Quantum mechanics and the geometry of spacetime

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Outline

- Brief review of the gauge/gravity duality
- Role of strong coupling in the emergence of the interior
- Role of entanglement in the shape of the geometry. Wormholes and entanglement.
- Vague similarity between tensor networks and spacetime geometry.

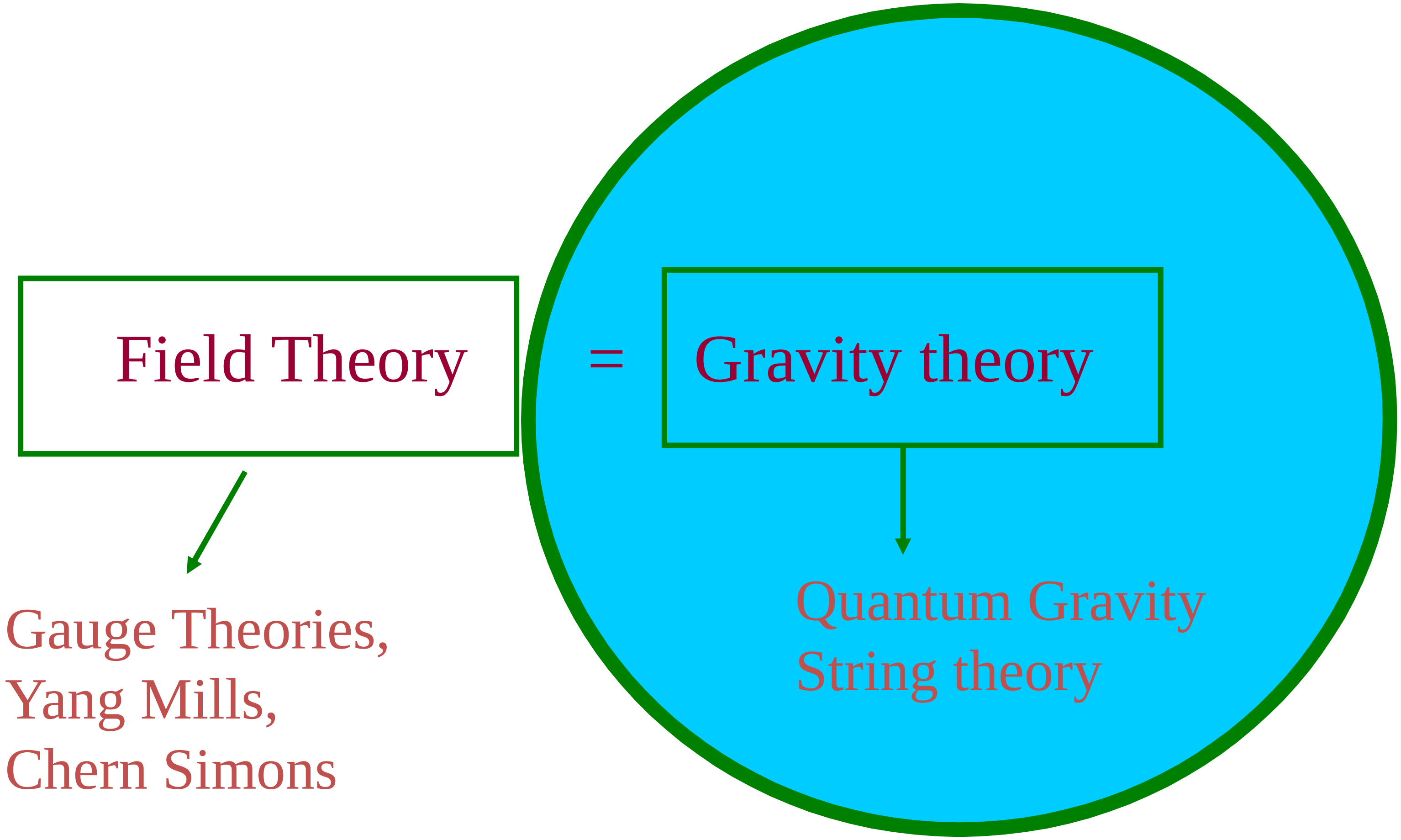
Gauge/Gravity Duality

(or gauge/string duality, AdS/CFT, holography)

Quantum Field Theory

Theories of quantum interacting particles

Dynamical Space-time (General relativity) string theory


Field Theory
=
Gravity theory
Gauge Theories,
Yang Mills,
Chern Simons
Quantum Gravity
String theory

Example

- Maximally supersymmetric SU(N) Yang Mills theory at large N. (Four dimensional gauge theory with some scalars and fermion fields, adjusted to that it has the maximum amount of supersymmetry).

SU(N) Super Yang Mills = string theory on $AdS_5 \times S^5$

JM

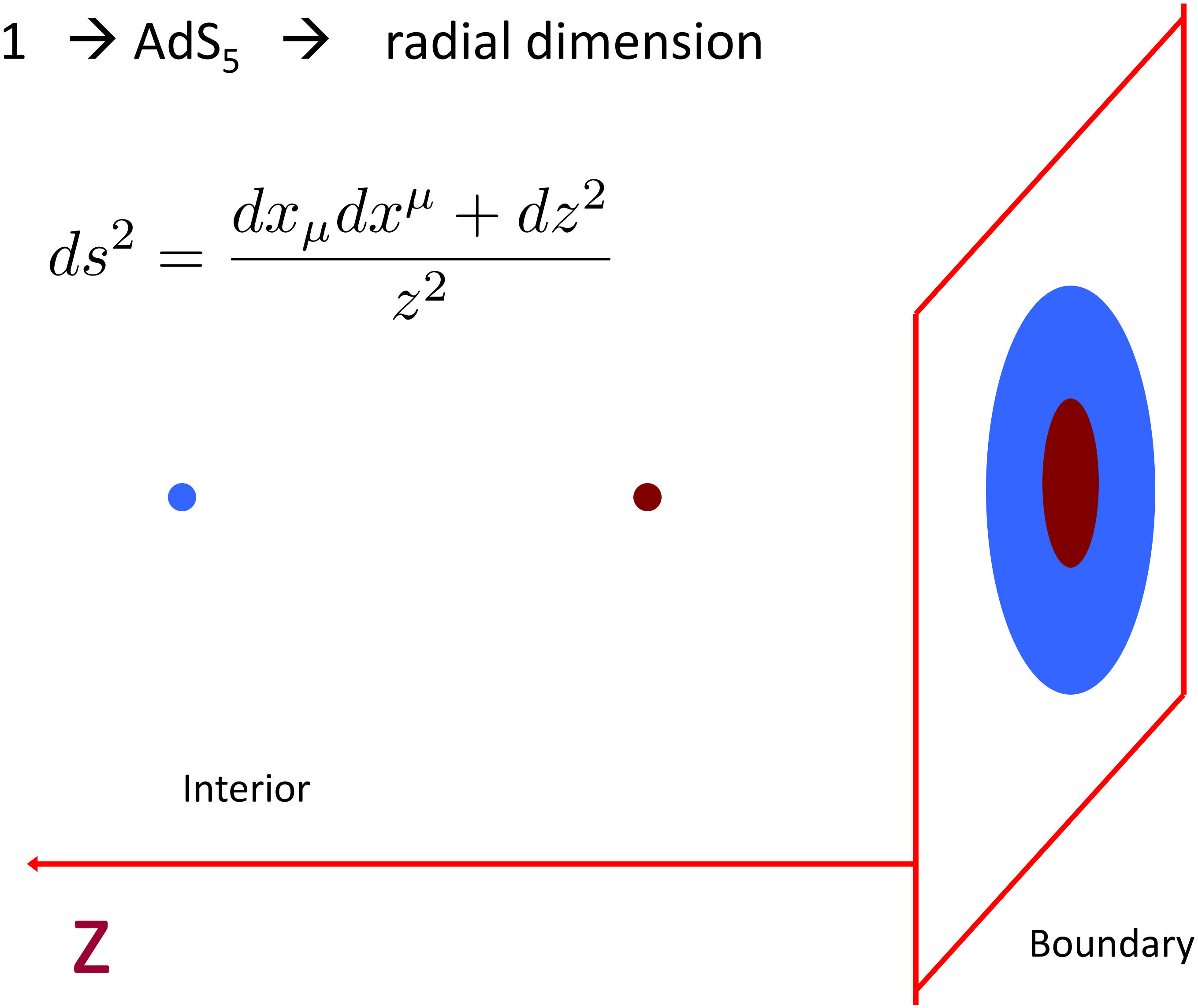
$\frac{R}{l_s}$ increases as the coupling constant of the gauge theory increase

Strong gauge theory coupling → large radius space = general relativity description

The extra dimension

3+1 $\rightarrow$ AdS_5 $\rightarrow$ radial dimension

$$ds^2 = \frac{dx_\mu dx^\mu + dz^2}{z^2}$$

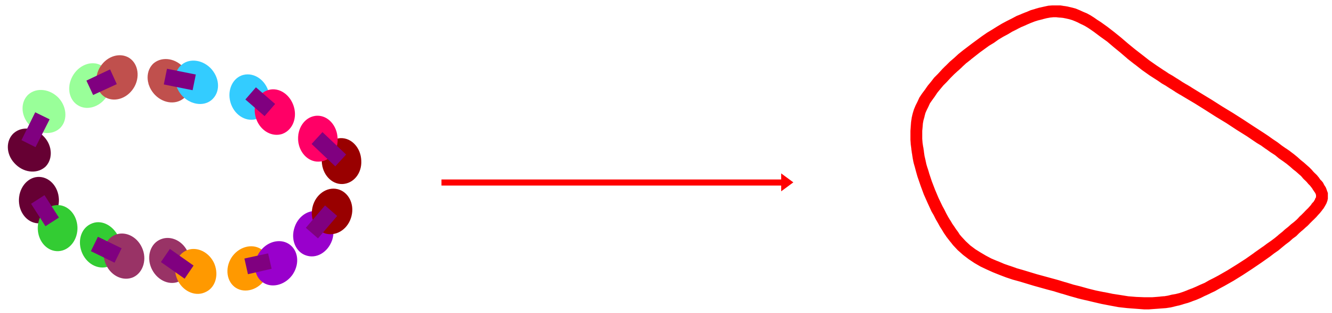


Large N

- Large number of colors → semiclassical geometry = small G_N

$$G_N \sim \frac{1}{N^2}$$

Gluon chains → strings



String Interactions $\sim \frac{1}{N} \sim \sqrt{G_N}$

Effective gluon interactions = $g^2 N$ = size of quantum effects in the boundary field theory.

$$(g^2 N)^p \sim \frac{\text{Radius of curvature of space}}{\text{Size of graviton}}$$

Effective gauge theory coupling

Large N → weakly coupled string theory in the bulk

Ordinary Einstein gravity theory → Strongly coupled field theory

The emergence of the bulk spacetime could be understood completely within the planar approximation (weakly coupled string theory, large N).

