that this kind of tumbling motion is common, at an angular speed of order 1 radian per 10^{10} yr. Could this motion be detected by the proposed SIM Planetquest mission, which has a proper-motion accuracy of 1 microarcsecond/yr?

7. What is the inclination between the plane of the ecliptic and the plane of the Galaxy?

8. Suppose that $N \gg 1$ measurements of the position of a star relative to several quasars are made at equal intervals Δt, all with the same uncertainties. The total timespan or baseline of the observations is therefore $T = N\Delta t$. Assuming that the trigonometric parallax is negligible, how does the accuracy of the resulting proper motion determination scale with the length of the baseline; in other words, if the accuracy of the proper motion is $\epsilon \propto T^{-a}$, what is a?

9. (this is problem 2.7 of BT) Astronauts orbiting an unexplored planet find that (i) the surface of the planet is precisely spherical and centered on $r = 0$; and (ii) the potential exterior to the planetary surface is $\Phi = -GM/r$ exactly, that is, there are no non-zero multipole moments other than the monopole. Can they conclude from these observations that the mass distribution in the interior of the planet is spherically symmetric? If not, give a simple example of a non-spherical mass distribution that would reproduce the observations.

10. What is the difference between "bootstrap" and "jack-knife"?