The Quantum Spacetime

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Abstract

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1 Opening

It is a great pleasure to be able to give this talk, to this illustrious audience. I will try to give a (somewhat biased) overview of quantum gravity, from generalities to more current developments in string theory.

2 Classical spacetime dynamics

Our current view of spacetime is based on the theory of general relativity, which states that spacetime is a dynamical object. It can support propagating "vibrations", or gravity waves.

General relativity had two surprising predictions: Black holes and the expanding universe. These predictions were so surprising that even Einstein had trouble with them. In fact, Einstein said to Lemaître (mabye here in Brussels): "Your math is correct, but your physics is abominable". I like this phrase because it is similar to what string theorists are sometimes told.

It is interesting that the evidence for these more surprising aspects of general relativity came earlier, and is stronger, than that for the more straightforward gravity waves.

3 Quantum spacetime

Since nature is quantum mechanical, and spacetime is part of the dynamics, we should quantize spacetime. For example, you can think about the gravitational field of quantum superposition of a particle at two different locations.

Quantizing linearized gravity waves is as easy as quantizing the free electromagnetic field. It is just a collection of harmonic oscillators. But the theory is not free. Including interactions perturbatively, one finds the unique structure of General Relativity by postulating that gravitons interact with the energy of gravity waves in a self consistent fashion. In other words, one can derive general relativity by postulating relativistic massless spin two particles interacting in a self consistent way.

Also if we have a curved background we can quantize fields moving in it, including the quantization of the gravitational field.

4 Two surprising predictions

This approximate approach gives two surprising predictions:

1) Black holes emit Hawking radiation. They have a temperature and an entropy.

2) Inflation produces the primordial fluctuations.

Both change the classical behavior of the system in surprising and physically important ways.

Experiments have essentially confirmed the inflationary predictions. This can be viewed as a confirmation of quantum gravity. Though, only of the non-dynamical modes of the geometry which are modified by the scalar field (or inflationary clock). Seeing primordial, or inflation generated, gravity waves would be a more direct test that spacetime geometry should be quantized.

Again, quantum effects lead to surprises. We have quantum mechanics at the longest observable distances!. And they are crucial for understanding the universe. Without them we would have a uniform universe. With them we have an essentially unique quantum initial state, and the complexity of the world arises through the measurement process or decoherence. At least for the fluctuations[1].

[1] Namely, currently we cannot predict the values of the constants of nature: number of gauge groups, ranks, gauge couplings, other couplings, etc. However, given these, the rest of the properties of the universe can be computed from the quantum state produced by inflation. Primordial fluctuations produce structure, which produces stars, etc...

5 Quantization at low energies

Let us go back to quantizing spacetime. The effective coupling among is proportional to the square of the typical energy of the interacting gravitons. $g_{eff}^2 = E^2 G_N = E^2 M_{pl}^2$ [2]. And, this is the size of the quantum gravity effects.

Gravity can be quantized as a low energy effective theory to any order in perturbation theory. We should introduce new parameters at each order in perturbation theory. This is similar to the Fermi theory of weak interactions. This works fine for low energies. It is OK during inflation, for example.

However, it fails completely when the energy is comparable to the Planck mass. Or when we require non perturbative precision. This is not just a problem of resuming the perturbation theory, but the fact that we have an infinite number of undetermined constants, which makes the theory ill defined.

6 UV completion in field theory

In quantum field theory, such effective field theories have a UV completion. In the case of the low energy field theory describing a condensed matter problem, this completion can be a lattice model, or the Schroedinger equation. For the fermi theory of weak interactions, it is the electroweak theory.

Going to short distances we find the local degrees of freedom which are the "fundamental" description of the theory.

7 UV completion in gravity ?

Could gravity be UV completed in a similar way?. We expect that the answer is no.

We start with a local classical lagrangian, so that one might expect a picture similar to the field theory one. But a big difficulty arises because we cannot devise a thought experiment that would allow us to explore short distances. If we collide high energy particles, we form black holes which get bigger as we increase the energy.

