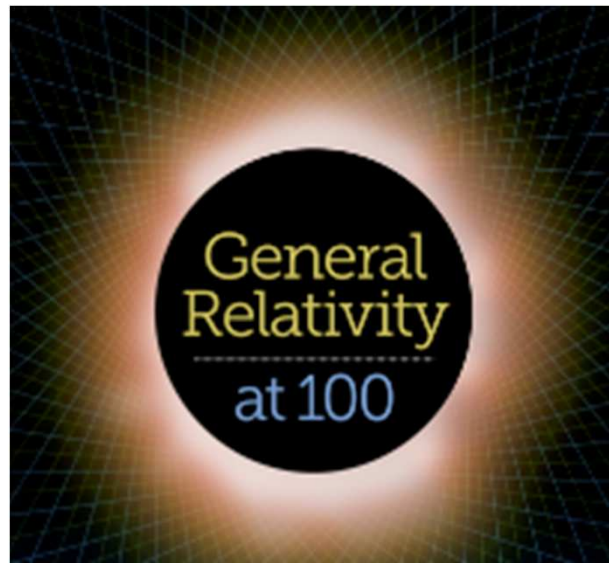


Quantum Gravity

Juan Maldacena

Institute for Advanced Study



November 2015

General Relativity produced two stunning predictions:

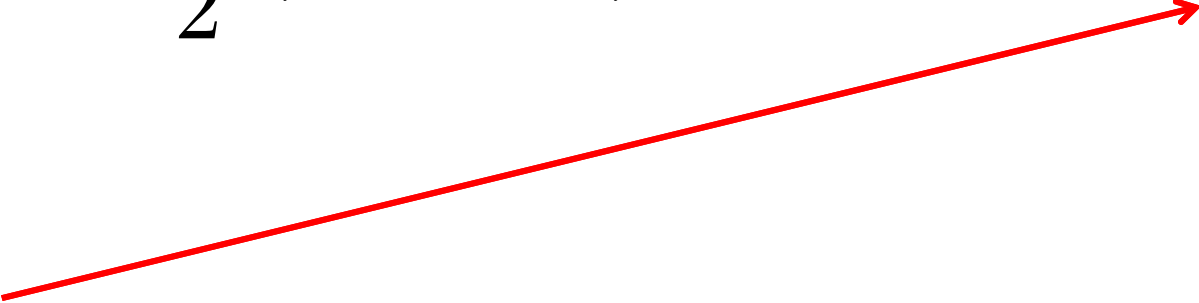
- Black holes
- Expanding universe

“Your math is great, but your physics is dismal”

(Einstein to LeMaitre)

Both involve drastic stretching of space and/or time

Quantization

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = 8\pi G_N T_{\mu\nu}$$


Matter \rightarrow quantum mechanical \rightarrow left hand side should be quantum mechanical also.

The stress tensor is an operator \rightarrow geometry also !

In most circumstances we can neglect the quantum fluctuations.

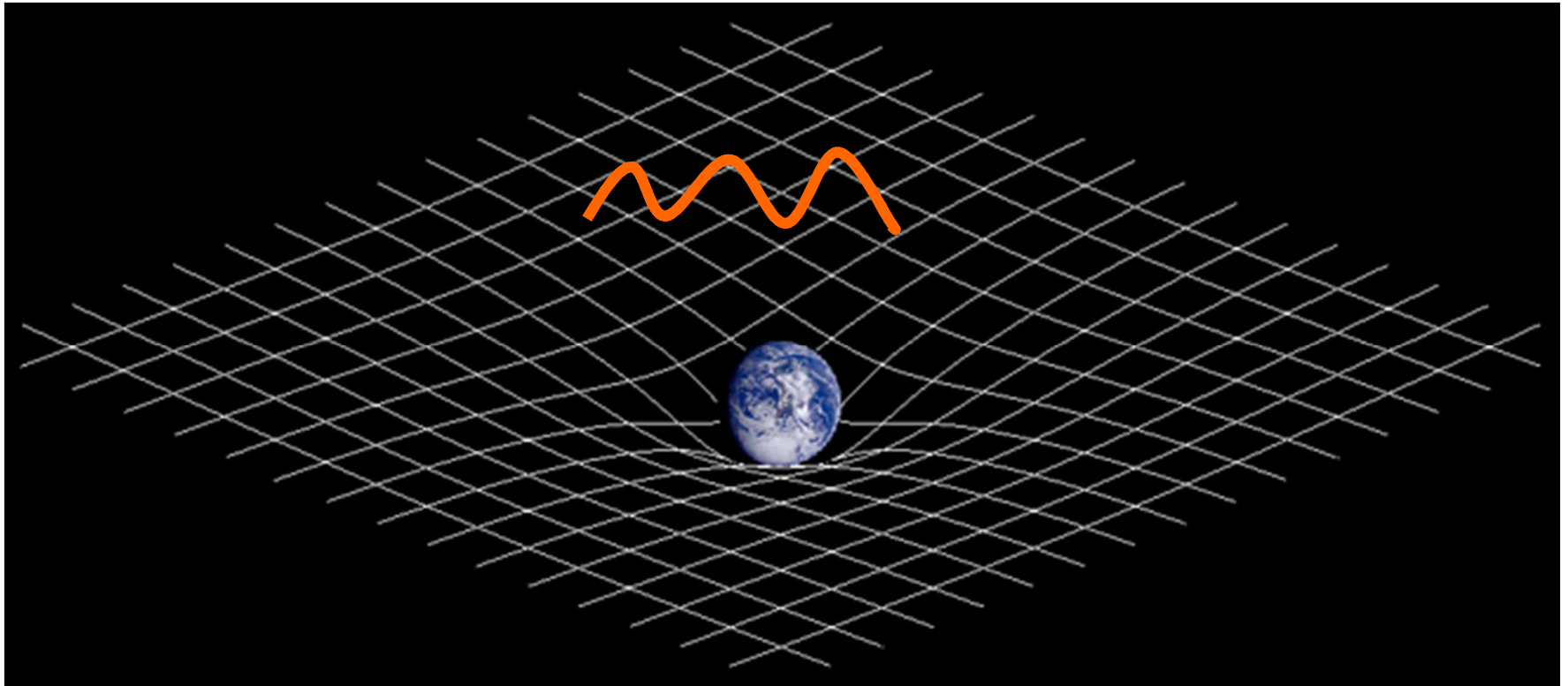
But sometimes they are crucial. E.g. beginning of the Big Bang.

