What is Quantum Field Theory?

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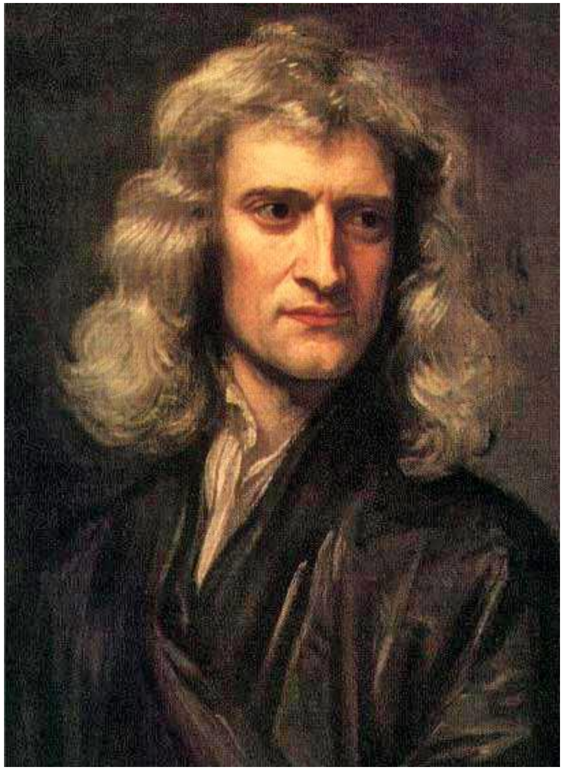
IAS

Quantum Field Theory

- Quantum field theory is the natural language of physics:
  - Particle physics
  - Condensed matter
  - Cosmology
  - String theory/quantum gravity
- Applications in mathematics especially in geometry and topology
- Quantum field theory is the modern calculus
  - Natural language for describing diverse phenomena
- Enormous progress over the past decades, still continuing
- Indications that it should be reformulated

Calculus vs. Quantum Field Theory

- New mathematics
- Motivated by physics (motion of bodies)
- Many applications in mathematics, physics and other branches of science and engineering
- Sign that this is a deep idea
- Calculus is a mature field. Streamlined – most books and courses are more or less the same



Calculus vs. Quantum Field Theory

- New mathematics (in fact, not yet rigorous)
- Motivated by physics (particle physics, condensed matter)
- Many applications in mathematics and physics
- Sign that this is a deep idea
- QFT is not yet mature – books and courses are very different (different perspective, order of presentation)
- Indications that we are still missing big things – perhaps QFT should be reformulated

Presentations of quantum field theory

- Traditional. Use a Lagrangian…
- More abstractly, operators and their correlation functions
  - In the traditional Lagrangian approach this is the outcome
- Others?

Abstract presentation of QFT

Use a collection of operators with their correlation functions

- These include point operators (local operators), line operators, surface operators, etc.
  - We do not distinguish between operators, observables and defects (this depends on the spacetime orientation)
  - Their correlation functions should be well defined – mutually local
- Place the theory on various manifolds
  - This can lead to more choices (parameters)
  - More consistency conditions
  - Can we recover this information from local measurements?

Lagrangian

- Natural starting point – quantize a classical system
  - Functional integral
  - Canonical quantization
  - Others
- Need to regularize (e.g. a lattice) to make it meaningful. Then need to prove the existence of
  - the continuum limit
  - the large volume limit

Lagrangian

- Pick "matter fields"
  - Target space of the matter fields
  - Non-derivative couplings (potential, Yukawa, …)
  - Metric on the target space
  - Wess-Zumino terms
- Pick a gauge group G. It can act on the matter fields.
  - Kinetic term
  - Chern-Simons term
  - Dependence on the global structure of G
  - Theta parameters (including discrete theta parameters)

