Planet formation mini-course. 2

Leiden University
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The planetesimal (Safronov) hypothesis

planets form by multi-stage process:

1. rock and ice grains condense out and settle to the disk midplane
2. formation of small (km-sized) solid bodies (***planetesimals,*** i.e. bodies big enough that they are unaffected by gas)
3. planetesimals collide and grow
4. a few planetesimals grow large enough to dominate evolution of the rest of the planetesimal disk ("planetary embryos")
5. planetary embryos gradually collide and grow into "planetary cores"
6. cores of intermediate and giant planets accrete gas envelopes before the gaseous disk disperses

requires growth by 45 orders of magnitude in mass through many different physical processes

Formation of planetesimals

- dust condenses out of the cooling gaseous disk (iron, silicates, nickel in inner solar system; ammonia and ice in outer solar system)
- dust settles to the midplane of the disk in a timescale of order $\alpha^{-1/2}$ orbital periods, where α is the particulate fraction, after coagulating to a size of order 1 meter ("rocks")
- the dust disk becomes gravitationally unstable when Toomre's parameter

$$Q = c\,\Omega/\pi G\Sigma < 1$$

Here c, Σ are velocity dispersion and surface density of particles. If solid mass fraction is 0.5%, Σ=15 g cm^{-2} at 1 AU which requires c < 20 cm/s or thickness $h = c/\Omega$ < 1000 km

Think of a layer of meter-sized rocks, 1000 km thick, colliding about once per year at around 20 cm/s

- For $Q \ll 1$ all wavelengths < $\lambda_c = 4\pi^2 G\Sigma/\Omega^2 = 1 \times 10^9$ cm are unstable. Maximum unstable mass is $M_c = \pi\Sigma(\lambda/2)^2 = 10^{19}$ gm, corresponding to radius of 10 km (Goldreich-Ward mechanism)

Formation of planetesimals

the Goldreich-Ward mechanism, continued:

- gas disk rotates slower than Keplerian by about 0.2%. This leads to strong shear at the surface of the particulate disk
- shear induces Kelvin-Helmholtz instability which leads to turbulent velocities of order $v - v_g \sim c^2/\Omega R \sim 5 \times 10^3$ cm s^{-1} which gives $Q > 100$ and suppresses gravitational instability
- possibly K-H instability can be suppressed if solid/gas surface density ratio enhanced by factors of 2-10 (Youdin & Shu 2002)

Formation of planetesimals

If the Goldreich-Ward mechanism doesn't work…

- particle velocities are turbulent ($v \sim 5 \times 10^3$ cm s^{-1} ~ 150 km/hr) but maybe collisions lead to sticking
- but:
  - rocks don't stick when they collide!
  - icy bodies fracture at these high speeds
  - largest inclusions in meteorites are a few cm

so maybe the process is just slower...