Results from three approaches

- 1) Effective field theory.
- 2) Well defined perturbation theory. Strings theory.
- 3) Some exact (non-perturbative) examples.

Quantum fields on a background geometry

Small deviations of the geometry give rise to gravity waves \rightarrow treat them as one more quantum field.



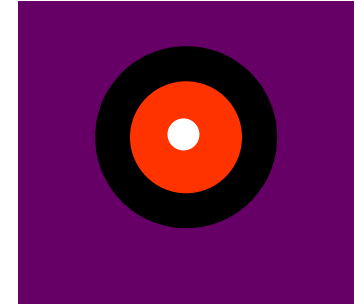
- Even this simple approximation gives surprising predictions!

Two surprising predictions

- Black holes have a temperature

$$T \sim \frac{\hbar}{r_H}$$

Hawking



We can have white ``black holes''

- An accelerating expanding universe also has a temperature

$$T \sim \hbar H = \frac{\hbar}{R_H}$$

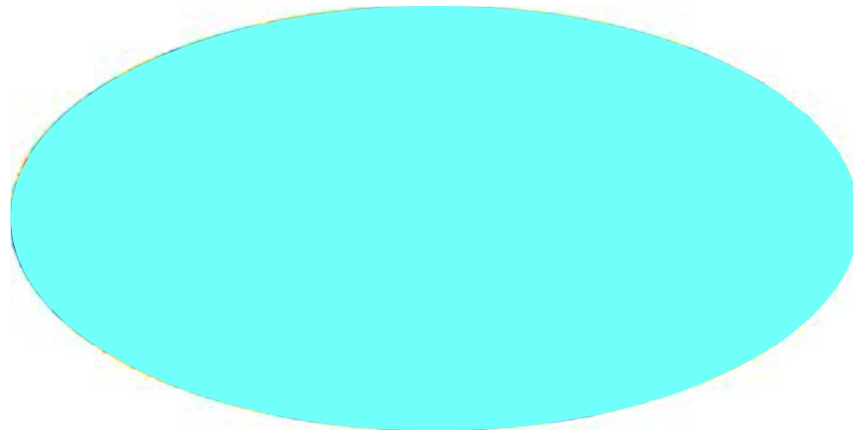
Chernikov, Tagirov,
Figari, Hoegh-Krohn, Nappi,
Gibbons, Hawking,
Bunch, Davies,

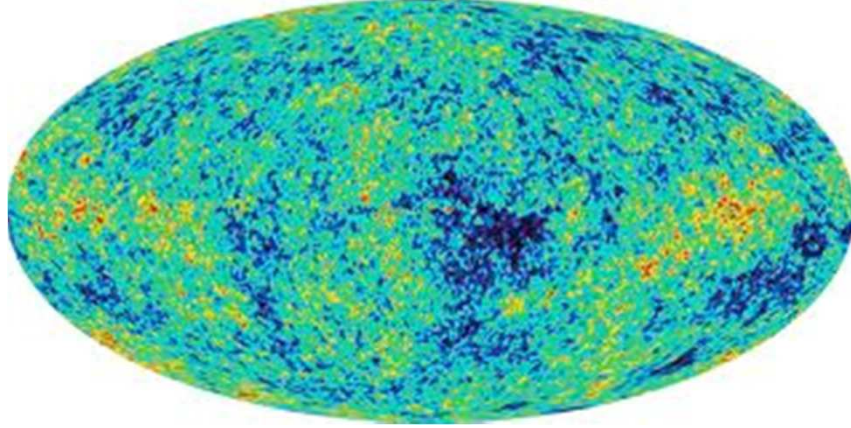
Very relevant for us!

Inflation

Starobinski, Mukhanov
Guth, Linde,
Albrecht, Steinhardt, ...

- Period of expansion with almost constant acceleration.
- Produces a large homogeneous universe



