Supersymmetric Gauge Theories in *3d*

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Intriligator and NS, arXiv:1305.1633

Aharony, Razamat, NS, and Willett, arXiv:1305.3924

3d SUSY Gauge Theories

- New lessons about dynamics of quantum field theory
- Since they can be obtained by studying a *4d* theory on a circle, they reflect properties of *4d* theories.
- Applications to condensed matter physics?
- New elements, which are not present in *4d*…

New elements in *3d N=2 SUSY*

- No asymptotic freedom bound on the number of matter fields.
- *U(1)* theories can exhibit interesting dynamics, hence we can examine the effect of Fayet-Iliopoulos terms.
- New SUSY coupling constants: Chern-Simons terms, real masses.
- New phases (topological)
- BPS (half-SUSY) particles; e.g. Skyrmions, vortices
- Vortex/monopole operators (analogs of twist fields in *2d* and ‘tHooft lines in *4d*). They cannot be written in a simple local fashion.

The Coulomb branch

- A *3d* N=2 gauge multiplet includes a scalar σ. When the theory is obtained by compactifying a theory on a circle it originates from A_4 (a Wilson line around the circle).
- The photon is dual to a compact scalar a.
- $X = e^{\sigma/g_3^2 + ia}$ (+ quantum corrections) is a chiral superfield. Its vev parameterizes a Coulomb branch.

Monopole operators

Microscopically *X* is defined as a monopole operator [Kapustin et al].

- A monopole operator leads to a singularity in *F*

$$dF = 2\pi\delta^{(3)}(x)$$

(equivalently, remove a ball around *x = 0* and put one unit of flux through its surface).

- In SUSY *F* is in a linear superfield $\Sigma = D\bar{D}V$

and the monopole operator is defined through

$$\bar{D}^2\Sigma = 2\pi\delta^{(3)}(x)\theta^2 \qquad ; \qquad D^2\Sigma = 0$$

Hence

$$\sigma \sim \frac{1}{|x|}$$

Monopole operators

- It is easy to see that in the dual variables

$$\bar{D}^2\Sigma = 2\pi\delta^{(3)}(x)\theta^2$$

has the same effect in the functional integral as the insertion of

$$X = e^{\sigma/g_3^2 + ia}$$

- Semiclassically, this insertion sets

$$\sigma \sim \frac{1}{|x|}$$

and thus pushes the scalar σ to infinity.

- This explains why the chiral operator X is associated with the Coulomb branch.

Monopole operators vs. vortices

- A BPS particle (vortex) exists at a point in space. It needs a central charge Z (say $Z > 0$). It is annihilated by Q_- and its conjugate $\bar{Q}_+$.
  2d rep: $\bar{Q}_- |a> = 0$

  $$|b> \sim Q_+ |a>$$

- A BPS (monopole) operators X is a chiral operator (at a point in spacetime). It is annihilated by $\bar{Q}_\pm$.
- $X|0>$ is not a BPS particle. But if an appropriate BPS state $|a>$ exists, it can be created by projecting on the lowest energy state

$$|a\rangle = \lim_{\tau\to\infty} e^{-(H-Z)\tau} X|0\rangle$$

3d SU(2) gauge theory

[Affleck, Harvey, Witten]

- Semiclassically, a Coulomb branch of vacua parameterized by

$$X \sim e^{\sigma/g_3^2+ia}$$

- The *4d* monopole is like a *3d* instanton and following Polyakov it leads to a superpotential

$$W = \frac{1}{X}$$

- This superpotential is exact because of an R-symmetry. (Recall, no axial anomaly in *3d*.)
- It leads to runaway – no vacuum.

4d SU(2) gauge theory on a circle

- Semiclassically, the Coulomb branch of vacua is compact $A_4 = \sigma \sim -\sigma \sim \sigma + \frac{2}{R}$. It is still parameterized by a chiral superfield X.
- Global symmetry
$$X \to \frac{1}{\eta X}$$
$$\eta = \Lambda^6 \sim e^{-\frac{8\pi^2}{g_4^2} + i\theta}$$
(This is the *4d* instanton factor.) It disappears in the *3d* limit.
- It relates the two points in the moduli space with unbroken *SU(2)*: $X = 0,\ \infty$ ($\sigma = 0,\ \frac{1}{R}$).