We need to tackle strongly coupled systems:

Approaches

1) Integrability: Use very special symmetries of maximally supersymmetric Yang Mills
→ Anomalous dimensions, amplitudes, etc. Review: see Beisert et al..

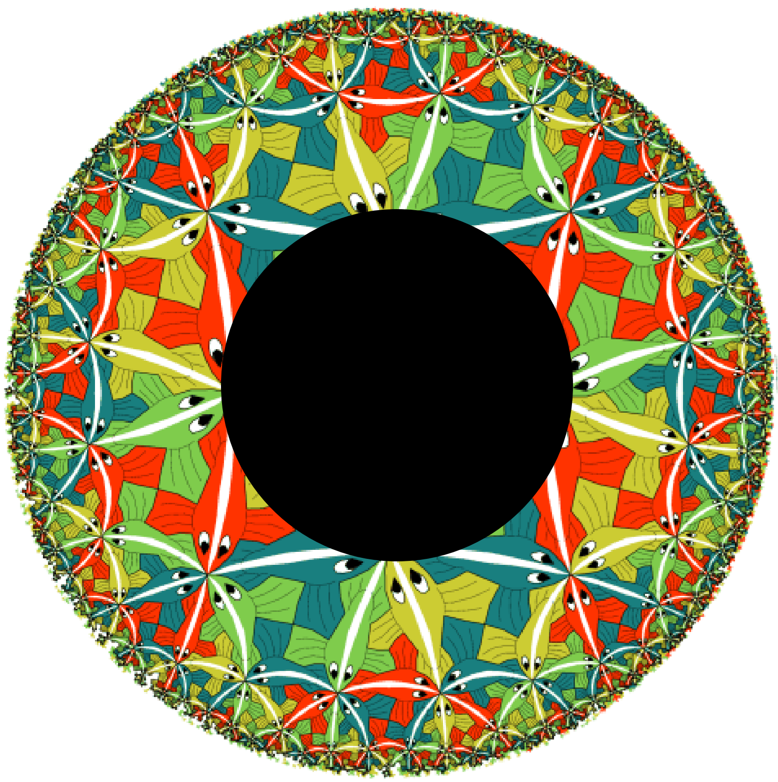
2) Use supersymmetry to compute special quantities. (Sphere partition functions, Wilson loops) Pestun, Kapustin, Jacobs, Willet, etc.

3) Computer simulations (matrix quantum mechanics) Hanada et al

4) Assume the duality and use it to gain intuition about strongly coupled systems (AdS/QCD, AdS/CMT)

Black holes in AdS

Thermal configurations in AdS.



Entropy:

$S_{GRAVITY}$ = Area of the horizon =
$S_{FIELD\ THEORY}$ =
Log[Number of states]

Strominger-Vafa

Evolution: Unitary

Long distance fluctuations of the horizon → hydrodynamics

Interior = ?

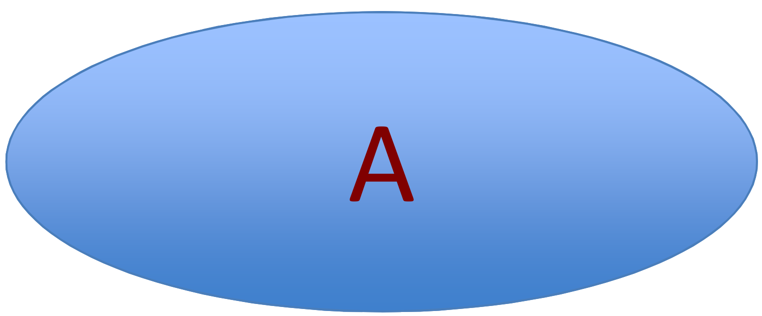
Conclusions

- Numerics can give interesting checks of the duality.
- It could help us understand better the emergence of a local theory in the bulk.
- The duality could also serve as an interesting test case for new numerical techniques.

Entanglement and the gauge/gravity duality

Entanglement in QFT

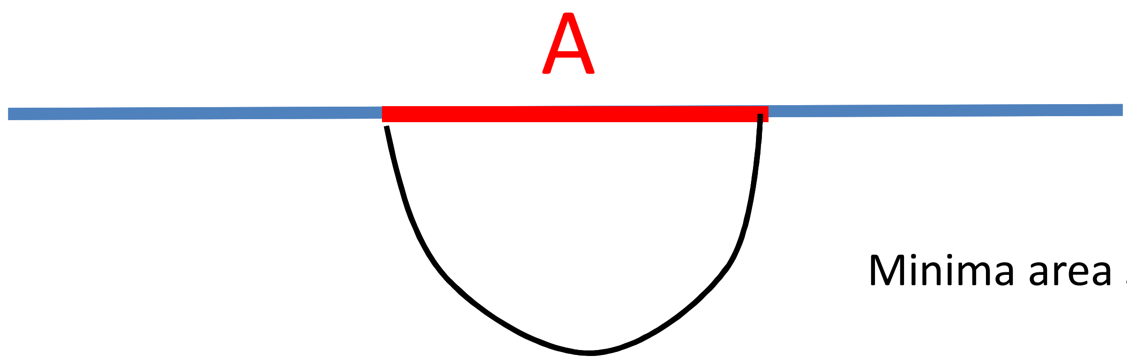
- In QFT the vacuum is a very entangled state of the fundamental degrees of freedom.
- Most of this entanglement is short range.



$$S_A = \frac{\mathrm{Area}}{\epsilon^2} + \cdots$$

- There is interesting QFT information in the dots (c, f theorems)

Cassini Huerta



Minima area surface in the bulk

Ryu-Takayanagi
(Hubeny, Rangamani ..)

Fursaev, Headrick, Lewkowycz, JM

$$S_A = \frac{A_{\rm minimal}}{4G_N}$$

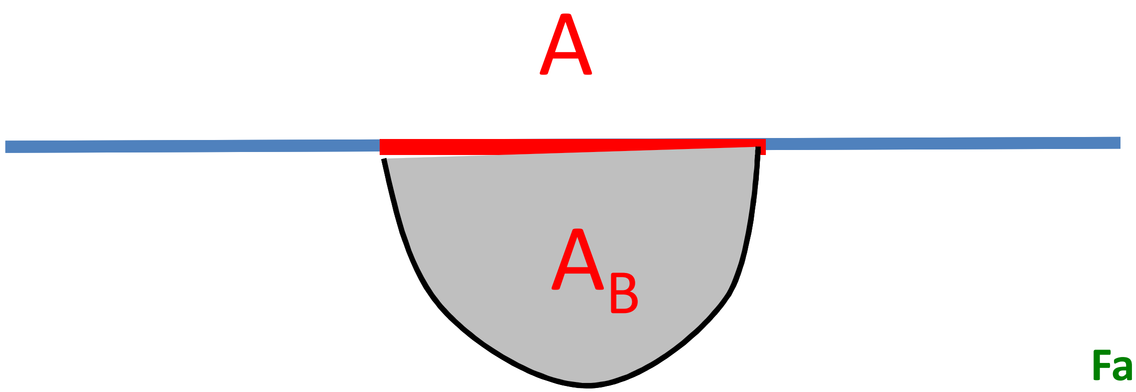
Leading order in G_N expansion

This a generalization of the Bekenstein-Hawking formula for black hole entropy

Quantum correction

$$S_A = \frac{\text{Area} + \alpha' \text{ corrections}}{4G_N} + S_q$$

$$S_q = S_{\text{Bulk entanglement}} + \cdots$$



Faulkner, Lewkowycz, JM

Define a bulk region A_B , inside the minimal surface.

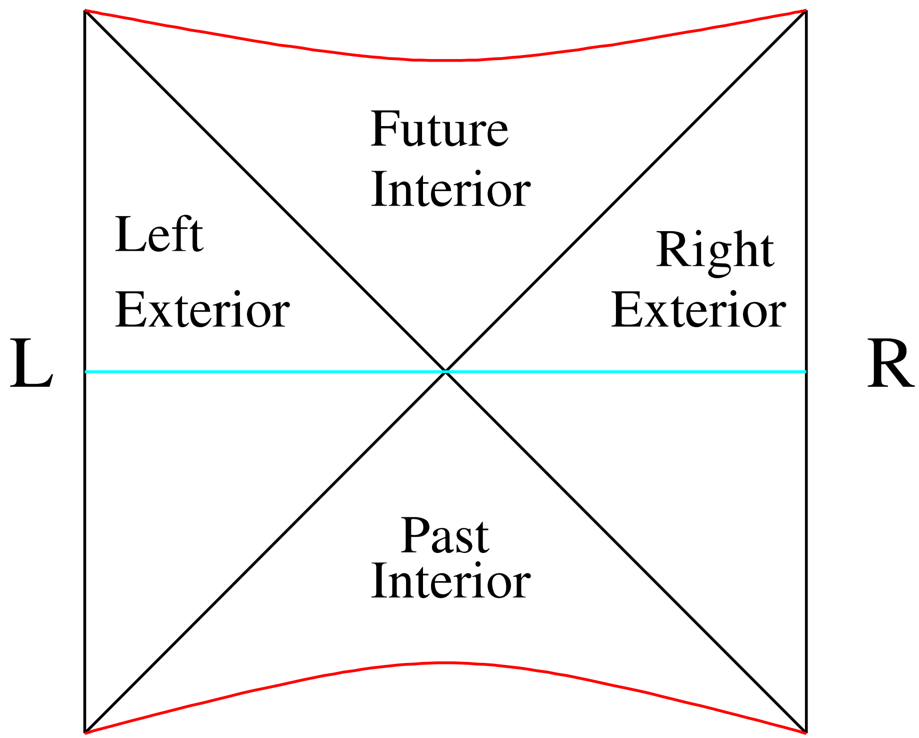
Compute the entanglement of the bulk quantum fields between A_B and the rest of the spacetime.