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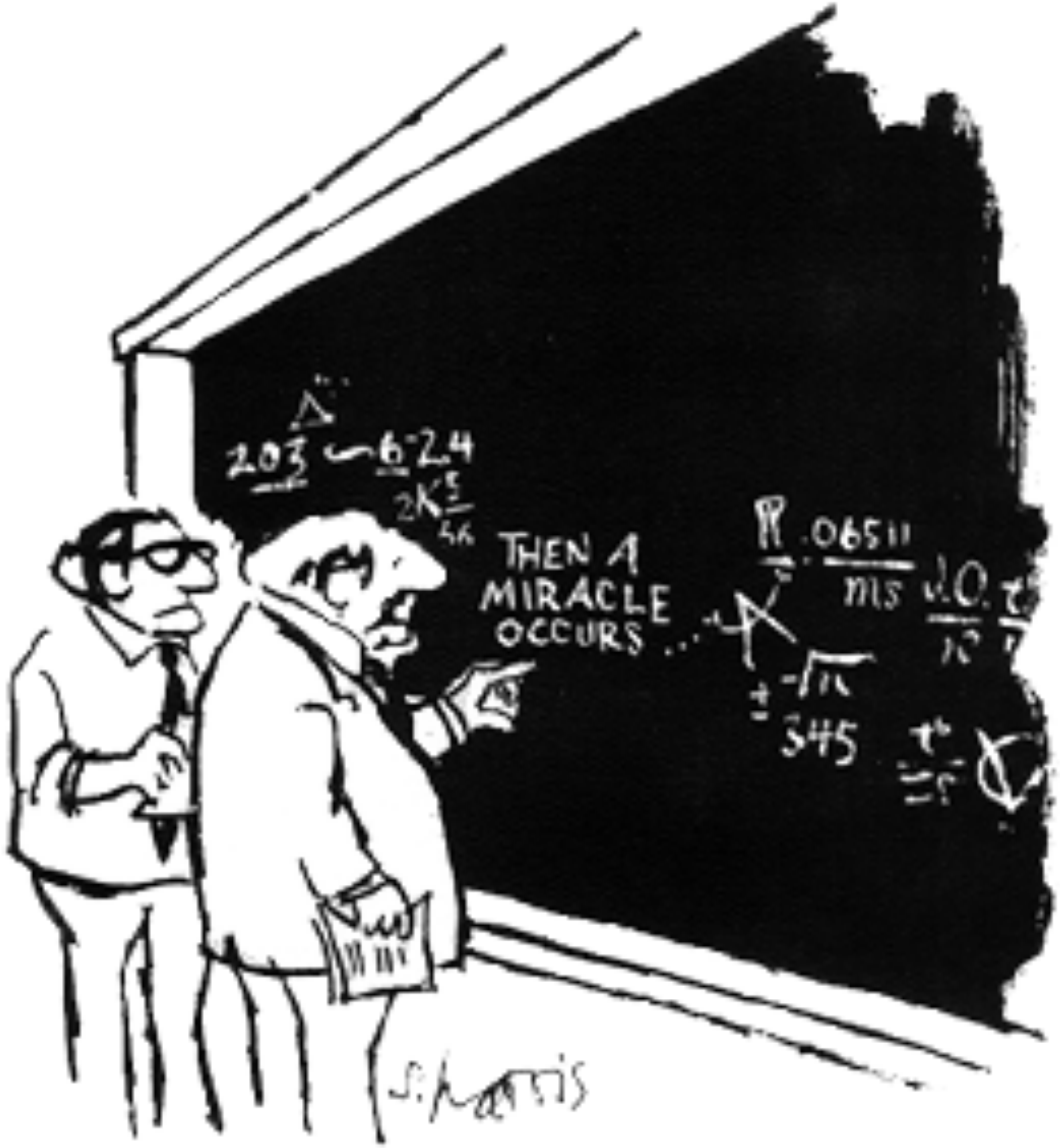
but it has to grow the particles to >> 1 meter in less than 100 years



"I think you should be more explicit here in step two."

Formation of planets

- once the meter hurdle is jumped, gas drag becomes unimportant
- further growth occurs through collisions

What is the collision cross-section between a test particle and a body of mass m and radius r? Without gravity,

$$dr/dt \sim \Sigma\Omega/\rho_p \sim 30 \text{ cm/yr at 1 AU}$$

so Earth would take 20 Myr to grow, and outer planets much longer

Formation of planets

Gravity enhances the cross section to

$$\sigma = \pi r^2(1+\Theta) \quad \Theta = 2Gm/rv^2 = v_{escape}^2/v^2$$

here Θ is the Safronov number (gravitational focusing). Hence

$$dr/dt \sim \Sigma\Omega/\rho_p (1+\Theta)$$

Moreover when $\Theta \gg 1$ collision debris doesn't have to stick

For $v \sim 5\times10^3$ cm/s, $\Theta \sim 1$ for $r \sim 50$ km

Formation of planets

Without gravity,

$$dr/dt \sim \Sigma\Omega/\rho_p \sim 30 \text{ cm/yr at 1 AU}$$

With gravity

$$r = \frac{r_0}{1 - t/t_0} \quad \text{where} \quad t_0 = \frac{v^2}{G\Sigma\Omega r_0} \sim 10^5 - 10^6 \text{ yr}$$

(for $v \sim 5\times10^3$ cm/s, $\Sigma \sim 15$ g/cm^2, $r_0 \sim 50$ km)

Planet reaches "infinite" radius in finite time (runaway growth)

This assumes that v remains small; however,

- near misses are more common than hits (a growing planet heats its food faster than it can eat it);
- inelastic collisions between small bodies are needed to keep v small