But the problem does not just appear when we go to the Planck scale, it also shows up at long distances in the form of information bounds, that are believed to hold for any quantum gravity theory. These bounds say that the total quantum information, or number of q-bits, we can store in a region of space is given by the area in Planck units. $S \leq \frac{\mathrm{Area}}{\mathrm{l_p^2}}$. You might be familiar with the fact that the entanglement entropy in quantum field theory has a similar expression, with l_p replaced by the UV cutoff. In QFT we can have q-bits in the interior which are not entangled with the exterior, so that total entropy in a region can be bigger than the area. This is not so in gravity. For example, if we have a dilute gas of particles, the entropy is naively expected to grow like the volume, thus, if have a sufficiently big gas of

[2] In units with $\hbar = c = 1$.

particles we would naively violate the entropy bound. So, what is wrong with such a large sphere of gas? Well, when it is about to violate the bound, it collapses into a black hole!.

8 Perturbative string theory

String theory is sometimes presented as UV completion of gravity, but it is not a completion in the same sense. It is a theory that perturbatively constructs the S-matrix.

We introduce a new length scale l_s, where new massive particles appear, and a dimensionless interaction constant g_s governing the quantum corrections. $G_N \sim g^2 l_s^2$.

These massive particles can be viewed as the oscillation modes of a string. There is a massless spin two particle, so we recover gravity at low energies. The amplitudes do not increase with energy and the quantum corrections are finite and calculable.

The simplest examples are ten dimensional and supersymmetric. It has no parameters. The coupling is the vacuum expectation value of some field.

9 Unification

In string theory we replace the classical notion of geometry by the new notion of stringy geometry, which is different at short distances. This stringy geometry has some very surprising features. For example, if we turn one of the dimensions into a circle of radius R, the physics is equivalent to that of a circle of radius $R' = l_s^2/R$. So, as we try to shrink a dimension, a new dimension grows to large size.

String theory also provides a unified description of matter and spacetime. Gravitons, gauge bosons, Higgs bosons, fermions, all come from the same string.

By going from ten to four dimensions on a compact six dimensional manifold we get gauge fields, chiral matter, and presumably the particle physics we see in nature.

10 Beyond perturbation theory?

There is a large amount of evidence that there is an exact theory whose approximation is perturbative string theory. This exact theory is usually also called "string theory", though it might not contain discernable strings, if we are at strong coupling.

At weak coupling, one can compute non-perturbative effects by considering D-branes. For example, the leading low energy correction to the scattering of gravitons in ten dimensions can be computed exactly.

The very strong coupling behavior of one theory is believed to be dual to other string theories, or to an eleven dimensional theory. And all string theories are connected by such dualities. These strong/weak coupling dualities give rise to mathematical identities which are very non-trivial. These can be checked to be true, giving evidence for the dualities.

11 Beyond perturbation theory

Many conceptually important problems in gravity seem to lie beyond perturbation theory:

1) Initial cosmological singularity, origin of big bang.

2) Graviton scattering at Planckian energies.

3) Describing black holes in a unitary fashion.

In fact, one gets confused at the very start. It is hard to define precise observables. In ordinary quantum mechanics, the position of a particle, or a spin projection, are well defined observables whose expectation values we can compute with arbitrary precision.

In gravity, nothing that can be measured by an observer living and measuring in a finite region seems to have this quality, since the observer has a finite number of degrees of freedom to store this information. Related to this, we can always have a quantum fluctuation of the metric where this region is completely absent.

In spacetimes with a simple asymptotic shape we can define precise observables. These include asymptotically flat space and asymptotically AdS space. Quantum fluctuations are suppressed at long distances and we can make precise measurements. Such as the measurement of the S-matrix in the flat space case. Knowing that they are well defined is nice. But, can we calculate them?.

12 Non perturbative Quantum Spacetimes

The existence of D-branes in string theory made it possible to discover some non-perturbative descriptions of some spacetimes. The examples include:

1) Matrix theory, which describes the S-matrix of some flat spacetimes.

2) Gauge gravity duality, describing AdS spaces (and other spacetimes with a timelike boundary).

In both cases we extract the spacetime physics by doing a computation in a well defined quantum mechanical system with no gravity. We will discuss this in more detail in the case of the gauge/gravity (or gauge/string) duality.

13 Hyperbolic space

A few words about a central player in this story. This is hyperbolic space the simplest and first example of a negatively curved space. We can see a nice drawing in figure 1. The fish all have the same proper size. The number of fish grows exponentially as we approach the boundary. It displays spatial version of inflation. This boundary is infinitely far away. Including time we get Anti-de-Sitter space. Thinking of the radial direction as time, we get de-Sitter (or ordinary inflation). We will discuss the Anti-de-Sitter case.

