Symmetries Come and Go

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50 Years of Quarks & Color

Common in physics

- Today’s signal is tomorrow’s background
- Today’s new deep concepts are tomorrow’s derived consequences

Examples:

- The periodic table ← Quantum Mechanics ← ???
- Kepler’s laws ← Newton’s gravity ← General relativity ← String theory?
- Many others

This talk: color and gauge symmetry

Physicists love symmetries

- Crystallography
- Lorentz
- Flavor $SU(2),\ SU(3)$
  - Consequence of light quarks. Quarks are deep.
- Color
  - Gauge symmetry is deep

Gauge symmetry is deep

- Largest symmetry (a group for each point in spacetime)
- Useful in making the theory manifestly Lorentz invariant, unitary and local (and hence causal)
- Appears in
  - Maxwell theory, the Standard Model
  - General Relativity
  - Many condensed matter systems
  - Deep mathematics (fiber bundles)

But

- Because of Gauss law the Hilbert space is gauge invariant. (More precisely, it is invariant under small gauge transformation; large gauge transformations are central.)
- Hence: gauge symmetry is not a symmetry.
  - It does not act on anything.
- A better phrase is gauge redundancy.

Gauge symmetry can appear trivial

- Start with an arbitrary system and consider some transformation, say a $U(1)$ phase rotation on some fields. It is not a symmetry.
- Introduce a Stueckelberg field $\phi(x)$, which transforms under the $U(1)$ by a shift.
- Next, multiply every non-invariant term by an appropriate phase $e^{i\,\phi(x)}$, such that the system has a local $U(1)$ gauge symmetry.
- Clearly, this is not a fundamental symmetry.

Gauge symmetries cannot break

- Not a symmetry and hence cannot break
- For spontaneous symmetry breaking we need an infinite number of degrees of freedom transforming under the symmetry. Not here.
- This is the deep reason there is no massless Goldstone boson.
- For weakly coupled systems (like Landau-Ginsburg theory of superconductivity, or the weak interactions) the language of spontaneous symmetry breaking is perfectly appropriate and extremely useful [Stueckelberg, Anderson, Brout, Englert, Higgs, …].

Global symmetries can emerge as accidental symmetries at long distance.

Then they are approximate.

Exact gauge symmetries can be emergent.

Examples of emergent gauge symmetry

- The example with the added field $\phi(x)$ above.
- Some σ-models can be described using gauge fields (e.g. the $\boldsymbol{CP}^N$ σ-model) and then they become dynamical.
  - This is common in condensed matter physics.
- Simple dualities
  - In *3d* a compact scalar is dual to Maxwell theory, whose gauge symmetry is emergent.
  - In *4d* Maxwell theory is dual to a magnetic Maxwell theory.

Duality in interacting field theories $\boldsymbol{N} = 4$ supersymmetry

- This is a scale invariant theory characterized by a gauge group G and a complex coupling constant $\tau = \frac{\theta}{2\pi} + \frac{4\pi}{g^2} i$ for each factor in G.
- For simply laced G the theory with τ is the same as with $\tau + 1$ (shift θ by 2π) and the same as with $-1/\tau$ (generating $SL(2, \boldsymbol{Z})$)…

Duality in interacting field theories $N = 4$ supersymmetry

- The duality is an exact equivalence of theories.
  - Same spectrum of states
  - Same spectrum of operators
  - Same correlation functions
- $\tau \to -1/\tau$ maps strong to week coupling.
- More technical:
  - This ignores certain global issues.
  - Some modifications for non-simply laced G.

Duality in interacting field theories
$N = 4$ supersymmetry

- The gauge symmetry of the dual description is emergent!
- Which of the two gauge symmetries is fundamental?
- Which set of gluons are elementary?
- Perhaps neither gauge symmetry is fundamental.