The core instability model

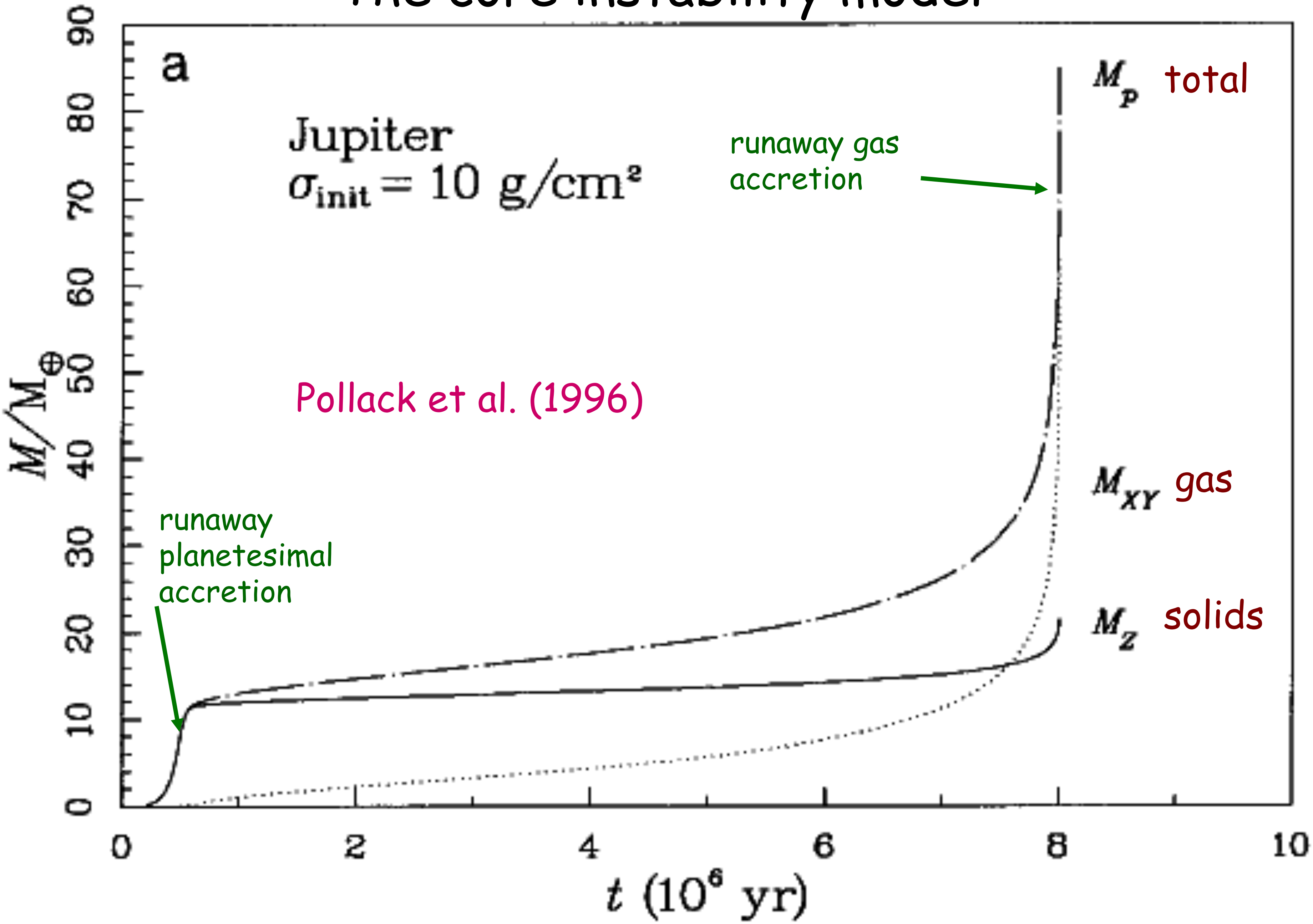
How do the gas giants acquire their huge envelopes?