Some lessons

- Entanglement is computed by a geometric construction.
- But many other observables are also computed by the bulk geometry.
- We think that the bulk geometry reflects more directly the patterns of entanglement of the gauge theory.

A more drastic example

Eternal AdS black hole

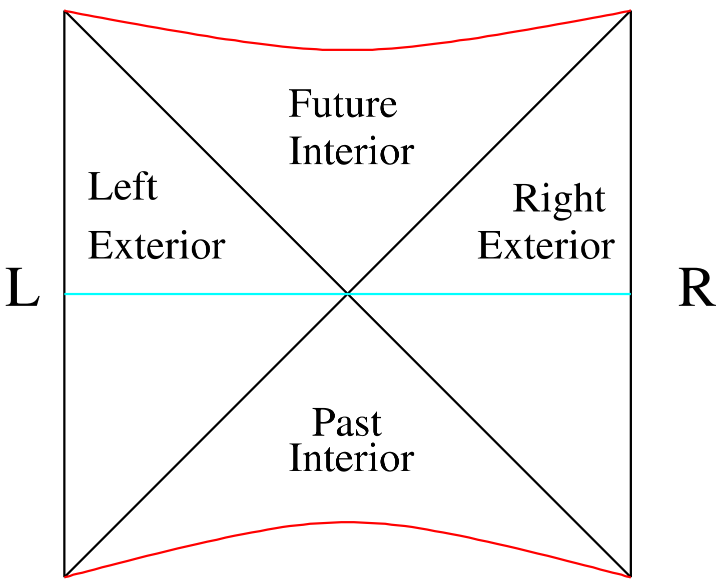


Entangled state in two non-interacting CFT's.

Israel
JM

$$|\Psi\rangle = \sum_n e^{-\beta E_n/2} |E_n\rangle_L^{CPT} \times |E_n\rangle_R$$

Eternal AdS black hole



$$\frac{\text{Area}}{4G_N} = \text{Entanglement entropy}$$

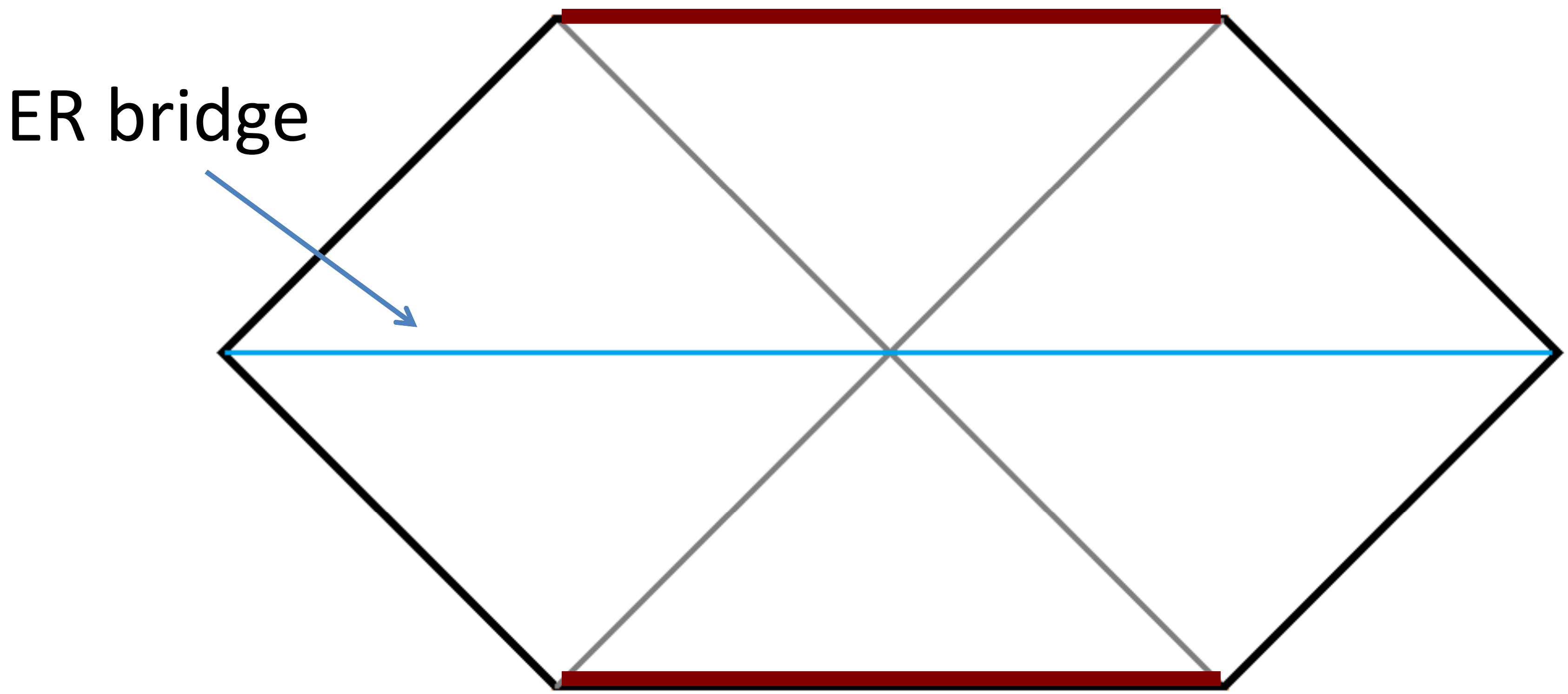
$$|\Psi\rangle = \sum_n e^{-\beta E_n/2} |E_n\rangle_L^{CPT} \times |E_n\rangle_R$$

- We generate a spatial connection despite the fact that the two quantum field theories are <u>not interacting</u> in any way.

- Just entanglement (of the right type) has created a geometric connection.