4d SU(2) gauge theory on a circle

- Non-perturbatively [NS, Witten]

$$W = \frac{1}{X} + \eta X$$

4d monopole acting as a *3d* instanton

4d instanton $\eta = \Lambda^6$ disappears in the *3d* limit. Breaks axial $U(1)$.

- This superpotential is exact (holomorphy, right limits at zero and infinity, and global symmetry).
- 2 vacua, as in *4d*. They run to infinity in the *3d* limit $\eta \to 0$.

Duality in *4d* N=1 SUSY Gauge Theory

Two dual theories are related by RG flow

- Two asymptotically free theories flow to the same IR fixed point
- An asymptotically free theory in the UV flows to an IR free field theory.

Characteristic example

- Electric theory: *SU(N_c)* with N_f quarks $Q,\ \tilde{Q}$
- Magnetic theory: *SU(N_f - N_c)* with N_f quarks $q,\ \tilde{q}$ and singlets *M* (mesons) with a superpotential

$$W = Mq\tilde{q}$$

Lessons

- Nontrivial mixing of flavor and gauge
  – The dual gauge group depends on the flavor
- Composite gauge fields
  – Gauge symmetry can be emergent
  – Gauge symmetry is not fundamental
- . . .

Going to *3d*

- Reducing the two dual Lagrangians to *3d* does not lead to dual theories.
  - Perhaps it is not surprising – radius going to zero does not commute with IR limit in *4d*
- Aharony, Giveon and Kutasov conjectured some dual pairs.
  - They were motivated by stringy brane constructions.
  - These were recently generalized and tested by various authors.

Aharony duality

- The electric theory: $U(N_c)$ with N_f quarks $Q, \tilde{Q}$
- The magnetic theory: $U(N_f - N_c)$ with N_f quarks, $q, \tilde{q}$ and singlets M (mesons) and $X^{\pm}$ (monopoles) with a superpotential

$$W = Mq\tilde{q} + X^+\tilde{X}^- + X^-\tilde{X}^+$$

$\tilde{X}^{\pm}$ are monopoles of the magnetic gauge group.

Aharony duality

- The Lagrangian of the magnetic theory includes monopole operators $\tilde{X}^{\pm}$. These are local operators, but they cannot be expressed simply in terms of the elementary fields.
- The monopoles of the electric theory are elementary in the magnetic theory.
- The monopoles carry nontrivial flavor quantum numbers. Hence, mixing of gauge and flavor.
- Similar dualities with orthogonal and symplectic gauge groups

Giveon-Kutasov duality

- The electric theory: $U(N_c)_k$ *(the subscript denotes the coefficient of the Chern-Simons term)* with N_f quarks $Q,\ \tilde{Q}$
- The magnetic theory: $U(|k|+N_f - N_c)_{-k}$ with N_f quarks $q,\ \tilde{q}$ and singlets M (mesons) with a superpotential

$$W = Mq\tilde{q}$$

Giveon-Kutasov duality

- No monopole operators in the Lagrangians
- Relation to level-rank duality
- This duality can be derived from Aharony duality (and vice versa) by turning on real mass terms and using renormalization group flow.
- Similar dualities with orthogonal and symplectic gauge groups

Duality in 3d

- Until recently, only a few tests – main motivation was brane constructions.
- New non-trivial tests involving the S^3 and the $S^2 x S^1$ partition functions [Kapustin, Willett and Yaakov, …]

Questions:

- Why doesn't the simple reduction from *4d* work?
- Why are these *3d* dualities so similar to the *4d* dualities?
- Are there additional dual pairs?
- What is the underlying reason for duality?