- escape speed from a growing solid protoplanet with the density of the Earth is

$$v_{esc} = 11 \text{ km/s } (M/M_{\oplus})^{1/3}$$

- growing protoplanet gradually acquires a static atmosphere
- when $v_{esc} \gg c$ the mass of the atmosphere can exceed the mass of the planet ⇒ instability
- in more detail:
  - energy balance is determined by competition between cooling by radiation and heating by accreting planetesimals
  - as planetesimal density falls, cooling wins and the accretion of gas runs away until the planet acquires all of the gas in its region of the disk
  - runaway sets in at about 10-20 $M_{\oplus}$
  - must occur before protoplanetary gas disk is dispersed

The core instability model



Planet migration

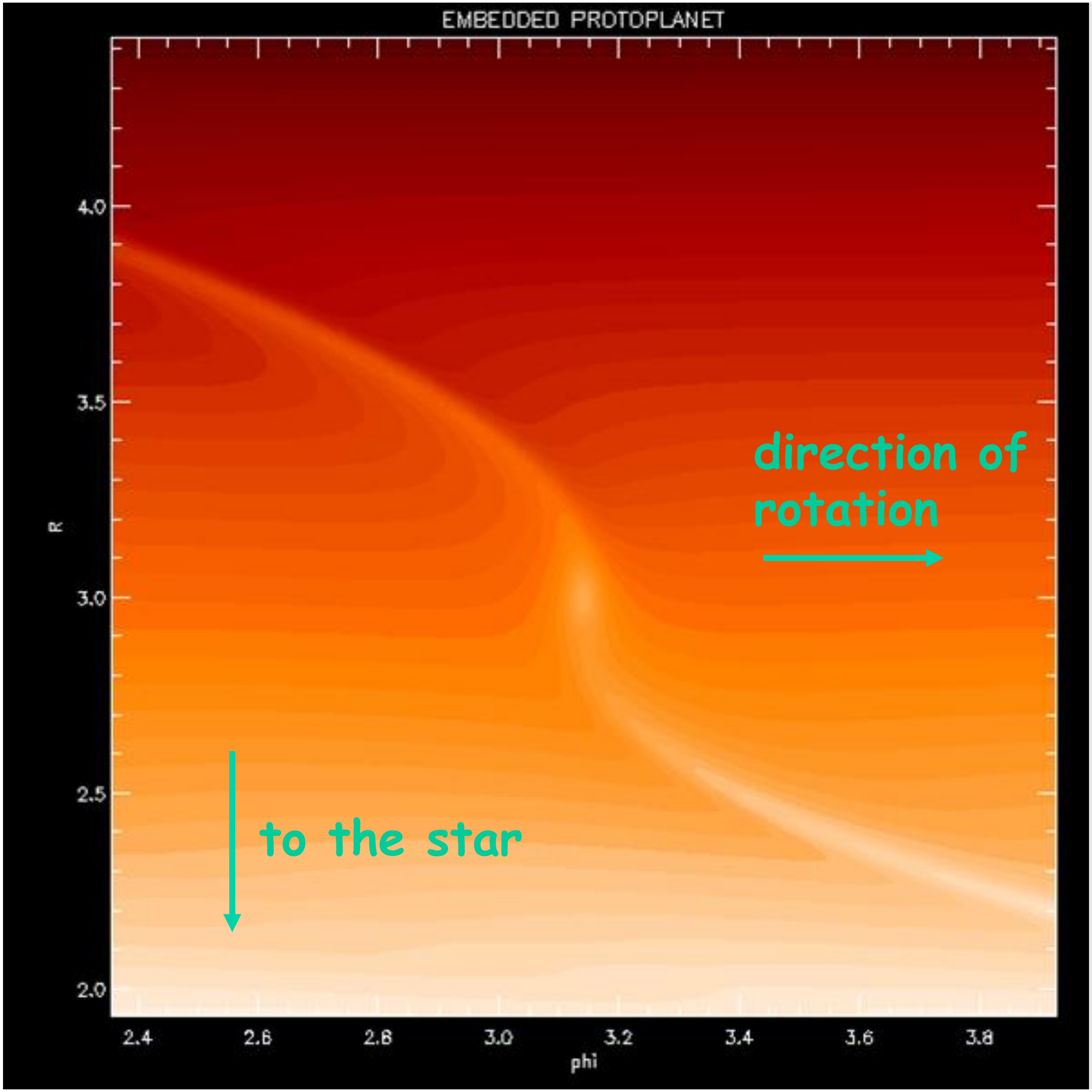
At $r < 0.1$ AU no elements condense so planetesimals cannot form. So why are there planets there?

Gravitational interactions between a planet and the surrounding gas disk leads to repulsive torques between them.