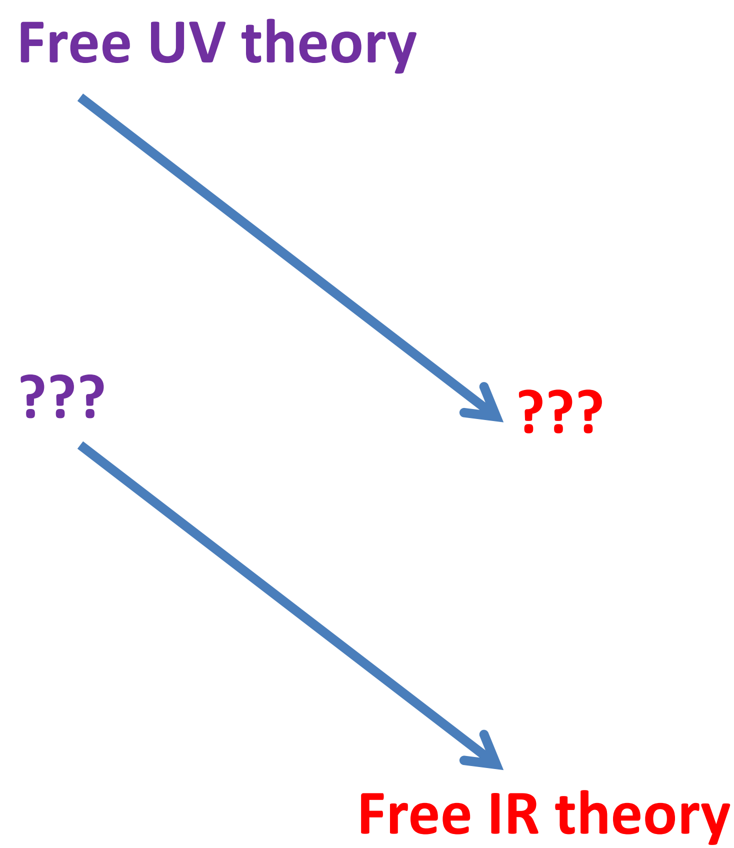
Lagrangian

Questions:

- Do we know all such constructions?
- Do we know all consistency conditions?
- Duality: When do different such constructions lead to the same theory?
- More below

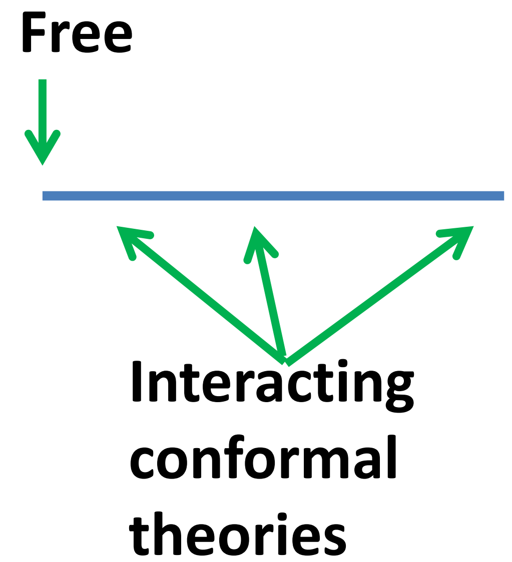
Lagrangians are meaningful and useful when they are weakly coupled

- Free UV theory perturbed by a relevant operator, e.g.
  - Asymptotically free 4d gauge theory (e.g. QCD)
  - . . .
- Free IR theory perturbed by an irrelevant operator, e.g.
  - 4d QED
  - Chiral theory of pions
  - . . .



Lagrangians are meaningful and useful when they are weakly coupled

- Family of conformal theories connected to a free theory, e.g.
  - 4d N=4 super Yang-Milles
  - 3d Chern-Simons theory (large level)
  - 2d sigma model with Calabi-Yau target space
  - . . .



Lagrangians are not good enough

- Strong coupling
- Not used in exact solutions
- Duality
- Theories without Lagrangian

Lagrangians are not good enough

Strong coupling

Given a weakly coupled theory, described by a Lagrangian, there is no clear recipe to analyze it at strong coupling.

This is one of the main challenges in Quantum Field Theories.

Examples:

- Gapped systems, e.g. with confinement in 4d gauge theory
- Interacting Conformal Field Theories
- Strongly coupled Topological Field Theories like 3d Chern-Simons theory with small k
- . . .

Lagrangians are not good enough

Not used in exact solutions

Rely on the more abstract presentation.

Use consistency to constrain the answer.

- Bootstrap methods in Conformal Field Theories use associativity of the operator product expansion…
- Integrable systems are solved using a large symmetry .
- Rational conformal field theories are solved using the combination of a large symmetry and consistency.
- . . .

Lagrangians are not good enough

Not used in exact solutions

Rely on the more abstract presentation.

Use consistency to constrain the answer.

- Supersymmetric theories are analyzed using holomorphy/BPS/chiral/topological observables.
- Some TQFT are solved using simple rules (not the Lagrangian).
- Some field theories are analyzed by embedding them in String Theory.
- Amplitudes are analyzed using consistency like unitarity.

Lagrangians are not good enough

Duality

Duality is intrinsically quantum mechanical.

It cannot be seen at weak coupling.

- Free field theories, e.g. $R \to \frac{1}{R}$, 3d Maxwell v. compact scalar, 4d Maxwell (small to large $\hbar$)
- Interacting conformal theories labeled by a dimensionless parameter. Here duality relates weak (described by a Lagrangian) and strong coupling, e.g.
    - 4d N=4 SYM
    - 2d sigma models with mirror Calabi-Yau target spaces

Lagrangians are not good enough

Duality

Duality is intrinsically quantum mechanical.