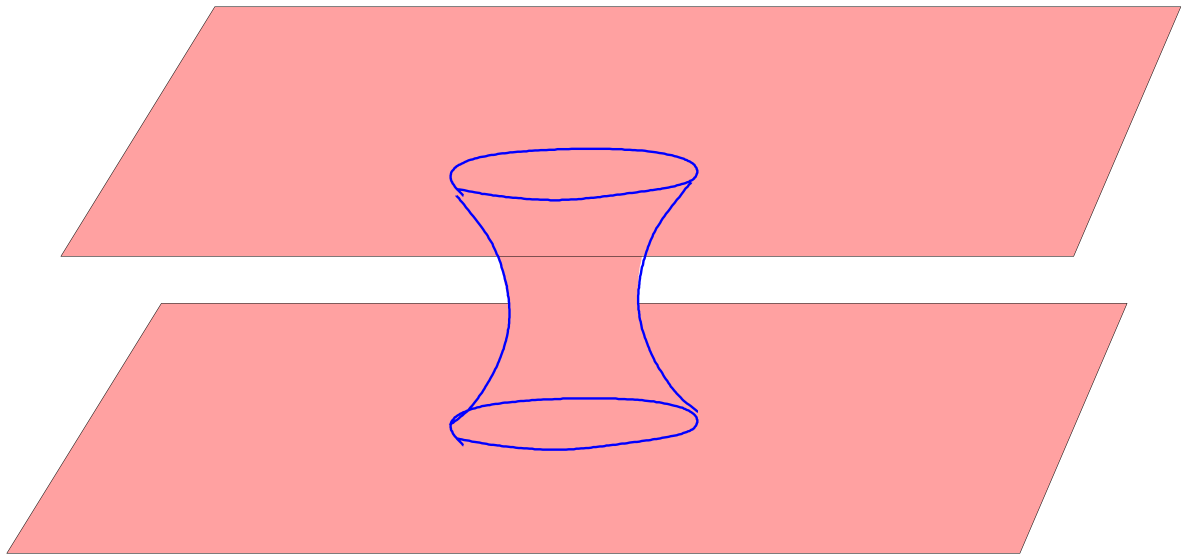
Wormholes

Eternal black hole

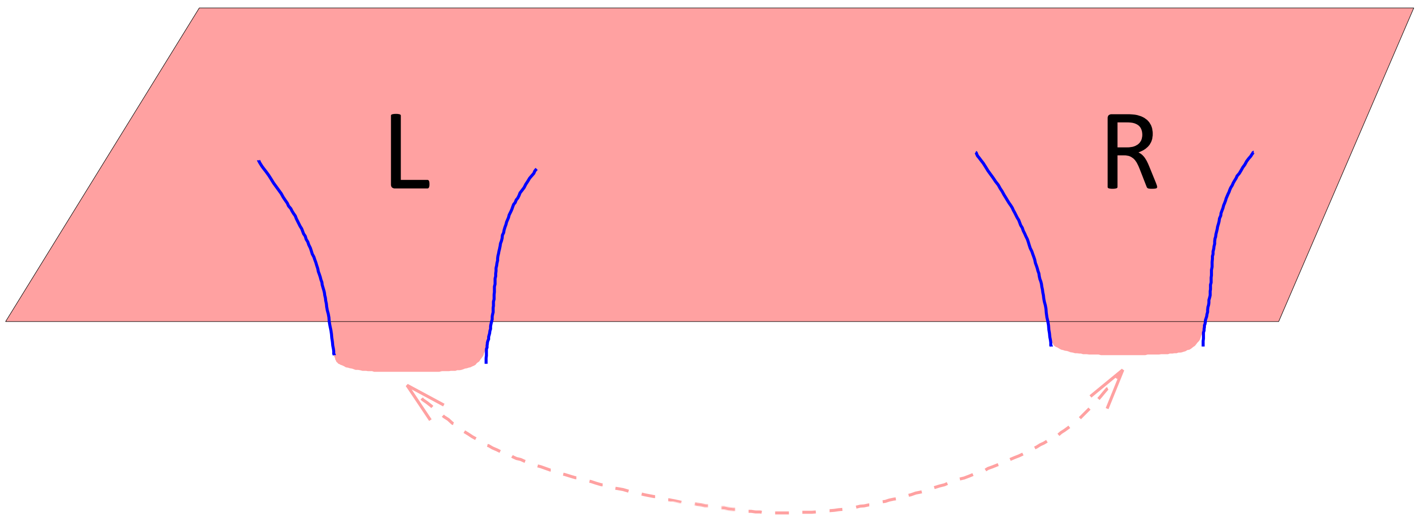


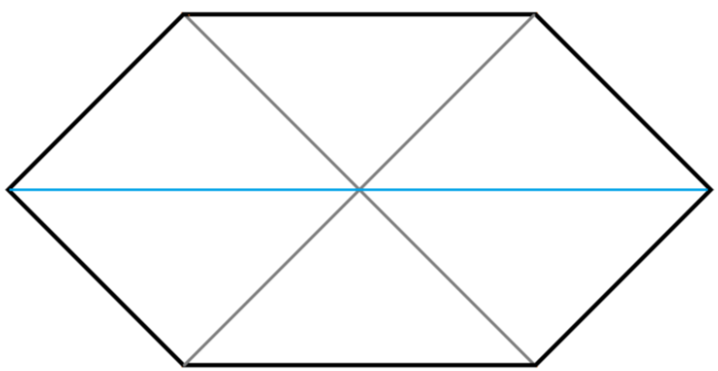
Maximally extended Schwarzschild geometry

Einstein-Rosen bridge



Wormhole interpretation.