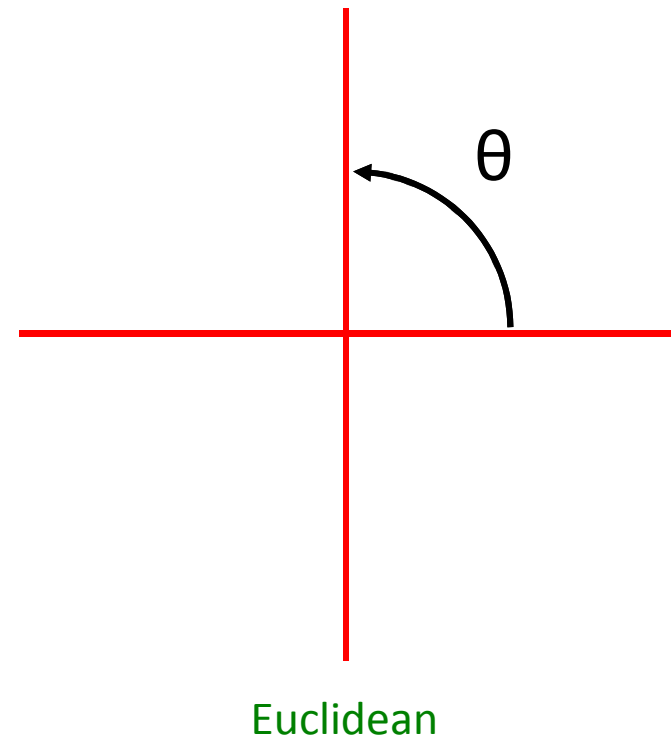
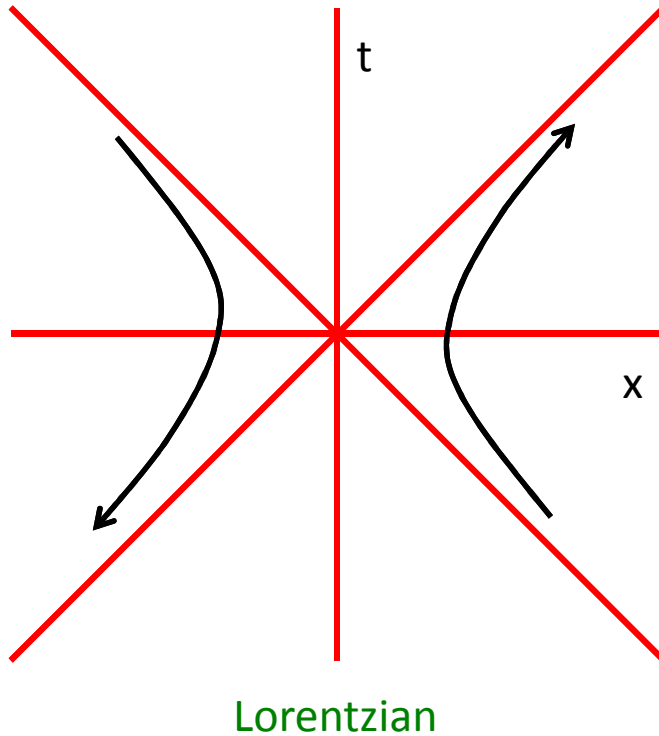


Quantum mechanics is crucial for understanding the large scale geometry of the universe.

Why a temperature ?

- Consequence of special relativity + quantum mechanics.

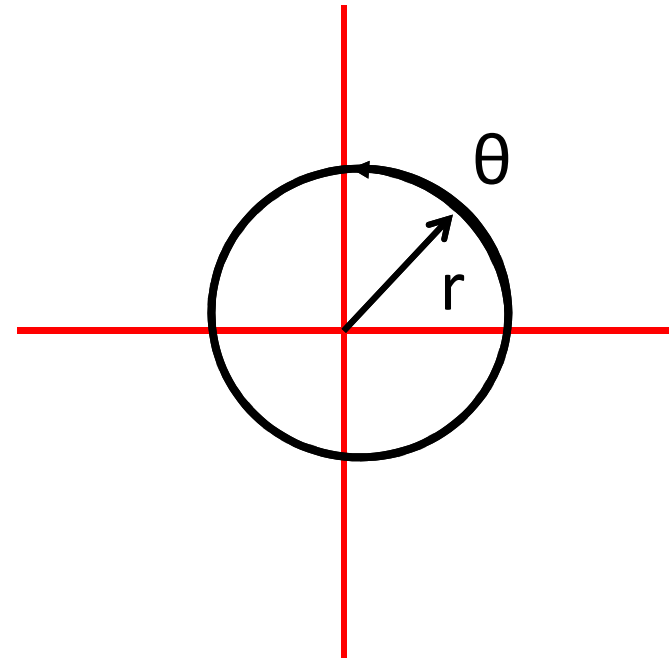
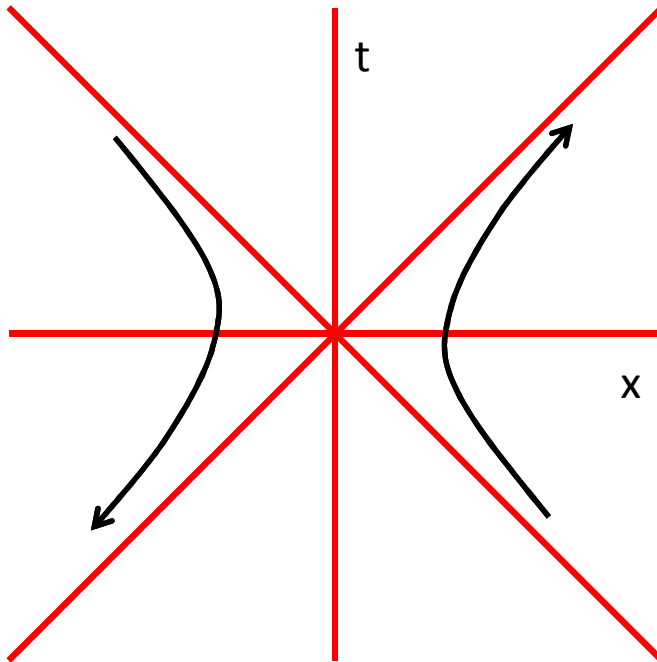
Why a temperature ?



Accelerated observer \rightarrow energy = boost generator.

Continue to Euclidean space \rightarrow boost becomes rotation.

Why a temperature ?



Continue to Euclidean space \rightarrow boost becomes rotation.

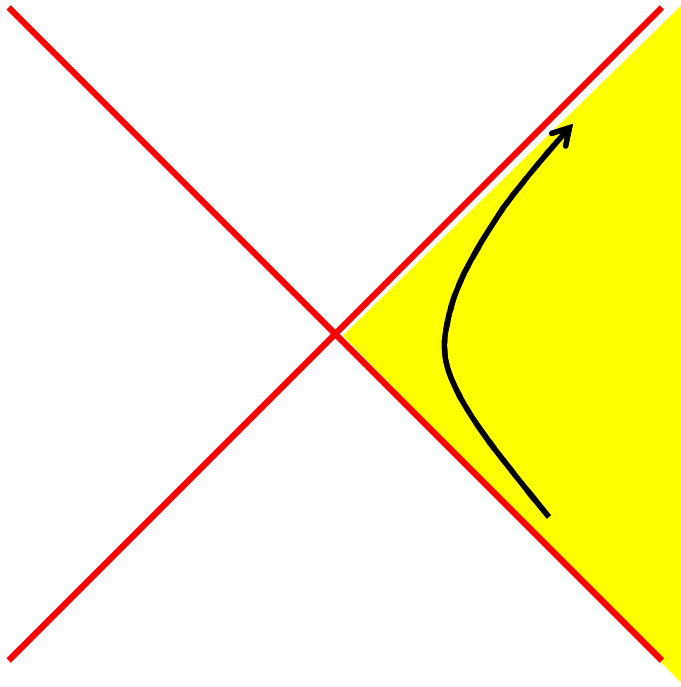
Angle is periodic \rightarrow temperature

$$\beta = \frac{1}{T} = 2\pi r = \frac{2\pi}{a}$$

Ordinary accelerations are very small, $g = 9.8 \text{ m/s}^2 \rightarrow \beta = 1 \text{ light year}$