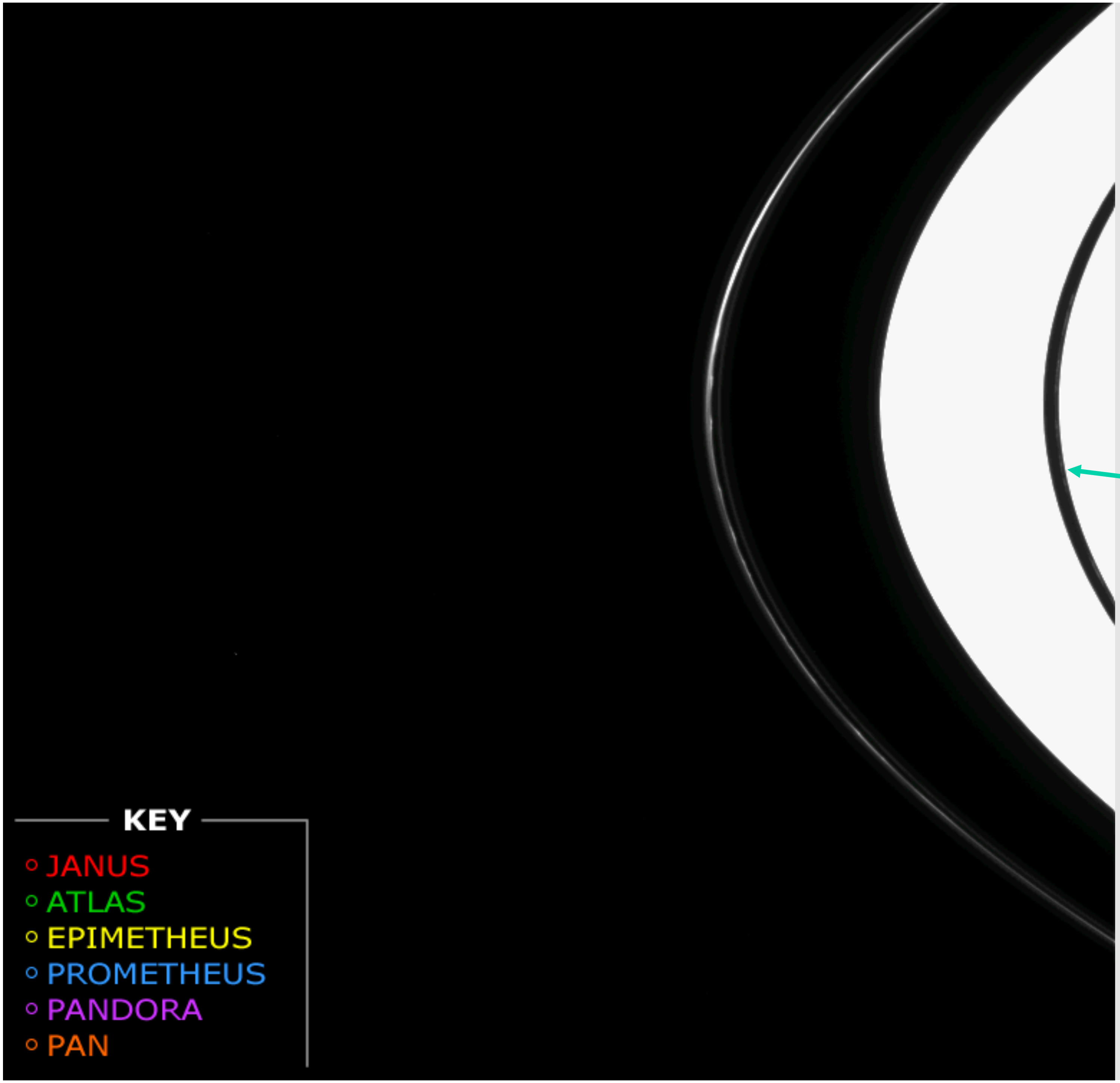
Repulsion is independent of viscosity, pressure, self-gravity, whether particles or gas, etc.