Non travesable

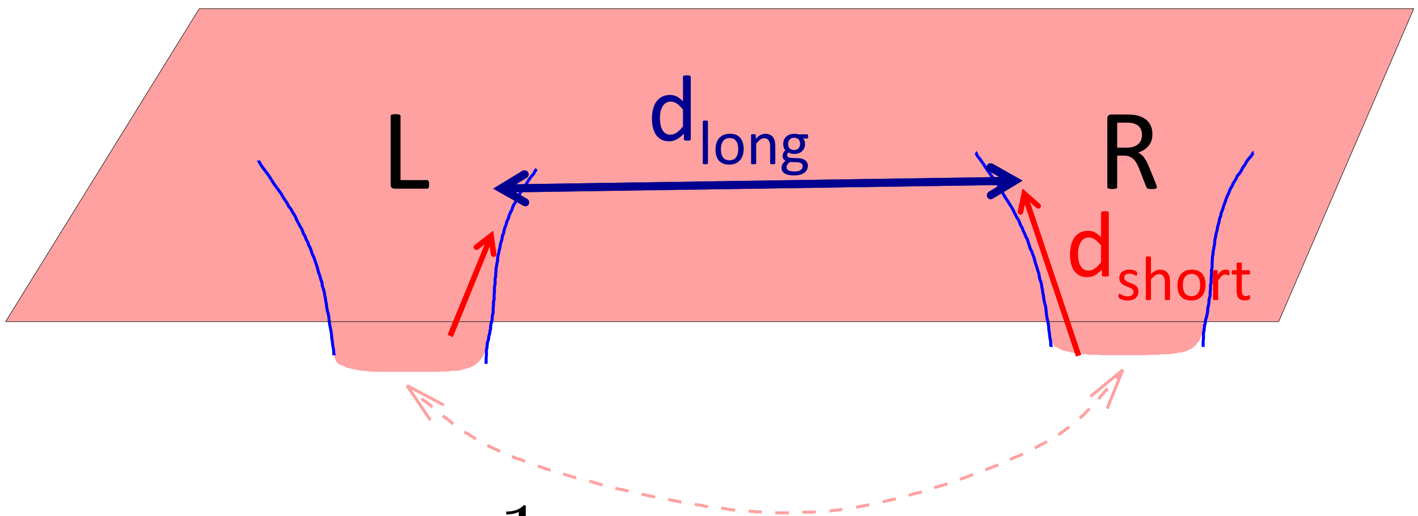
No signals

No causality violation

Fuller, Wheeler, Friedman, Schleich, Witt, Galloway, Wooglar

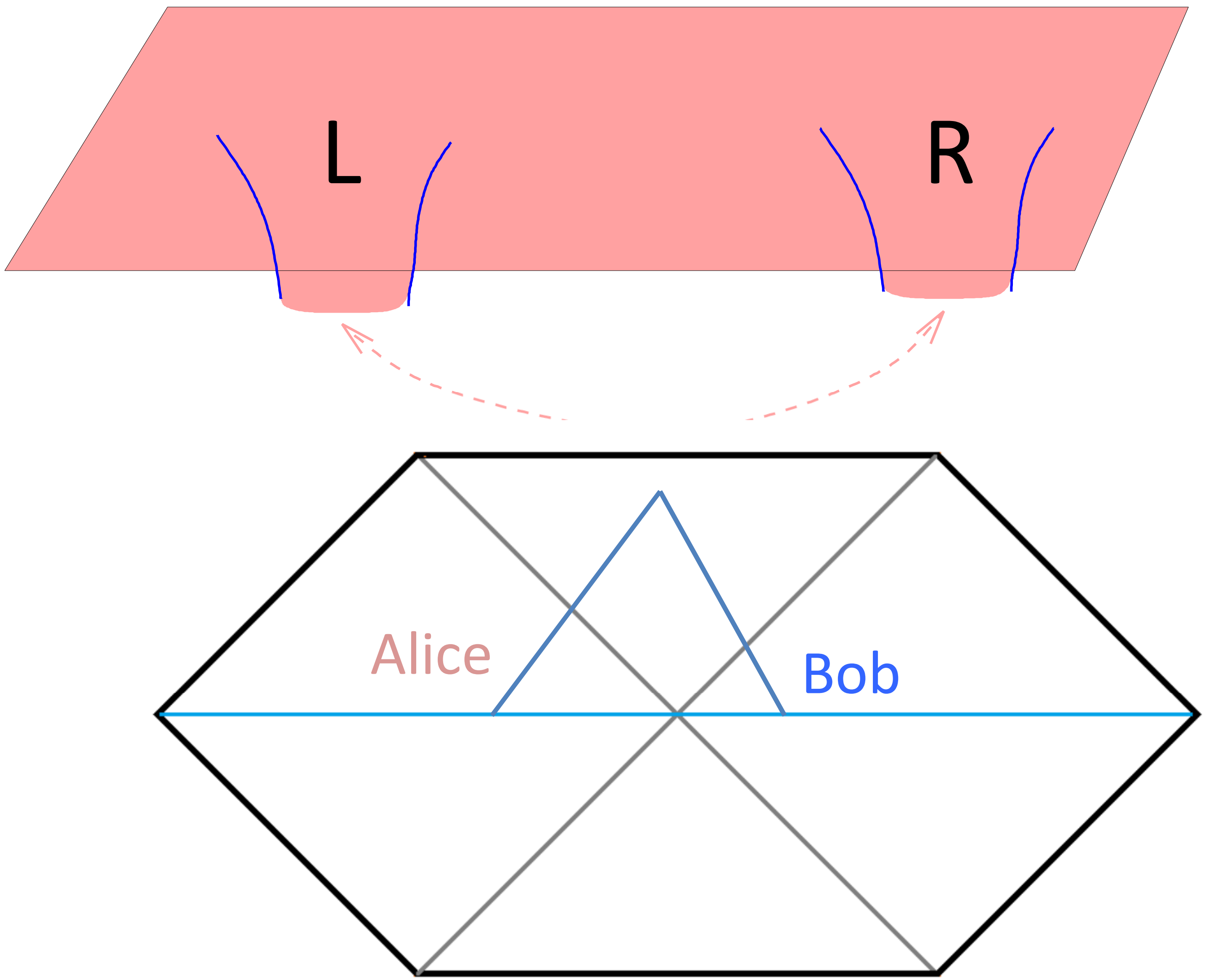
ER = EPR

Entangled black holes in the same spacetime

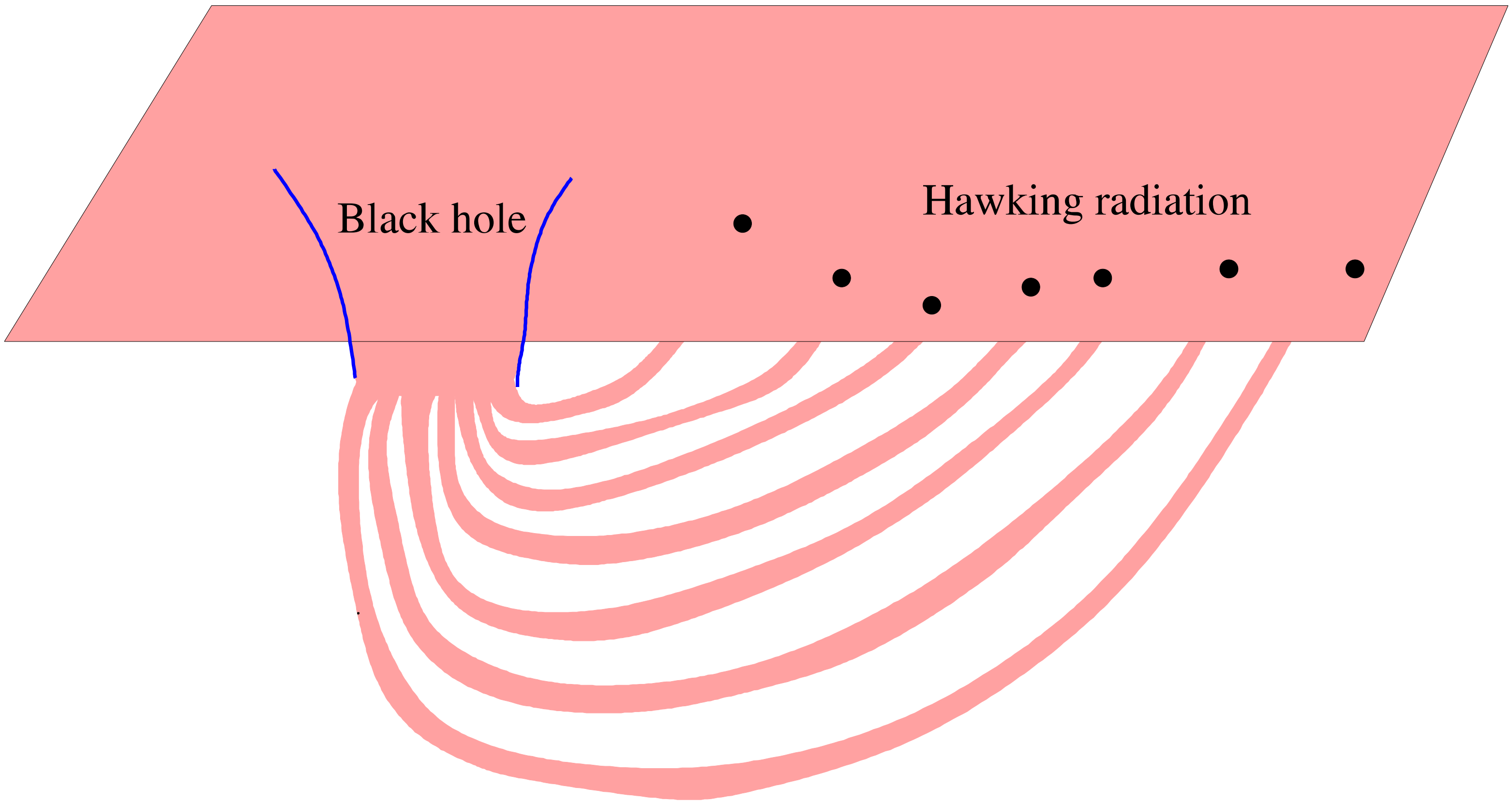


$$\langle \phi\phi \rangle = \frac{1}{d^2_{short}}$$

How to arrange a forbidden meeting



Black hole + radiation ?



We do not really know what this picture means, except to say that it is possible to send signals behind the horizon by manipulating the radiation.

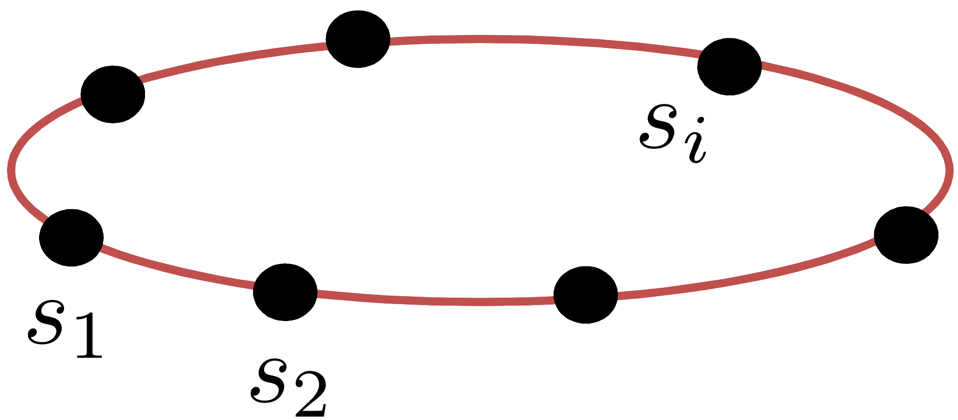
Conclusions

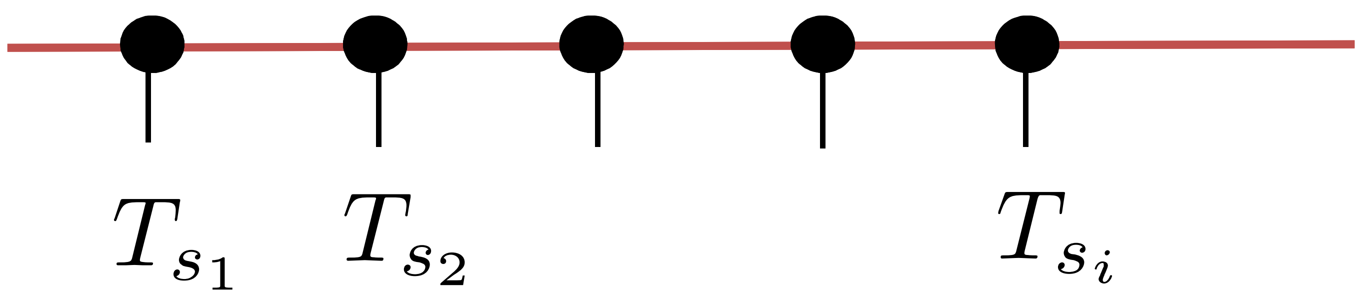
- Entanglement can be computed simply at strong coupling → Minimal areas.
- Generalizations of the Hawking Bekenstein black hole entropy formula.
- ER = EPR: Entanglement can produce a geometric connection (view it as a constraint on quantum gravity theories).

Tensor networks and geometry

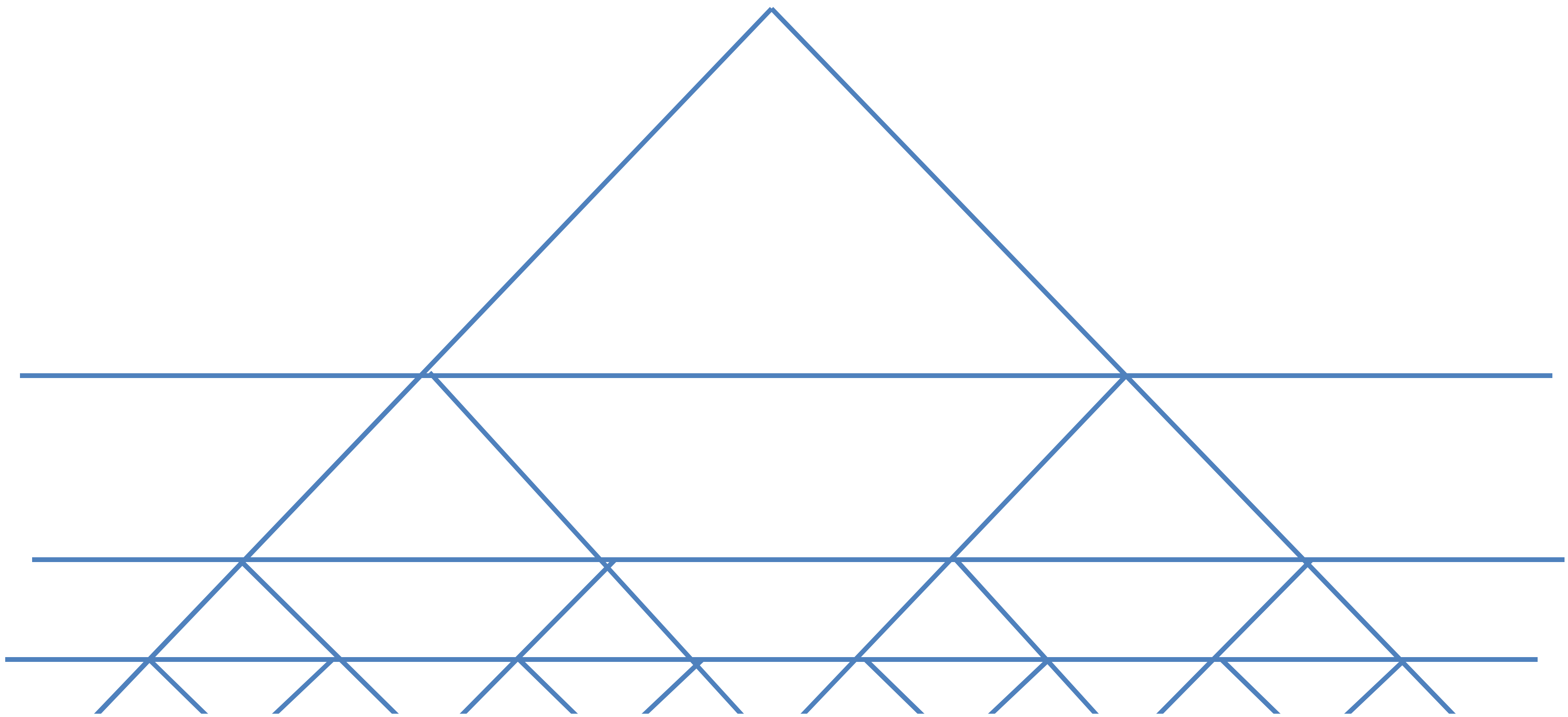
Matrix product states

$$\Psi(s_1, \cdots, s_n) = Tr[T_{1,s_1} T_{2,s_2} \cdots T_{n,s_n}]$$





Scale invariant wavefunctions (MERA)



