Think Globally, Act Locally

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Quantum Fields beyond Perturbation Theory, KITP 2014

Ofer Aharony, NS, Yuji Tachikawa, arXiv:1305.0318
Anton Kapustin, Ryan Thorngren, arXiv:1308.2926, arXiv:1309.4721
Anton Kapustin, NS, arXiv:1401.0740

Coupling a QFT to a TQFT

In many cases such a coupling affects the local structure, e.g.:

- Free matter fields coupled to a Chern-Simons theory in *3d*.
- Orbifolds in *2d* CFT

Often the local structure is not affected, but there are still interesting consequences: spectrum of line and surface operators, local structure after compactification…

In the paper, many examples in various dimensions both in the lattice and in the continuum.

Here, we focus on one *4d* example in the continuum.

Line operators

Some line operators are boundaries of surfaces.

1. If the results depend on the geometry of the surface, this is not a line operator.
2. In some cases the dependence on the surface is only through its topology.
3. Genuine line operators are independent of the surface.

$$W(\gamma)\mathcal{T}(\gamma') = e^{2\pi i \ell(\gamma,\gamma')/n}\mathcal{T}(\gamma')W(\gamma)$$

Here at least one of the line operators needs a surface. Hence, the apparent lack of locality.

Genuine line operators of the form $W(\gamma)^n\mathcal{T}(\gamma)^m$ with appropriate n and m are relatively local.

Higher-form global symmetries

Continuous q-form global symmetry – transformation with a closed q-form $\epsilon^{(q)}$ ($q = 0$ is an ordinary global symmetry with constant ϵ).

Discrete q-form global symmetry $\int \epsilon^{(q)} \in 2\pi\mathbb{Z}$.

Example:

An ordinary gauge theory with group G is characterized by transition functions $g_{ij} \in G$ with $g_{ij}g_{jk}g_{ki} = 1$.

If no matter fields transforming under C , the center of G, 1-form discrete global symmetry $g_{ij} \to h_{ij}g_{ij}$ with $h_{ij} \in$ C and $h_{ij}h_{jk}h_{ki} = 1$.

Higher-form global symmetries

1-form discrete global symmetry $g_{ij} \to h_{ij} g_{ij}$ with $h_{ij} \in$ C and $h_{ij} h_{jk} h_{ki} = 1$.

When compactified on a circle, this 1-form global symmetry leads to an ordinary global symmetry C.

It is common in thermal physics – the Polyakov loop is the order parameter for its breaking.

We gauge C by relaxing $h_{ij} h_{jk} h_{ki} = 1$ (analog of gauging an ordinary global symmetry by letting ϵ depend on position).

The resulting theory is an ordinary gauge theory of $G/$C.

A simple TFT – a *4d* $\mathbb{Z}_n$ gauge theory

[Maldacena, Moore, NS; Banks, NS]

1. Can describe as a $\mathbb{Z}_n$ gauge theory.
2. Can introduce a compact scalar $\phi \sim \phi + 2\pi$ and a $U(1)$ gauge symmetry $\phi \to \phi + n\lambda$ (with $\lambda \sim \lambda + 2\pi$).
3. Can also introduce a $U(1)$ gauge field A with Lagrangian

$$\frac{i}{2\pi} H \wedge (d\phi + nA)$$

   H is a 3-form Lagrange multiplier. $U(1) \to \mathbb{Z}_n$ manifest.

4. Can dualize ϕ to find

$$\frac{in}{2\pi} B \wedge F$$

   with $F = dA$ and $H = dB$.

4d $\mathbb{Z}_n$ gauge theory

$$\frac{in}{2\pi} B \wedge F$$

5. Can dualize A to find

$$\frac{i}{2\pi} F \wedge (d\hat{A} + nB)$$

F is a 2-form Lagrange multiplier.

Gauge symmetry:

$$\hat{A} \rightarrow \hat{A} + d\hat{\lambda} - n\Lambda$$
$$B \rightarrow B + d\Lambda$$

6. Can keep only $\hat{A}$ with its gauge symmetries
7. Locally, can fix the gauge $\hat{A} = 0$ and have only a $\mathbb{Z}_n$ 1-form gauge symmetry.

Observables in a *4d* $\mathbb{Z}_n$ gauge theory

$$\frac{in}{2\pi} B \wedge F$$

Two kinds of Wilson operators

$$W_A(\gamma) = e^{i \oint_\gamma A}$$

$$W_B(\Sigma) = e^{i \oint_\Sigma B}$$

Their correlation functions

$$\langle W_B(\Sigma) W_A(\gamma) \rangle = e^{2\pi i \ell(\Sigma,\gamma)/n}$$

No additional ('t Hooft) operators using $\hat{A}$ or ϕ – they are trivial.

An added term in a *4d* $\mathbb{Z}_n$ gauge theory

$$\frac{i}{2\pi}F\wedge(d\hat{A}+nB) \qquad\qquad \frac{in}{2\pi}B\wedge F$$

In any of the formulations we can add the term [Gukov, Kapustin; Kapustin, Thorngren]

$$\frac{ipn}{4\pi}B\wedge B=\frac{ip}{4\pi n}d\hat{A}\wedge d\hat{A}$$

With the modified gauge transformations:

$$\hat{A}\to\hat{A}+d\hat{\lambda}-n\Lambda$$

$$B\to B+d\Lambda$$

$$A\to A-p\Lambda$$

Consistency demands $\frac{pn}{2}\in\mathbb{Z}$ and $p\sim p+2n$.