Interacting gauge theories

Start at short distance with a gauge group G. Depending on the details we end up at long distance with:

- IR freedom – a free theory based on G (same theory)
- A nontrivial fixed point. Interacting conformal field theory – no notion of particles.
- An approximately free (IR free) theory of bound states
- An empty theory – gap (possibly topological order)

All these options are realized in QCD for various numbers of flavors. (The approximately free theory is a theory of pions.)

Duality in interacting field theories $\boldsymbol{N} = 1$ supersymmetry

Here there is also a dual description based on another gauge theory with gauge group $\tilde{G}$ (magnetic theory).

- When the original theory (electric) is IR free the dual theory is strongly coupled.
- When the electric theory flows to a non-trivial fixed point so is the magnetic theory. The two theories are in the same universality class…

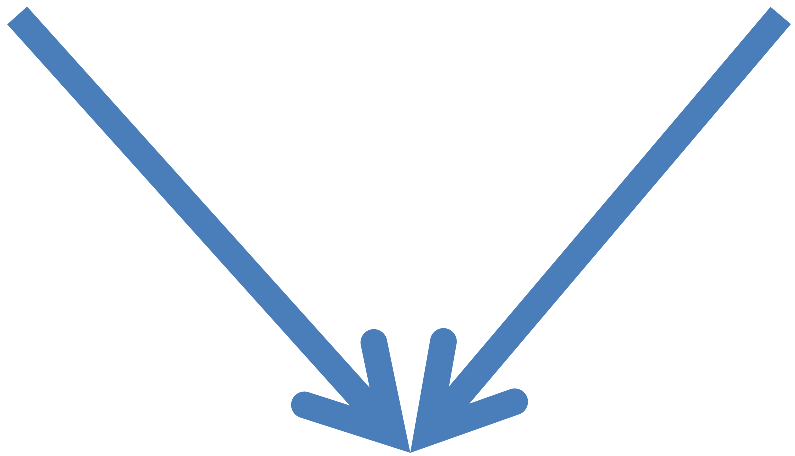
Duality in interacting field theories
$\boldsymbol{N} = 1$ supersymmetry

Electric theory

G

Magnetic theory

$\tilde{G}$



Non-trivial IR fixed point

Duality in interacting field theories
$\boldsymbol{N} = 1$ supersymmetry

A third option:

Electric theory

Based on G

Approximately free theory (IR free)

Based on $\tilde{G}$

Duality in interacting field theories $\boldsymbol{N} = 1$ supersymmetry

In the UV an asymptotically free theory based on G

In the IR an IR free theory based on $\tilde{G}$

At low energies QCD has pions. This theory has a non-Abelian gauge theory.

- The gauge fields of $\tilde{G}$ are composite.
- Their gauge symmetry is emergent.
- There is no ambiguity in the IR gauge symmetry – approximately free massless gauge fields.

Duality in interacting field theories $\mathcal{N} = 1$ supersymmetry

In all these cases

- As the original electric theory becomes more strongly coupled, the magnetic theory becomes more weakly coupled.
- When the electric theory confines the magnetic theory exhibits spontaneous gauge symmetry breaking (meaningful because it is weakly coupled).
- Clear physical demonstration of dynamical properties of gauge theories.

Many more examples of emergent gauge symmetries

- Many known examples based on different
  - gauge groups and matter representations
  - spacetime dimensions
  - amount of supersymmetry
- They exhibit rich physical phenomena.
- They lead to interesting mathematics (many applications).
- Duality and emergent gauge symmetry are ubiquitous.

Emergent general covariance and emergent spacetime

- So far we discussed duality between two field theories
- String-string duality
  - T-duality
  - S-duality
  - U-duality
- String-fields duality
  - Matrix models for low dimensional string theories
  - BFSS M(atrix) model
  - AdS/CFT
  - More generally gauge-gravity duality

Conclusions

- Gauge symmetries can come and go.
- They can emerge.
- It is often convenient to use them to make the description manifestly Lorentz invariant, unitary and local.
  - But there can be different such descriptions.
- Gauge symmetry is not fundamental.
- Look for a formulation of field theory that makes the duality manifest.
- We should not be surprised by duality!