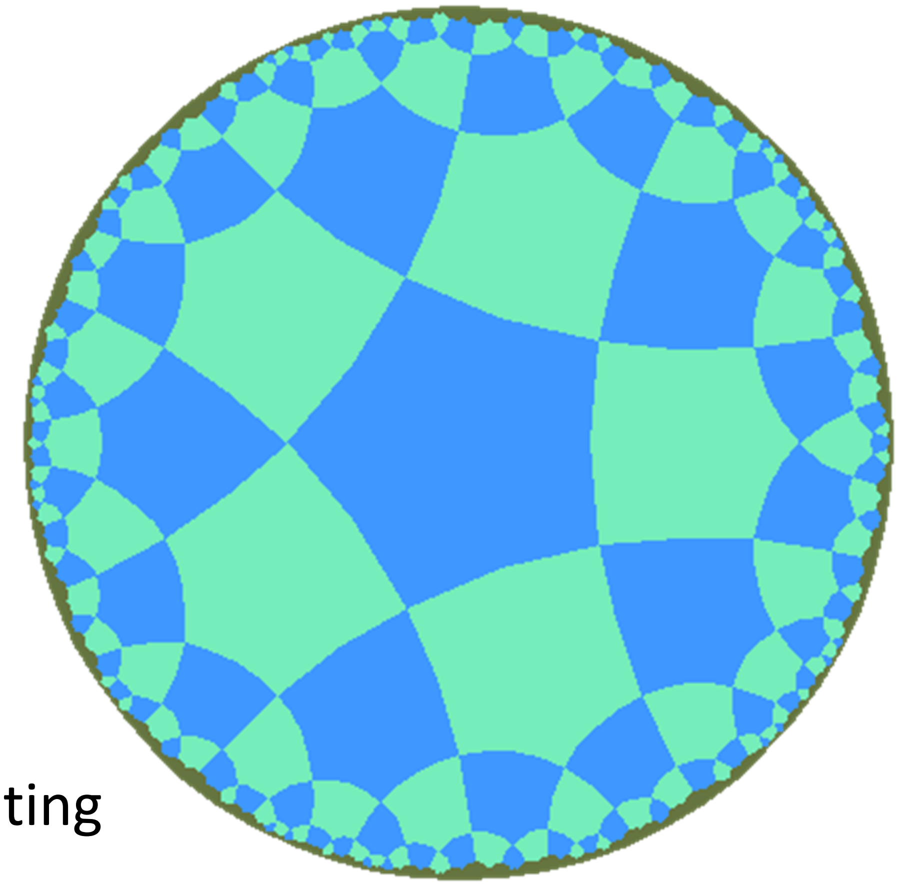
Each vertex is a five index tensor.   Each line is an index contraction.

Indices → not ``real'' states.

Vidal

This is similar to the geometry of AdS

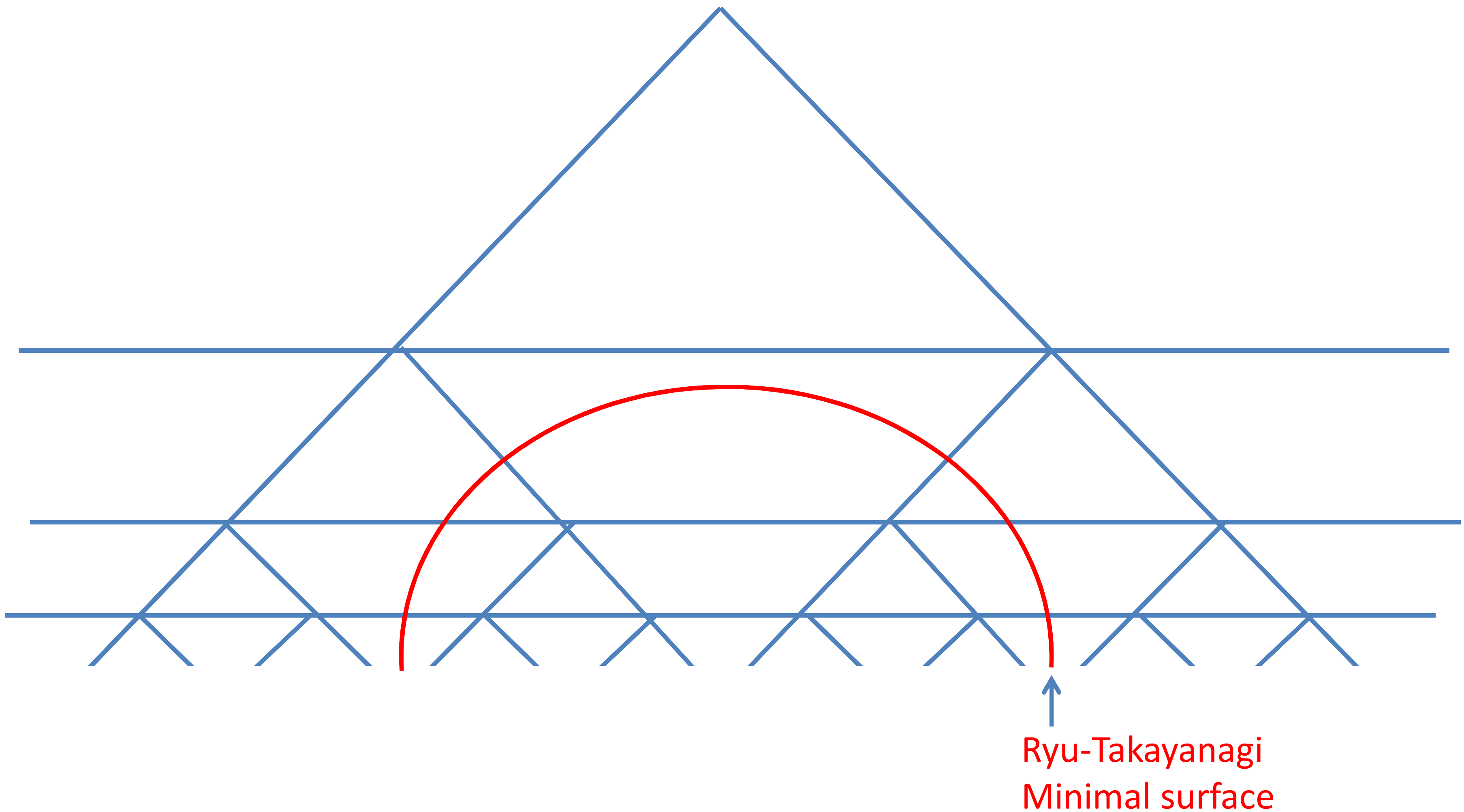
Swingle



Think of the tensors as representing
the AdS vacuum wavefunction.

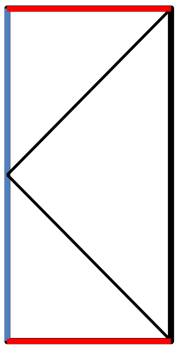
Tensor index contractions → entanglement

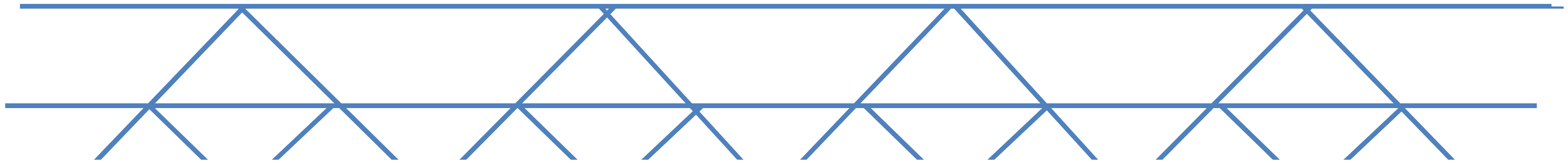
Entanglement & structure of space



Conformal invariant system in a state with a mass gap.

eg: AdS space with an end of the world brane in the IR

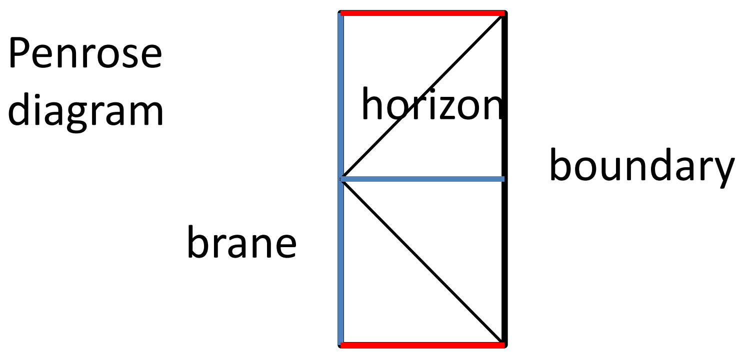


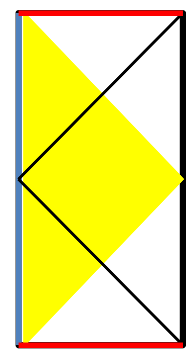


Time dependence

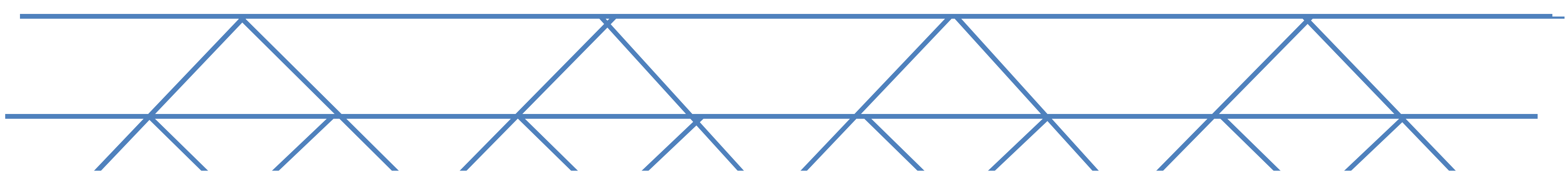
Start with a state with a gap and evolve it.