Bisognano Weichman, Unruh

Entanglement & temperature

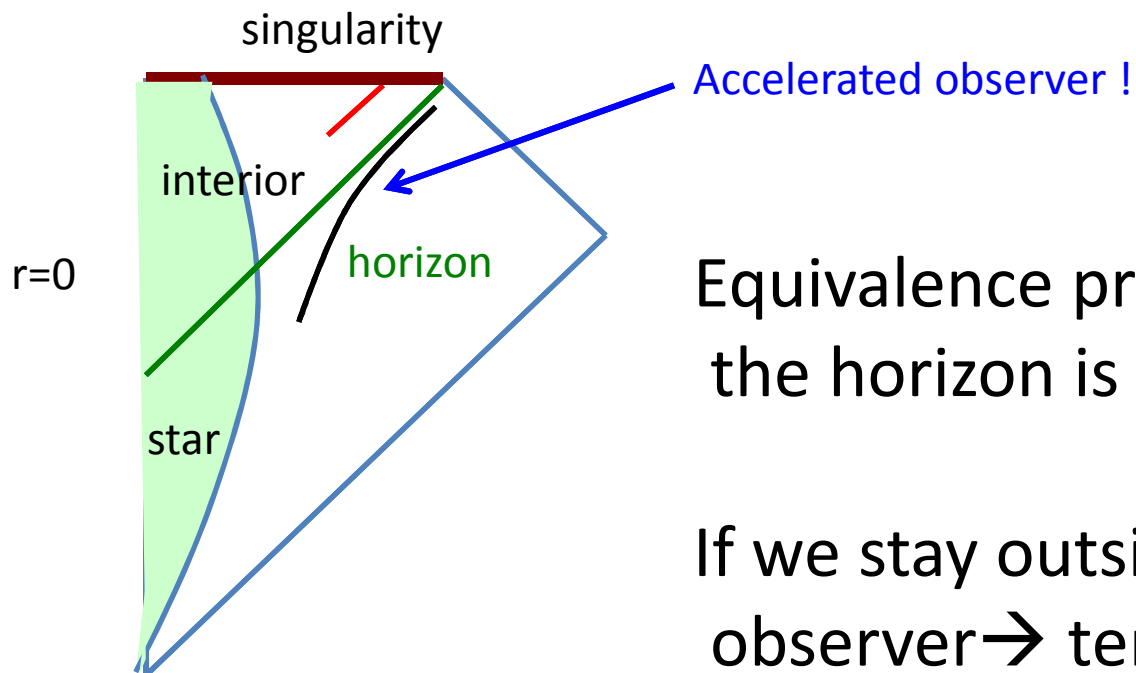


Horizon: accelerated observer only has access to the right wedge.

If we only make observations on the right wedge \rightarrow do not see the whole system \rightarrow get a mixed state (finite temperature).

Vacuum is highly entangled !

Black hole case



Equivalence principle: region near the horizon is similar to flat space.

If we stay outside → accelerated observer → temperature.

Black hole from collapse

Low energy effective theory

- Gravity becomes stronger at higher energies.

$$g_{\text{eff}}^2 \sim \frac{(\text{energy})^2}{M_{\text{Planck}}^2} = \frac{l_{\text{Planck}}^2}{(\text{distance})^2}$$

$$l_{\text{Planck}} \sim 10^{-35} m, \quad M_{\text{Planck}} = 10^{16} \text{TeV}$$

- Quantum effects are very small at low energies.

Limitations of the effective field theory expansion

- Expand in powers of g_{eff}^2 .
- Infinities \rightarrow counterterms \rightarrow new parameters
- Not a well defined theory.


2) A gravity theory with a well defined perturbative expansion

- Gravity $m=0$, spin 2 particles. $\delta g_{\mu\nu}$
- Add $m=0$, spin > 2 particles \rightarrow “weird” theory of gravity (with non-zero cosmological constant).
- Add $m>0$, spin > 2 particles \rightarrow ?
One example: String theory.

Vasiliev

Veneziano, ..., Green, Schwarz,....