Two main effects:

- imbalance between inner and outer torques leads to migration, usually inward (can create planets at very small semi-major axes)
- gap formation (terminates accretion and mass growth)

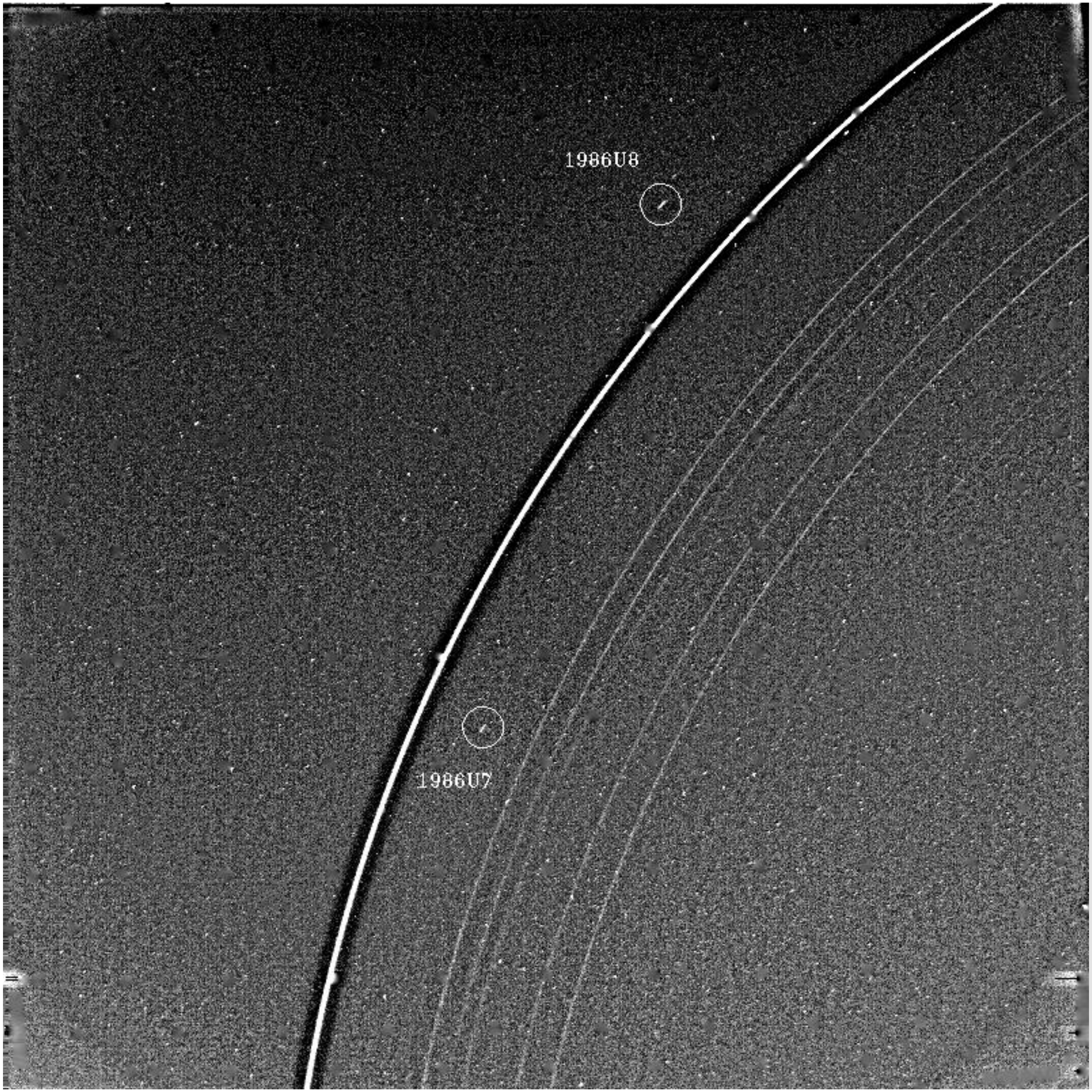


R. Nelson, University of London



small satellites can open gaps or create narrow rings

Huygens gap



Cordelia and Ophelia at Uranus

Types of migration

Type I: low mass planet only weakly perturbs the disk

- timescale of order $\Omega^{-1} (\Sigma R^2/M_\odot)(M_p/M_\odot)$
- very rapid, ~ 10^4 years for Jupiter in minimum solar nebula
- usually inward