It cannot be seen at weak coupling.

- Two different weakly coupled theories that flow to the same strongly coupled IR fixed point (universality), e.g.
    - 4d N=1 SYM
    - 2d linear sigma models flowing to mirror Calabi-Yau sigma models
- A weakly coupled theory flows at strong coupling to another (dual) weakly coupled theory, e.g. 4d N=1 SYM

Lagrangians are not good enough

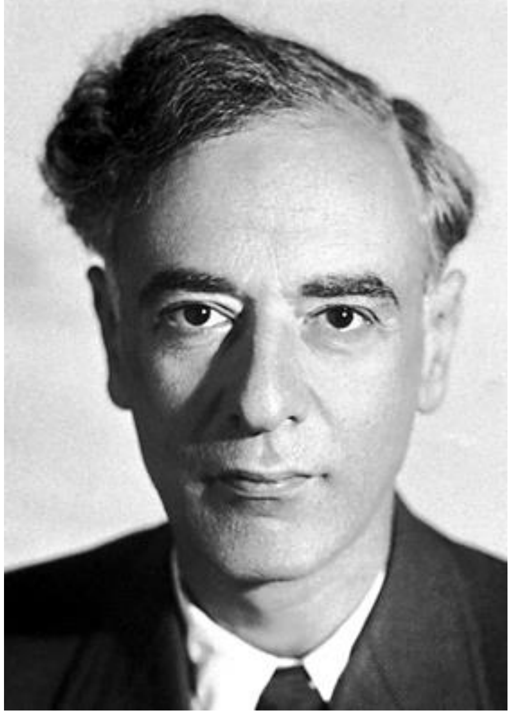
Theories without Lagrangians

- Theories without (Lorentz invariant) Lagrangians
  - Chiral fermions on the lattice
  - Selfdual bosons (even free) – intrinsically quantum mechanical
- Theories that are not in the strong coupling limit of a weakly coupled theory (which can be described by a Lagrangian) – no semi-classical limit
  - Some nontrivial fixed points in 4d
  - Nontrivial fixed points in 5d and 6d
  - As the list of such theories keeps growing and their marvelous properties are being uncovered, it is wrong to dismiss them.

Suggest that QFT should be reformulated

- Lagrangians are not good enough
  - Not useful at strong coupling
  - Not used in exact solutions
  - Duality – more than one Lagrangian of the same theory
  - Sometimes no Lagrangian
- Not mathematically rigorous
- Extensions of traditional local QFT
  - Field theory on a non-commutative space
  - Little string theory
  - Others?

Clarifying a historical comment



Landau (1960):

“We are driven to the conclusion that the Hamiltonian method for strong interaction is dead and must be buried, although of course with deserved honor."

- Hamiltonian and not Lagrangian
- He thought that the theory is wrong; not just its formulation

Here:

- The theory and its current formulations are right
- Hamiltonian exists
- We should look for better formulations

How should we think about it?

Of course, I do not know!

- A good place to start is Topological Quantum Field Theory
  - It is simple
  - Enormous progress during the recent decades
- But what about theories with local degrees of freedom?
- Suggest to look at topological observables in a non-topological field theory
- Theories with global symmetries have topological observables.
- Will follow A. Kapustin, NS, arXiv:1401.0740; D. Gaiotto, A. Kapustin, NS, B. Willett, arXiv:1412.5148

Global vs. Local Symmetries

Global

- Intrinsic
- Can be accidental in IR – approximate
- Classify operators
- Can be spontaneously broken
- If unbroken can classify states
- Useful in classifying phases
- ‘t Hooft anomalies
- Not present in a theory of gravity

Local (gauge)

- Ambiguous – duality
- Can emerge in IR – exact
- All operators are invariant
- Not really a symmetry
- Hence it cannot be broken (Higgs description meaningful only at weak coupling)
- Cannot be anomalous
- Appears essential in formulating the Standard Model and in Gravity

Global Symmetries

- Ordinary global symmetries
  - Act on local operators
  - The charged states are particles
- Generalized global symmetries
  - The charged operators are lines, surfaces, etc.
  - The charged objects are strings, domain walls, etc.
- It is intuitively clear and many people will feel that they have known it. We will make it more precise and more systematic.
- We will repeat all the things that are always done with ordinary symmetries
- The gauged version of these are common in physics and in mathematics