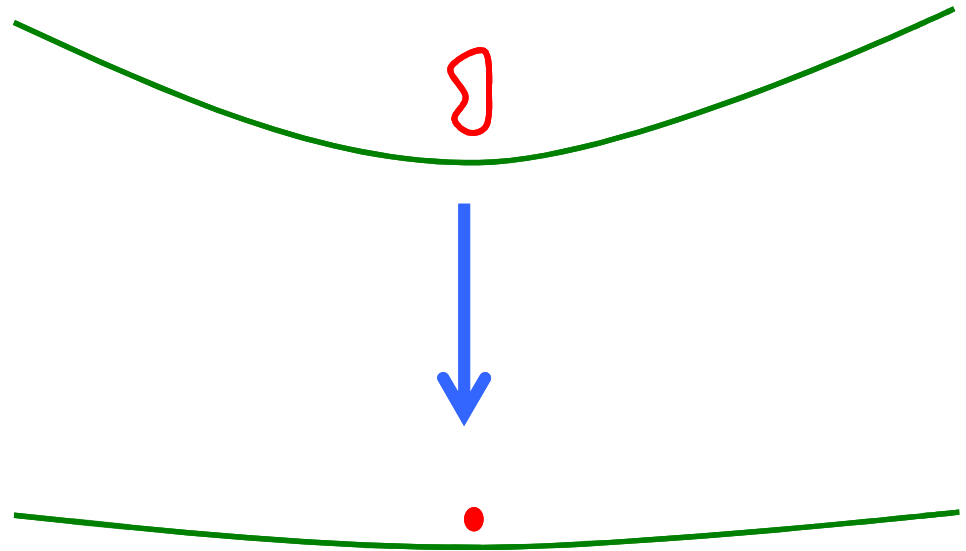
String theory

- Free particles \rightarrow Free strings 
- Different vibration modes of the string are different particles with different masses.
- Lowest mode $\rightarrow m=0$, spin =2 \rightarrow graviton.
- Theory reduces to gravity at low energies.

Gravity from strings

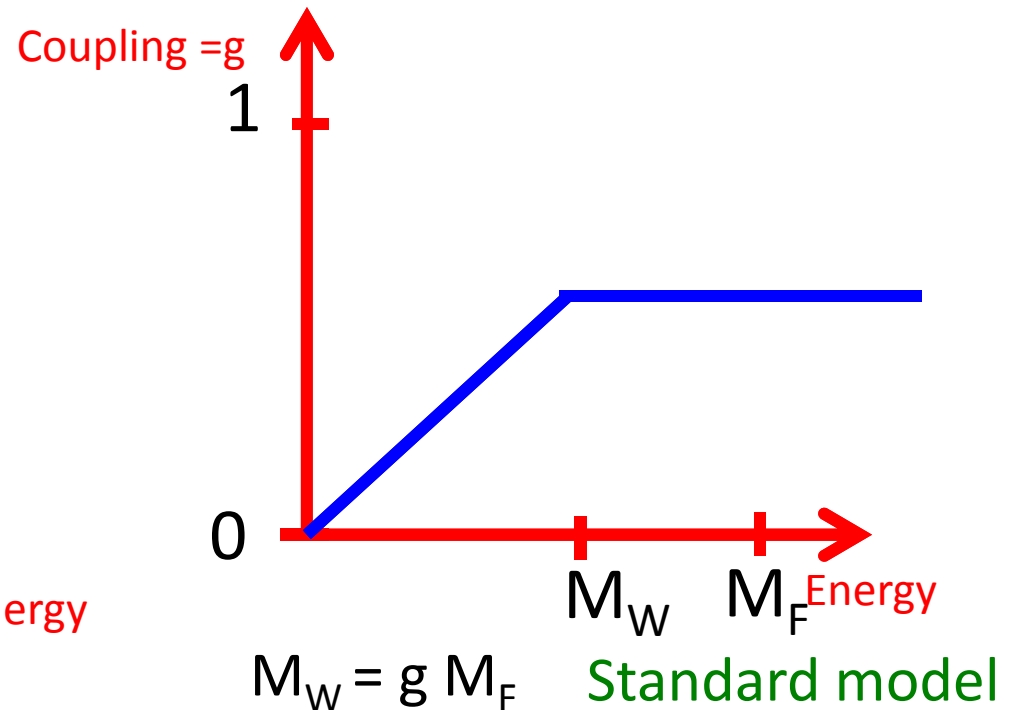
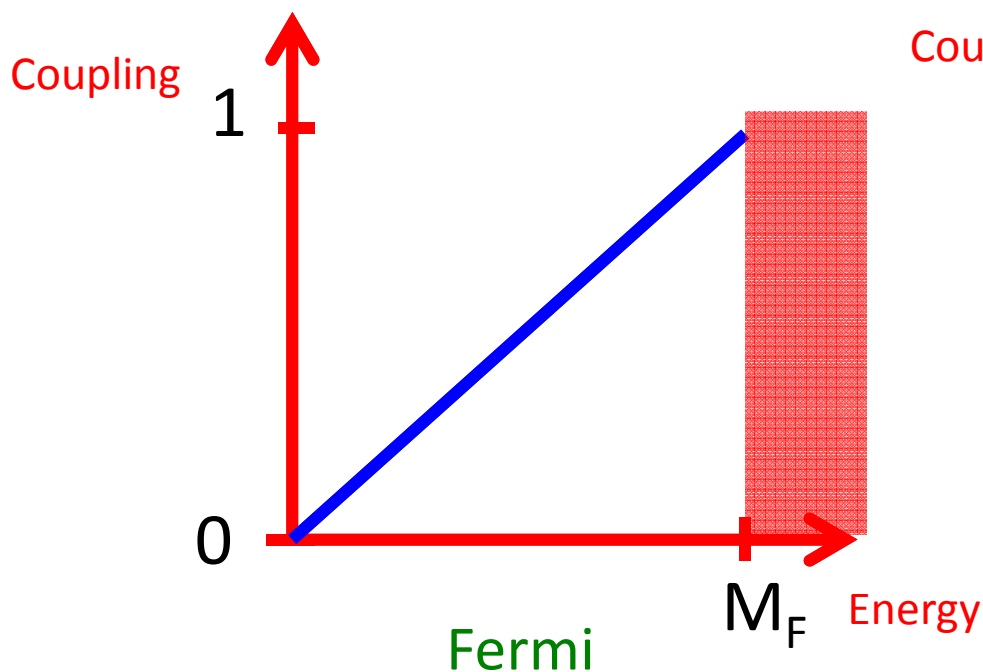
$$\frac{\text{Radius of curvature}}{\text{size of string}} \gg 1$$