Eg. Brane in Ads that falls into a black hole

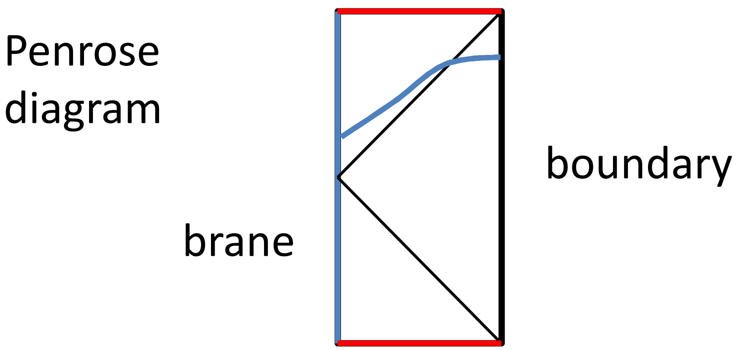


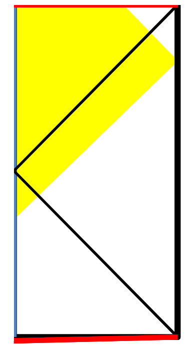


t=0



Hartman & JM





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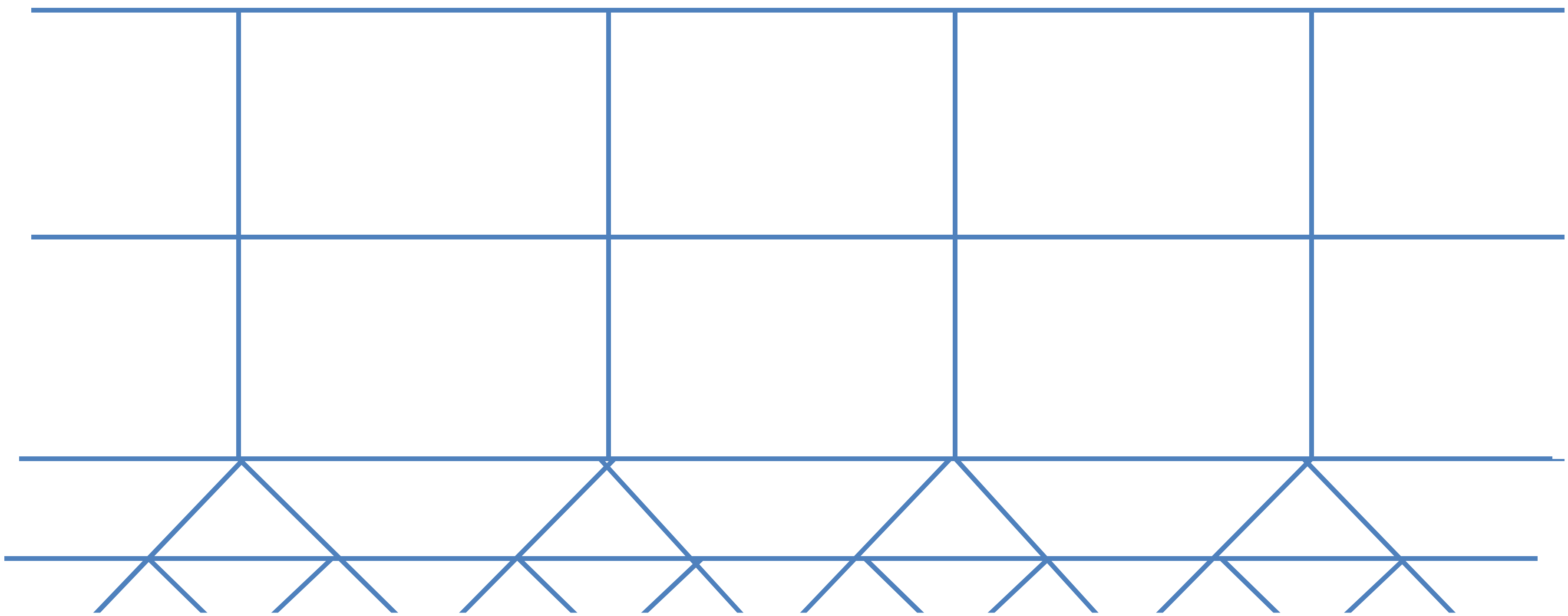
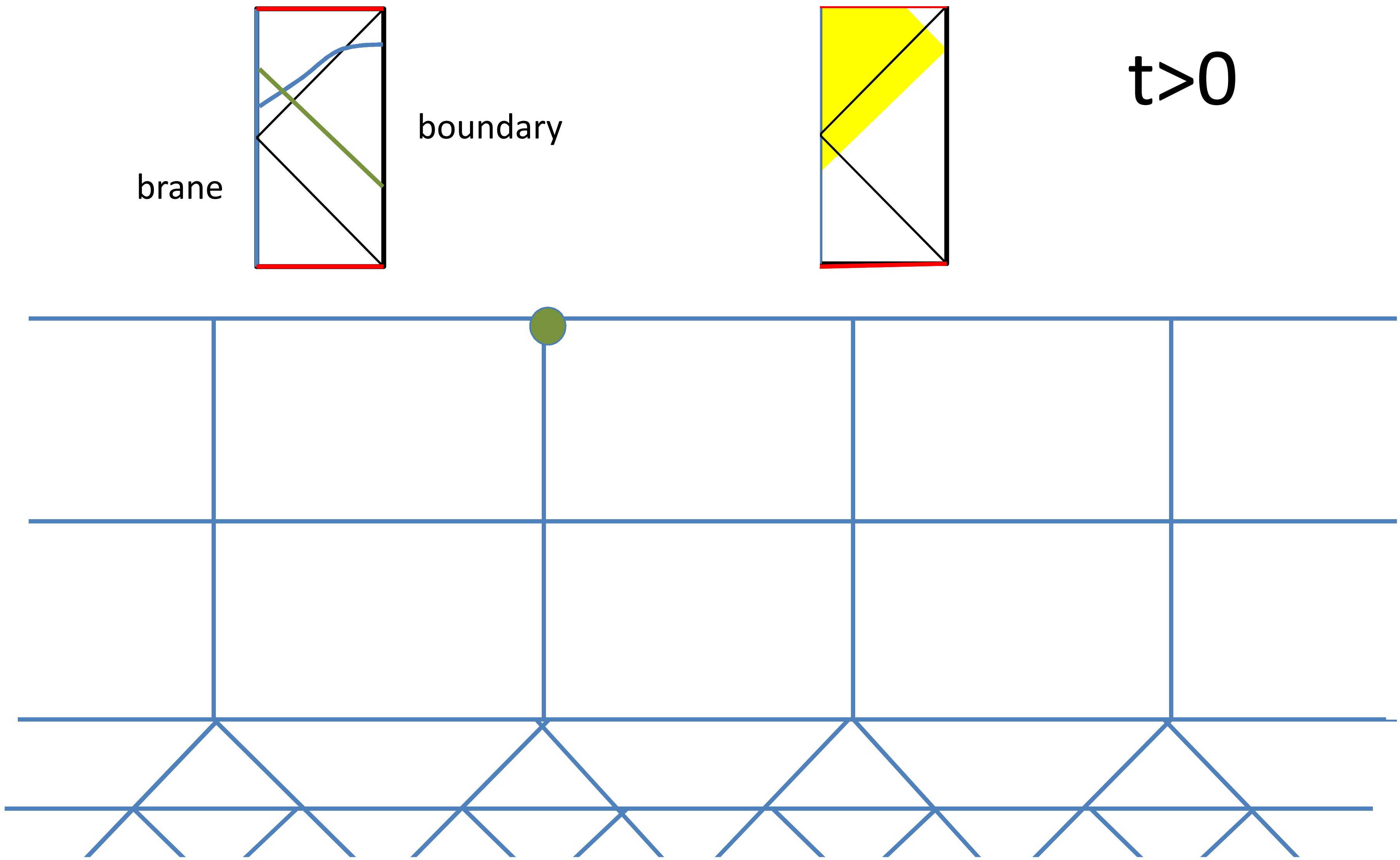
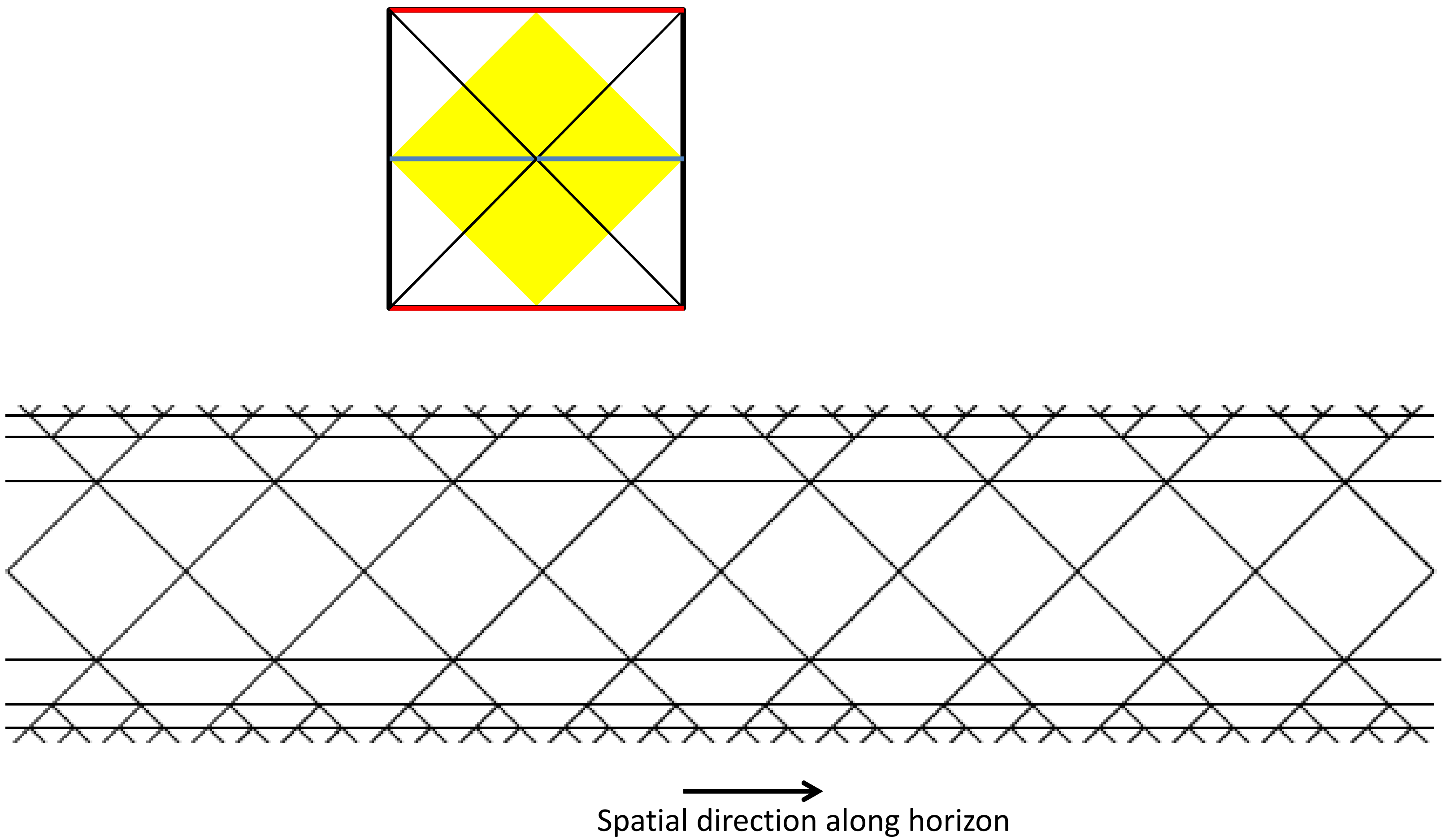
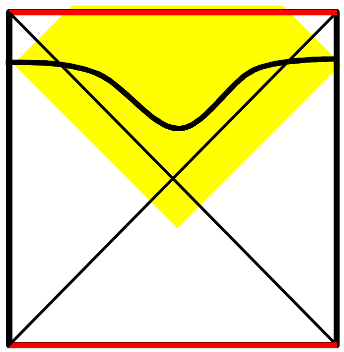