Ordinary global symmetries

- Generated by operators associated with co-dimension one manifolds M

$$U_g(M)$$

  $g \in G$ a group element

- The correlation functions of $U_g(M)$ are topological!
- Group multiplication $U_{g_1}(M)U_{g_2}(M) = U_{g_1 g_2}(M)$
- Local operators $O(p)$ are in representations of G

$$U_g(M)O_i(p) = R_i^j(g)O_j(p)$$

  where M surrounds p (Ward identity)

- If the symmetry is continuous,

$$U_g(M) = e^{i \int j(g)}$$

$j(g)$ is a closed form current (its dual is a conserved current).

q-form global symmetries

- Generated by operators associated with co-dimension $q+1$ manifolds M (ordinary global symmetry has $q=0$)
$$U_g(M)$$
$g \in G$ a group element
- The correlation functions of $U_g(M)$ are topological!
- Group multiplication $U_{g_1}(M)U_{g_2}(M) = U_{g_1 g_2}(M)$. Because of the high co-dimension the order does not matter and G is Abelian.
- The charged operators $\mathrm{V}(L)$ are on dimension q manifolds L. Representations of G – Ward identity
$$U_g(M)V(L) = R(g)V(L)$$
where M surrounds L and $R(g)$ is a phase.

q-form global symmetries

If the symmetry is continuous,

$$U_g(M) = e^{i \int j(g)}$$

$j(g)$ is a closed form current (its dual is a conserved current).

Compactifying on a circle, a q-form symmetry leads to a q-form symmetry and a $q-1$-form symmetry in the lower dimensional theory.

- For example, compactifying a one-form symmetry leads to an ordinary symmetry in the lower dimensional theory.

No need for Lagrangian

- Exists abstractly, also in theories without a Lagrangian
- Useful in dualities

q-form global symmetries

As with ordinary symmetries:

- Selection rules on amplitudes
- Couple to a background classical gauge field (twisted boundary conditions)
  - Interpret 't Hooft twisted boundary conditions as an observable in the untwisted theory
- Gauging by summing over twisted sectors (like orbifolds)
  - New parameters in gauge theories – discrete θ-parameters (like discrete torsion) [Aharony, NS, Tachikawa]
  - Dual theories often have different gauge symmetries. But the global symmetries must be the same
  - Non-trivial tests of duality including non-BPS operators

q-form global symmetries Characterizing phases

- In a confining phase the electric one-form symmetry is unbroken.
  - The confining strings are charged and are classified by the unbroken symmetry.
  - Area law of Wilson loop – order parameter $\langle W \rangle$ vanishes when it is large – symmetry unbroken.
  - Ordinary global symmetry after compactification. It is unbroken [Polyakov, Susskind].

q-form global symmetries
Characterizing phases

- In a Higgs or Coulomb phase the electric one-form symmetry is broken.
  - Renormalizing the perimeter law to zero, the large size limit of $\langle W \rangle$ is nonzero – vev "breaks the symmetry."
  - No strings
  - Ordinary global symmetry after compactification. It is broken [Polyakov, Susskind].

q-form global symmetries

Low energy behavior when broken

- A continuous broken symmetry leads to a massless Nambu-Goldstone boson.
  - Example: a photon in a Coulomb phase (cf. [Rosenstein, Kovner])

$$\langle 0|F_{\mu\nu}|\epsilon,p\rangle = (\epsilon_\mu p_\nu - \epsilon_\nu p_\mu)e^{i\,px}$$

- A discrete broken symmetry leads to a TQFT.
  - Example: a spontaneously broken $\boldsymbol{Z}_n$ one-form symmetry leads to a $\boldsymbol{Z}_n$ gauge theory

Higher Form SPT Phases

Consider a system with an unbroken symmetry with anomalies.

- 't Hooft anomaly matching forces excitations (perhaps only topological excitations) in the bulk, or only on the boundary.
- Symmetry Protected Topological Phase
- Domain walls between vacua in different SPT phases must have excitations.
- For examples, $\boldsymbol{N}=1$ SUSY $SU(N)$ gauge theory has N vacua in different SPT phases (the relevant symmetry is the one-form $\boldsymbol{Z}_N$ symmetry) and hence there is $U(k)_N$ on the domain walls between them [Dierigl, Pritzel]. This $U(k)_N$ was originally found by [Acharya, Vafa] using string considerations.

Conclusions

- Higher form global symmetries are ubiquitous.
- They help classify
  - extended objects (strings, domain walls, etc.)
  - extended operators/defects (lines, surfaces, etc.)
- As global symmetries, they must be the same in dual theories.
- They extend Landau’s characterization of phases based on order parameters that break global symmetries.
  - Rephrase the Wilson/’t Hooft classification in terms of broken or unbroken one-form global symmetries.
- Anomalies
  - ‘t Hooft matching conditions
  - Anomaly inflow
  - Degrees of freedom on domain walls

Thank you for your attention