From $SU(n)$ to $SU(n)/\mathbb{Z}_n$ gauge theories

$G = SU(n)$ gauge theory

- The Wilson line W is a genuine line operator
- The ‘t Hooft line $\mathcal{T}$ needs a surface (the Dirac string). Hence, the nonlocality in the commutation relations

$$W(\gamma)\mathcal{T}(\gamma') = e^{2\pi i \ell(\gamma,\gamma')/n}\mathcal{T}(\gamma')W(\gamma)$$

- If no matter fields charged under the $\mathbb{Z}_n$ center:
  - Only the topology of the surface is important. (Like disorder operator in the Ising model with vanishing magnetic field.)
  - Global 1-form discrete symmetry C$= \mathbb{Z}_n$

From $SU(n)$ to $SU(n)/\mathbb{Z}_n$ gauge theories

Next, we gauge the 1-form symmetry C= $\mathbb{Z}_n$ to find an $SU(n)/\mathbb{Z}_n$ g.t.

Can use any of the formulations of a C= $\mathbb{Z}_n$ gauge theory.

Extend $SU(n)$ to $U(n)$ by adding $\hat{A}$ and impose the 1-form gauge symmetry $\hat{A} \to \hat{A} - n\,\Lambda$ to remove the added local dof.

We can also add a new term in this theory – a discrete θ-term

$$\frac{ip}{4\pi n} d\hat{A} \wedge d\hat{A} \qquad \frac{pn}{2} \in \mathbb{Z} \qquad p \sim p + 2n$$

(Interpreted as an $SU(n)/\mathbb{Z}_n$ theory, it is identified with the Pontryagin square ${w_2}^2$ of the gauge bundle [Aharony, NS, Tachikawa]. Here, a manifestly local expression for it.)

$SU(n)/\mathbb{Z}_n$ gauge theory – operators

Use e.g.

$$\frac{i}{2\pi} F \wedge (d\hat{A} + nB) + \frac{ipn}{4\pi} B \wedge B \qquad \frac{pn}{2} \in \mathbb{Z}$$

$$B \to B + d\Lambda$$

$$\hat{A} \to \hat{A} - n\Lambda$$

$$F \to F - pd\Lambda$$

The surface operator

$$e^{i \oint_\Sigma B} = e^{-\frac{i}{n} \oint_\Sigma d\hat{A}}$$

measures the ‘t Hooft magnetic flux (w_2 of the bundle) through Σ.

It is a manifestly local expression – integral of a local density.

(More complicated expression for torsion cycles.)

$SU(n)/\mathbb{Z}_n$ gauge theory

Wilson $W(\gamma) = \left(W_f^{SU(n)}(\gamma) e^{\frac{i}{n}\oint_\gamma \hat{A}} \right) e^{i\int_\Sigma B}$ $\partial\Sigma = \gamma$

't Hooft $\mathcal{T}(\gamma) = e^{i\oint_\gamma A + ip\int_\Sigma B}$

The dependence on Σ is topological.

Genuine line operators (dyonic) $\mathcal{T}(\gamma) W(\gamma)^{-p}$

The parameter p is a discrete θ-parameter [Aharony, NS, Tachikawa].

- It labels distinct $SU(n)/\mathbb{Z}_n$ theories.
- It can be understood either as a new term in the Lagrangian $\frac{i\,p}{4\pi n} d\,\hat{A} \wedge d\,\hat{A}$ (it is the Pontryagin square w_2^2 of the gauge bundle), or in terms of the choice of genuine line operators.

Similarities to *2d* Orbifolds

- In orbifolds we start with a system with a global symmetry.
  - Background gauge field – twisted boundary conditions
  - Gauging the symmetry by summing over these sectors
  - This removes operators and includes others
  - Discrete torsion: different coefficients for the sectors
- In *4d* gauge theories the global symmetry is a 1-form symmetry
  - Twisted sectors are bundles of a quotient of the gauge group
  - Gauging the 1-form global symmetry – summing over sectors
  - This changes the line and surface operators
  - Discrete θ-parameter – different coefficients for the sectors

Conclusions

It is interesting to couple an ordinary QFT to a TQFT.

- The resulting theory can have a different local structure. More generally, it has different line and surface operators.
- When placed on manifolds other than $\mathbb{R}^D$ the effects are often more dramatic.
- Such a coupling to a TQFT can describe the difference between a theory with gauge group G and a theory with gauge group G/C, e.g. $SU(n)$ and $SU(n)/\mathbb{Z}_n$.
- It allows us to describe easily additional coupling constants (like discrete θ-parameters) as integrals of local densities.
- Such added TQFT are crucial in duality; e.g. in *2d* Ising, *4d* lattice gauge theories, *3d* and *4d* SUSY theories (not in this talk).

Thank you for your attention