Reduces to Einstein gravity



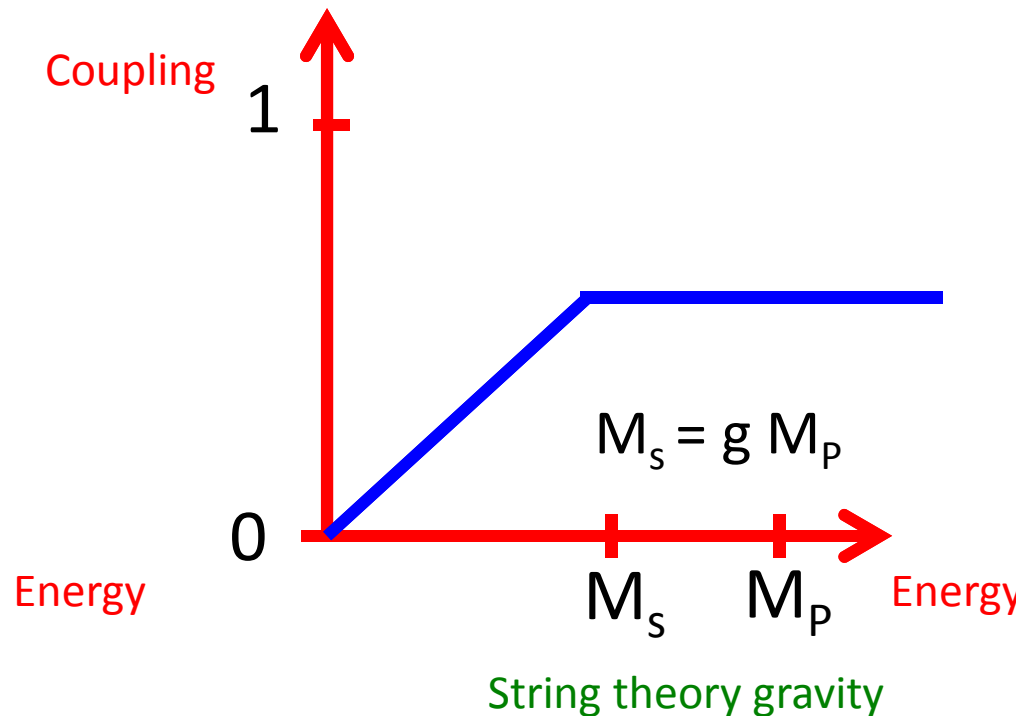
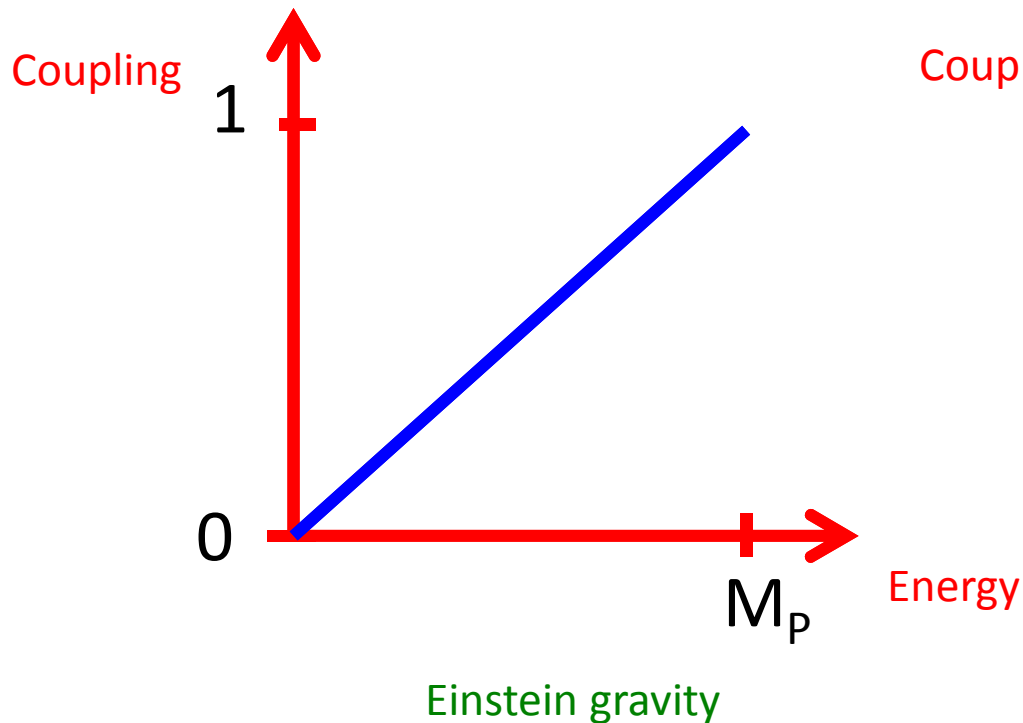
Analogy: Weak interactions vs. gravity

- In the Fermi theory of weak interactions.
- $G_N \rightarrow G_F = 1/M_F^2$



Weak interactions vs. gravity

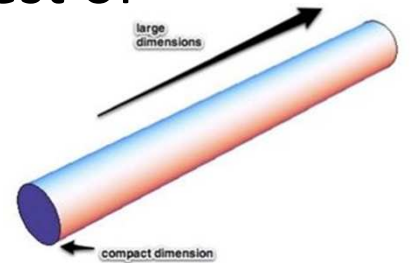
- In string theory it is similar.



M_S = mass of the lightest spin>2 particle

The beauty of string theory

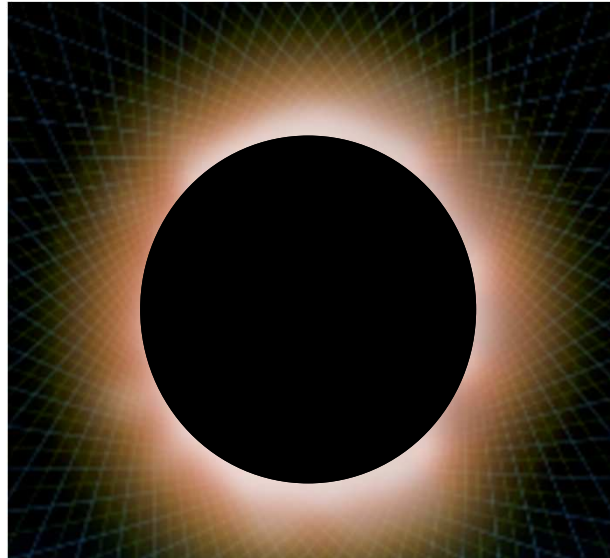
- Well defined theory of quantum gravity. No ultraviolet divergences.
- Dimension of (flat) spacetime is fixed to 10.
- Four large + 6 small dimensions might explain the rest of the forces.
- Incorporates chiral gauge interactions and grand unification.
- Supersymmetry and supergravity (small fermionic extra dimensions).