Type II: bigger planet opens a gap in the disk

- planet evolves with the disk on the disk's viscous evolution timescale (acts like a disk particle)
- probably ~ 10^4 - 10^5 yr timescale
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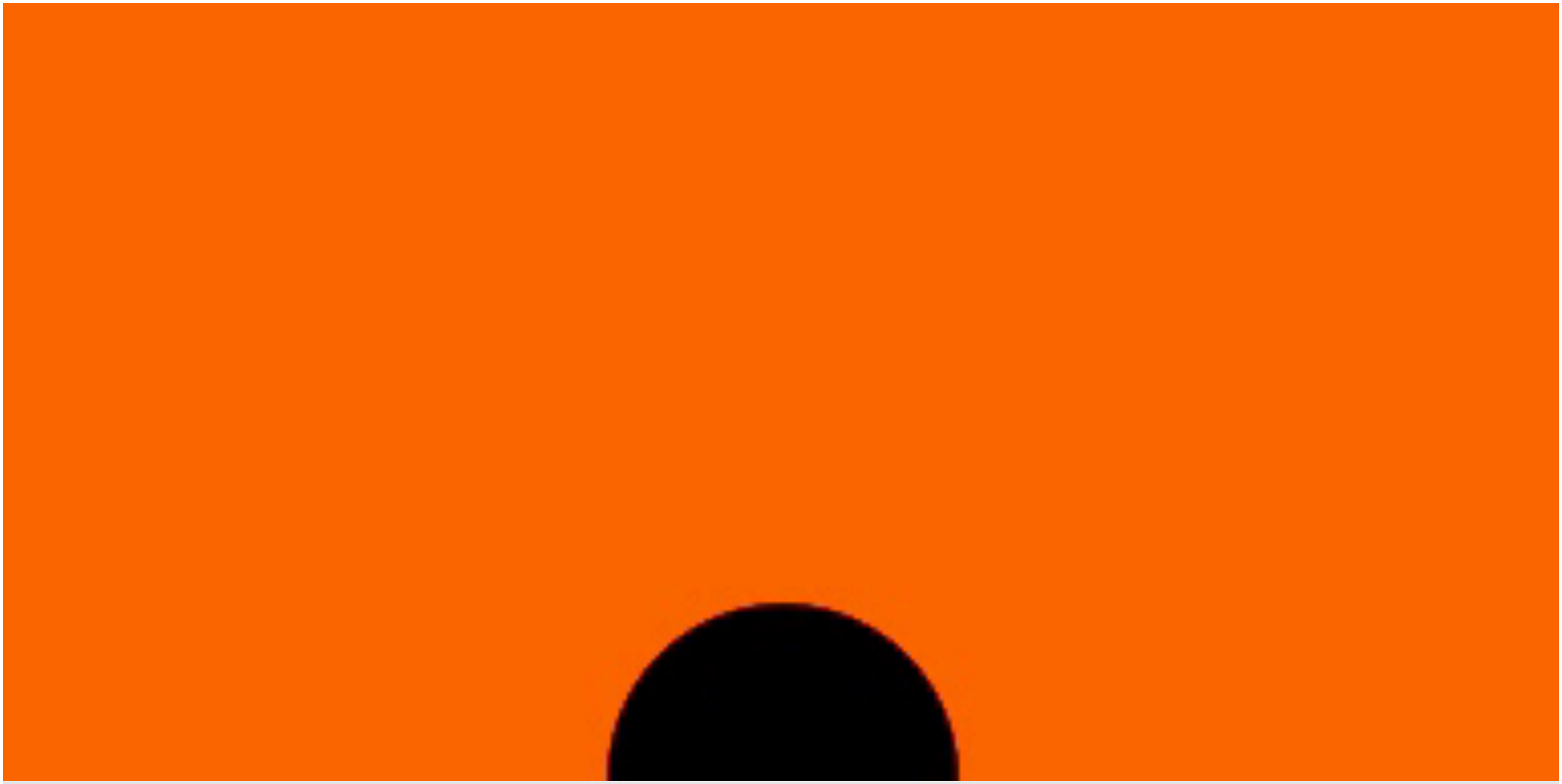

from Masset (2002)

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Both Type I and Type II migration have timescales much shorter than the age of the protoplanetary gas disk (10^6 – 10^7 yr)

Migration

- migration from larger radii offers a plausible way to form giant planets at small radii, but:
  - why did the migration stop?
  - why are the planetary semimajor axes distributed over a wide range?
  - why did migration not occur in the solar system?