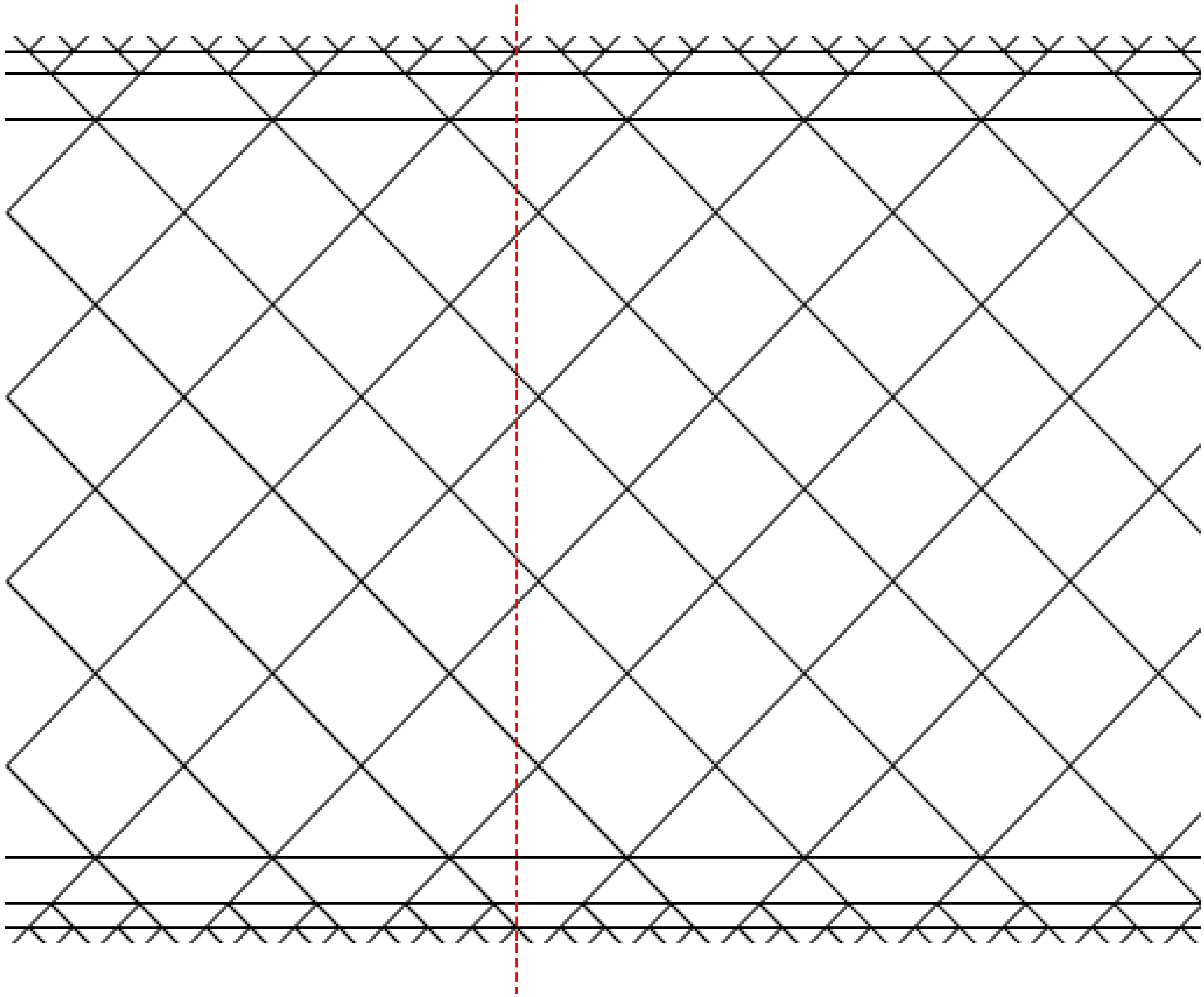
Time evolution produces a wavefunction that can be represented as a geometry which
Is simply longer.

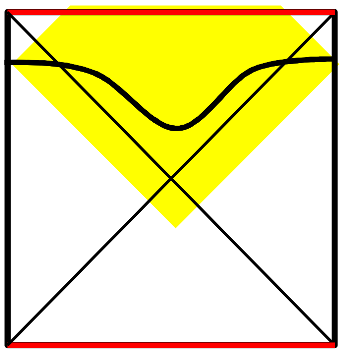


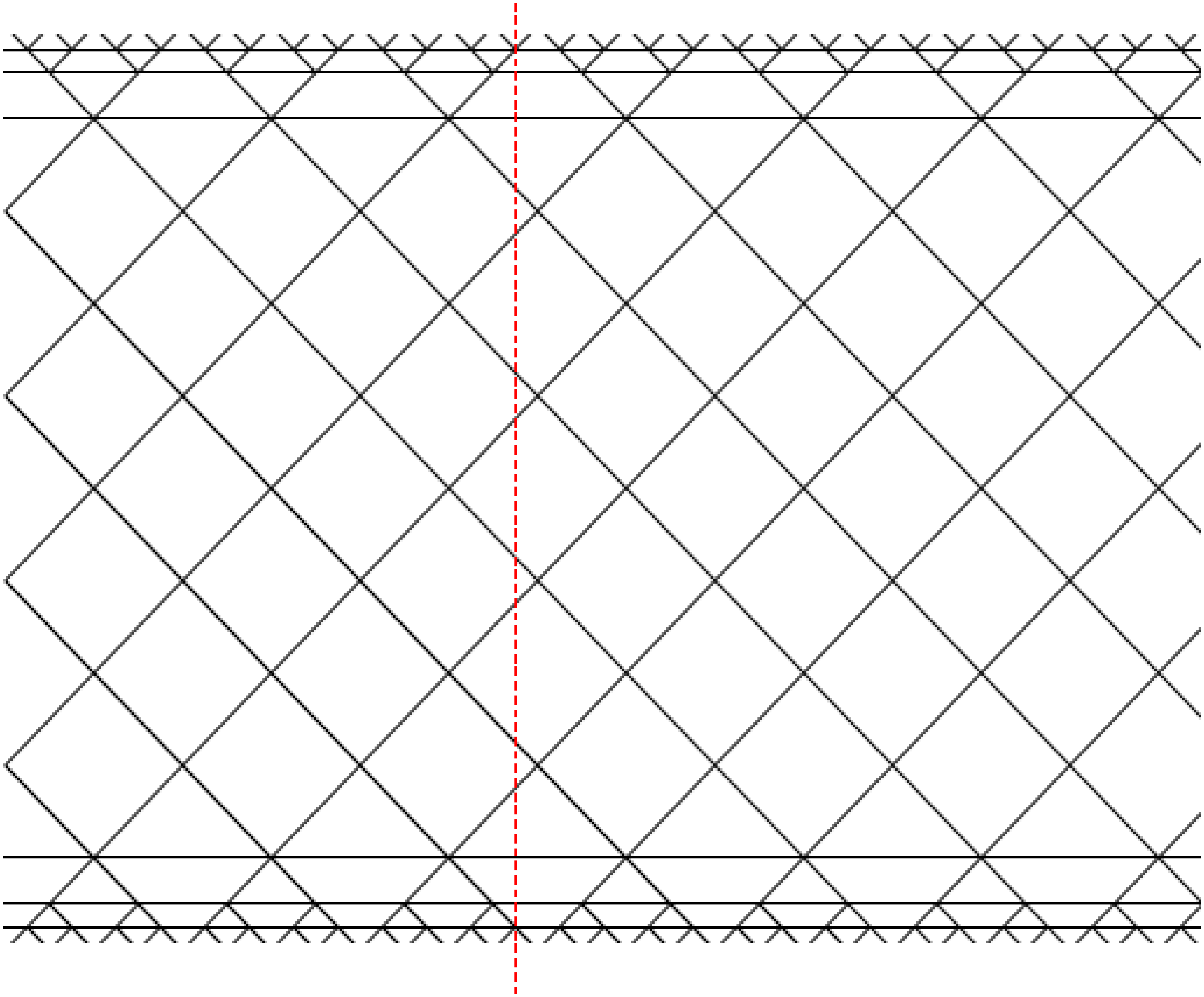
Addition of particles → changes in the tensors


Spatial direction along horizon









Captures better the entanglement patern.
Seems more similar to the ``nice slices'', which expand. The two horizons moving away…

Conclusions

- There are similarities between tensor networks and geometry.
- Both are constructions of the wavefunction and are constrained by the patterns of entanglement.
- In the case of black holes, it might help for understanding the interior.
- Growth of interior → need to add more tensors for describing the increasing complexity of the wavefunction.