3) Exact description in some cases.

Why do we care ?

- Puzzles with aspects of black holes.
- Big Bang and the multiverse



Black holes have a temperature

Wheeler's question to Bekenstein

Do they obey the laws of thermodynamics ?

Black hole entropy

$$T \sim \frac{\hbar}{r_H}$$

Special relativity near the black hole horizon

$$r_H \leftrightarrow M$$

Einstein equations

$$dM = TdS$$

1st Law of thermodynamics

$$S = \frac{(\text{Area})}{4\hbar G_N}$$

Black hole entropy

Bekenstein, Hawking

2nd Law \rightarrow area increase from Einstein equations and positive null energy condition.

Hawking,....,

A. Wall: including quantum effects. Monotonicity of relative entropy

General relativity and thermodynamics

- Black hole seen from the outside = thermal system.
- Is there an exact description where information is preserved ?
- Yes, but we need to go beyond perturbation theory...

3) Exact description in some cases.

String theory: beyond perturbation theory

- String theory started out defined as a perturbative expansion.
- String theory contains interesting solitons: D-branes.
- D-branes inspired some non-perturbative definitions of the theory in some cases.

Polchinski

Matrix theory: Banks, Fischler, Shenker, Susskind

Gauge/gravity duality: JM, Gubser, Klebanov, Polyakov, Witten

Gauge/Gravity Duality

(or gauge/string duality, AdS/CFT, holography)

Theories of quantum
interacting particles



Quantum dynamical
Space-time
(General relativity)
string theory

Gravity in asymptotically Anti de Sitter Space



Anti de Sitter = hyperbolic space
with a time-like direction

Gravity in asymptotically Anti de Sitter Space

Duality



Gravity,
Strings

Quantum interacting particles
quantum field theory



Strings from gauge theories.

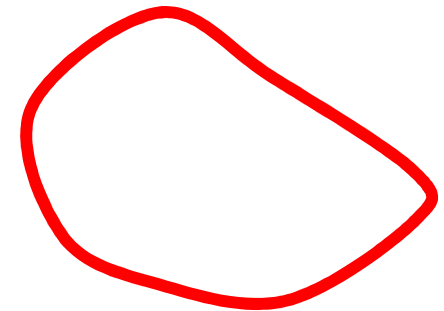
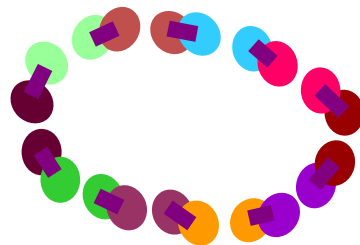
- In large $SU(N)$ gauge theories, Faraday lines \rightarrow dynamical strings.

Strings from gauge theories

Gluon: color and anti-color 

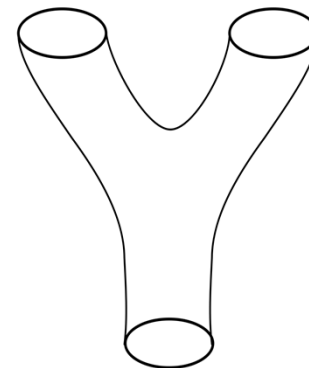
Take N colors $SU(N)$

Large N limit



t' Hooft '74

String coupling $\sim 1/N$



Experimental evidence for strings in strong interactions

Experimental evidence for strings in strong interactions

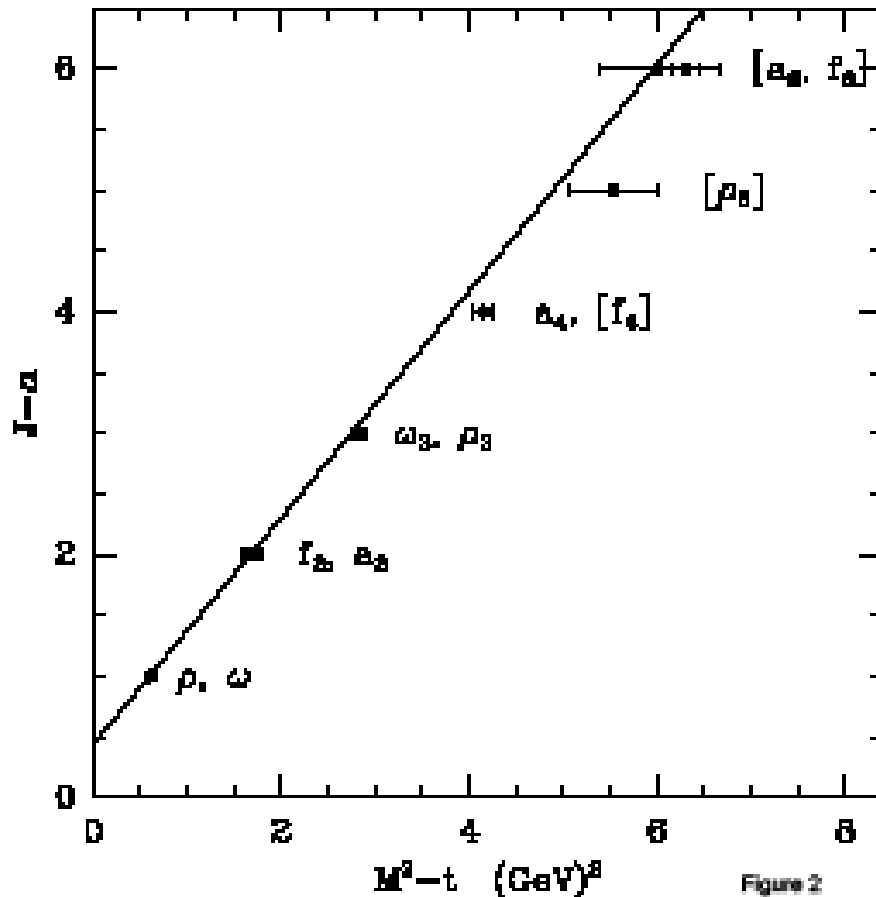
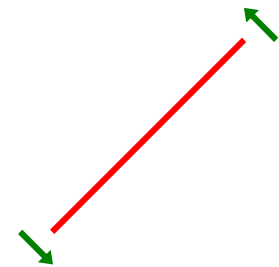