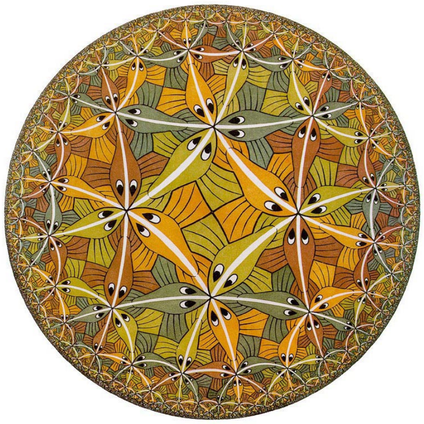

Figure 1:

14 Quantum Hyperbolic space.

We can get Anti-de-Sitter from string theory via a compactification on a suitable internal space. We have $AdS_d \times M^{10-d}$, with M^{10-d} a compact manifold. The string theory defines its quantum geometry perturbatively.

The gauge/gravity duality says that the physics of this quantum space is the same as the physics of an ordinary quantum field theory on the boundary. There are various examples. The simplest one involves a gauge theory similar to quantum chromodynamics, but with more supersymmetries.

The spacetime becomes classical and described by Einstein gravity when the gauge theory has a large number of colors and it is strongly coupled.

15 Emergent space

It is very easy to understand how the extra dimension emerges. Normally, in order to specify the state of an excitation we need to give its position. In a scale invariant theory, we also have to give its size. Thus, an excitation is a certain blob of some size. Blobs with different sizes are related by a scale transformation. Thus, we have an extra coordinate: the size of the blob.

In the interior, these blobs correspond to a particle of a fixed size that is at various positions along the extra direction. Changing the size of the boundary blob corresponds to changing the radial position of a particle in AdS. In figure 2 the red blob is smaller. It is described by a particle closer to the boundary. This picture is backed up by the conformal group representation theory.

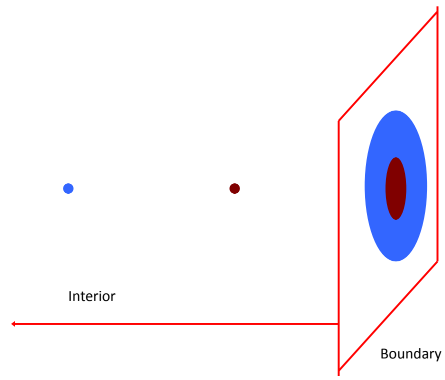

Figure 2:

16 Black holes in AdS

Any state in the interior has a corresponding state on the boundary. What about a black hole in the interior. It corresponds to a thermal system on the boundary. Its entropy, which is equal to the area of the horizon in gravity, corresponds to the ordinary statistical entropy in the boundary theory. The long distance dynamics of the black hole is related to hydrodynamics on the boundary theory. In fact, one can get the ordinary Navier Stokes equation from Einstein's equation by looking at the long wavelength excitations of such black holes. This picture makes it possible to compute transport properties of strongly coupled theories (see talks by Horowitz and Sachdev). A wave falling into the black hole corresponds to dissipation and thermalization on the boundary theory.

17 The Information problem

Since the information problem was the main historical reason for thinking about the gauge/gravity duality, let me describe it in some more detail, since thinking about it will probably lead to important new insights.

In a nutshell, the problem is the following. You form a black hole from a pure state. The black hole evaporates in a thermal fashion. In the end you got thermal radiation and the information about the pure state was lost.

This is incompatible with quantum mechanical evolution from the point of view of the outside observer.

18 The information problem

The perturbative gravity description seems to lose track of information. It seems lost to all orders in perturbation theory. This was most clearly seen in two dimensional models in the early 90's.

My opinion, which is not shared by everyone, is that this is not a problem by itself, since one needs non-perturbative accuracy in order to really test whether information is lost or not. Thus, to check whether information is lost one needs a non-perturbative method for computing the evolution of the state.

In conclusion, the Hawking argument does not show that information is lost, since it is not accurate enough. This can be understood without resorting to any duality. However, it does raise the important question of whether it is lost or not. The gauge gravity duality shows that information is not lost, since the boundary theory is unitary.

However, it does not give a clear bulk description for how information is recovered.

Also, one would like to get a description of the black hole interior from the point of view of the gauge theory.

19 Lessons

Let us mention some other lessons of this description of quantum spacetime.

- Spacetime is emergent. This means that spacetime is not fundamental, there isn't an operator in the theory which is the "space shape" operator, as there is a position operator in for a quantum mechanical particle. Note also that in string theory even the dimension of spacetime is an approximate concept. As we vary the parameters we can go from ten dimensional to eleven dimensions.