Compactify a *4d* theory on a circle

- Main difference between a *4d* theory and its *3d* counter-part is an anomalous *U(1)* symmetry in *4d*. Instantons explicitly break the symmetry.
- When the *4d* theory is placed on a circle *4d* instantons still break the *U(1)* symmetry. They typically lead to a term in the effective superpotential [NS, Witten]

$$W = \eta X$$

  - *X* is a monopole operator.
  - $\eta \sim e^{-8\pi^2/g_4^2 + i\theta}$ is the *4d* instanton factor.
  - η goes to zero in the *3d* limit.

Compactify a *4d* dual pair

In order to break the anomalous symmetries in the *3d* theories we should add to the *3d* Lagrangian of the electric theory ηX and to the *3d* Lagrangian of the magnetic theory $\tilde{\eta}\tilde{X}$.

- This leads to two dual theories in *3d*.
- All the tests of duality in *3d* are satisfied.
- One might not like the presence of the monopole operators in the Lagrangians.

Example

- Electric theory: $SU(N_c)$ with N_f quarks $Q, \ \tilde{Q}$ with a superpotential

$$W = \eta X$$

- Magnetic theory: $SU(N_f - N_c)$ with N_f quarks $q, \ \tilde{q}$ and singlets M (mesons) with a superpotential

$$W = Mq\tilde{q} + \tilde{\eta}\tilde{X}$$

$$\eta\tilde{\eta} \sim 1$$

Deforming the dual pair by relevant operators

- We can deform the dual pair with all the relevant operators present in *4d.*
  - This preserves the duality
  - Given our construction, this fact is trivial and does not lead to new dualities.
- New relevant operators – real masses for the quarks. They lead to new interesting dual pairs.

Example 1: Real mass for one of the flavors – the electric theory

- Start with $N_f + 1$ flavors and turn on real mass for one of the flavors (opposite signs for the quarks and anti-quarks).
- The low energy electric theory is $SU(N_c)$ with N_f quarks. There is no monopole operator in the Lagrangian (no η-term); $W=0$.
- This is standard SQCD.

Example 1: Real mass for one of the flavors – the magnetic theory

- The magnetic gauge group is Higgsed:

$$SU(N_f+1-N_c) \to U(N_f-N_c)$$

- The light elementary matter fields are:
  - N_f dual quarks, $q, \ \tilde{q}$
  - $SU(N_f - N_c)$ singlets $b, \ \tilde{b}$ with $U(1)$ charges $\pm(N_f - N_c)$
  - Neutral fields M (mesons) and X (monopole)
-
$$W = Mq\tilde{q} + Xb\tilde{b} + \tilde{X}^+ + \tilde{X}^-$$

$\tilde{X}^\pm$ are monopole operators of $U(N_f - N_c)$.

Example 2: Real masses for all the anti-quarks

- The electric theory: *$SU(N_c)$* with *N_f* fundamentals *Q*. No additional fields, *$W=0$*.
- Depending on the signs of the masses there might or might not be a Chern-Simons term.
- For even *N_f* we can let *$N_f/2$* of the anti-quarks have positive real mass and *$N_f/2$* of them have negative real mass. Then there is no Chern-Simons term.
- The magnetic theory: *$SU(N_f - N_c)$* with *N_f* fundamentals *q*. No additional fields, *$W=0$*.

Conclusions

- A *4d* dual pair leads to a *3d* dual pair (with monopole operators in the Lagrangians).
- Turning on real masses, we find many more dual pairs:
  - We reproduced all known examples (some still in progress)
  - Many new dualities (with or without monopole operators in the electric or magnetic Lagrangians)
- This explains:
  - Why naïve dimensional reduction of the dual pair does not work
  - Why the known examples are similar to the *4d* examples.

Conclusions

- This two step process (reduce with a monopole operator and flow down) has to work. It follows from the assumption of *4d* duality.
- Alternatively, the fact that it works leads to new tests of *4d* duality.
- It seems that all *3d* dualities follow from *4d* dualities.
- More generally, *3d* dynamics is part of *4d* dynamics.
- It raises many new questions…