- Holographic bounds are obeyed, and are essential for making sense of the relationship.

- The boundary conditions at the AdS boundary specify the system, they specify the Lagrangian of the dual theory. Thus, in a sense, this is a realization of Mach's principle. The shape of space far away determines they physics of the system in the interior.

One might wonder whether any theory has a gravity dual, or which are the theories that have gravity duals. As we reduce the number of colors we might get a more and more strongly coupled theory in the bulk, one that deviates in a strong way from Einstein'st theory. If one is willing to accept such theories as possible states in quantum gravity, then one concludes that quantum gravity includes all quantum systems.

But, what are the particular systems that have a weakly coupled and Einstein gravity dual?. We have two necessary conditions, we need a large number of degrees of freedom, and strong interactions. How generic such theories are, is not understood. We just know a variety of examples. These include an example involving simple matrix harmonic oscillators with an anharmonic coupling term. This is an ordinary non-relativistic quantum mechanical system that has a gravity dual.

As in condensed matter theory one would like to classify all conformal field theories. This goal might be too ambitious, due to the large expected number of theories. Maybe the particular class with gravity duals with a macroscopic spacetime, could be classified. The greeks classified regular polyhedra. Now, we can certainly classify highly supersymmetric field theories. How should we think about the less supersymmetric ones...?

We saw that approaching the AdS boundary meant going to short distances in the boundary theory. But the AdS boundary is infinitely far away in the bulk. This implies that there is a UV/IR connection, which relates long distances in the gravity theory to short distances on the boundary theory.

There is close connection between the de-Sitter, inflationary computations, and the present anti de Sitter discussion. In fact, there is a simple analytic continuation that allows one to compute the inflationary perturbations from the anti-de-Sitter ones.

One can say that the scale invariance seen in cosmic fluctuations is related to the scale invariance of critical phenomena, that we encounter in conformal field theories. For inflating universes, at the moment this is just a hint, and not a complete and proper connection. But one would hope to be able to apply some of these ideas to cosmology.

In fact, the Euclidean field theory partition function is equal to the Hyperbolic space analog of the Hartle Hawking, or no boundary, wavefunction, originally proposed for the de Sitter case.

20 String theory and the real world.

As we mentioned, string theory has four dimensional vacua that have features similar to those of nature: gauge fields, chiral matter, inflation, etc. This I would call top down unification. In fact, there seem to be so many vacua[3] that one with the right cosmological constant is very likely to exist.

In addition, we have seen a kind of different kind of unification. In fact, the physics that governs QCD is the same as the one governing spacetime. The precise QCD string is not known, but there is a lot of evidence that it should be part of what we now call "string theory". These strings have certainly been experimentally observed. Let me make a historical analogy. Newton observed that the force that makes an apple fall is the same as the one moving the heavens. We can now say that the quantum spacetime, is the same "stuff" as the one that gives the apple its mass: the strong interactions.

[4]

[3] More properly, we should call them long lived metastable states.

[4] Let us make a comment of how to think about the application of the known examples of the gauge gravity duality to the study of quantum gravity, strong interactions and condensed matter. Let us suppose that, instead of physicists trying to understand nature, we were doctors trying to cure cancer. Now in order to learn about cancer, one might want to study fruit flies. Obviously, fruit flies and humans are different. In the same way the field theories that have gravity duals are not the same as the ones in nature. Nor the gravity theories that have known field theory duals are the ones in nature. (At least the currently understood ones). They are analogous to the fruit fly, they have components or aspects which we believe are the same as those in natural systems, but are easier to understand, and to "take apart". They are "model organisms"

21 Some unsolved problems

I would now like to close with some of the big challenges that we are facing in our field.

One important problem is to describe spacetimes with cosmological, or big bang, singularities. To describe, means to be able to give probabilities for what comes out of the singularity. Related to this, is the description of the interior of black holes. Also vaguely connected to this is the measure problem in eternal inflation (see Guth's talk).

It is also important to make some concrete prediction, that we could experimentally check for the spectrum of particles/inflation/susy/dark energy from string theory. The simple predictions of string theory involve high energies and are hard to check experimentally. Current experiments involve low energy physics and are hard to predict from the theory.

Hopefully we will have more surprises and unexpected predictions that will be simpler to check. Maybe predictions about questions that we now think are unrelated to quantum spacetime!.

for the study of quantum gravity or strongly interacting quantum systems.