Figure 2

From E. Klempt hep-ex/0101031



Rotating String model

$$m^2 \sim TJ_{\max} + \text{const}$$

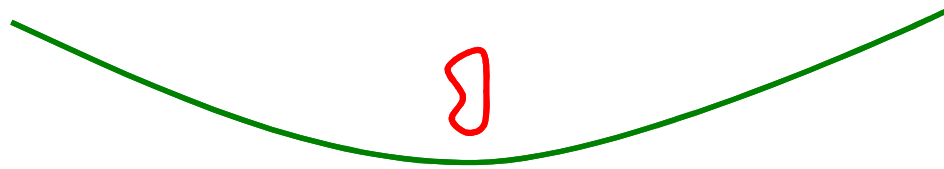
Strings of chromodynamics are the
strings of string theory.

Quantum gravity in one higher
dimension

Strings of supersymmetric
chromodynamics (in 4d) are the
strings of ten dimensional string
theory.

Gravity from strings

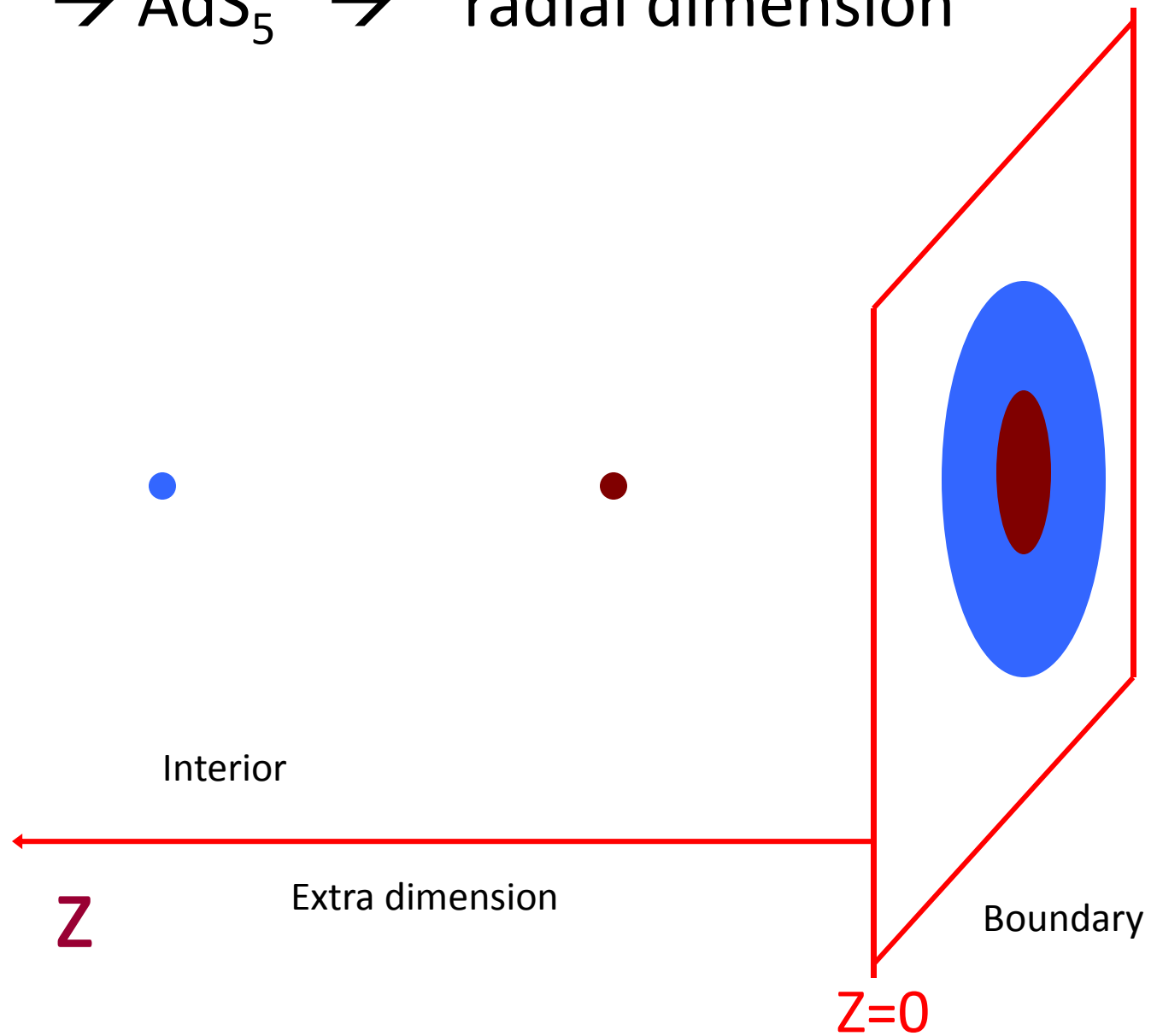
$$\frac{\text{Radius of curvature}}{\text{size of string}} \sim (\text{effective field theory coupling})^{\text{positive}}$$

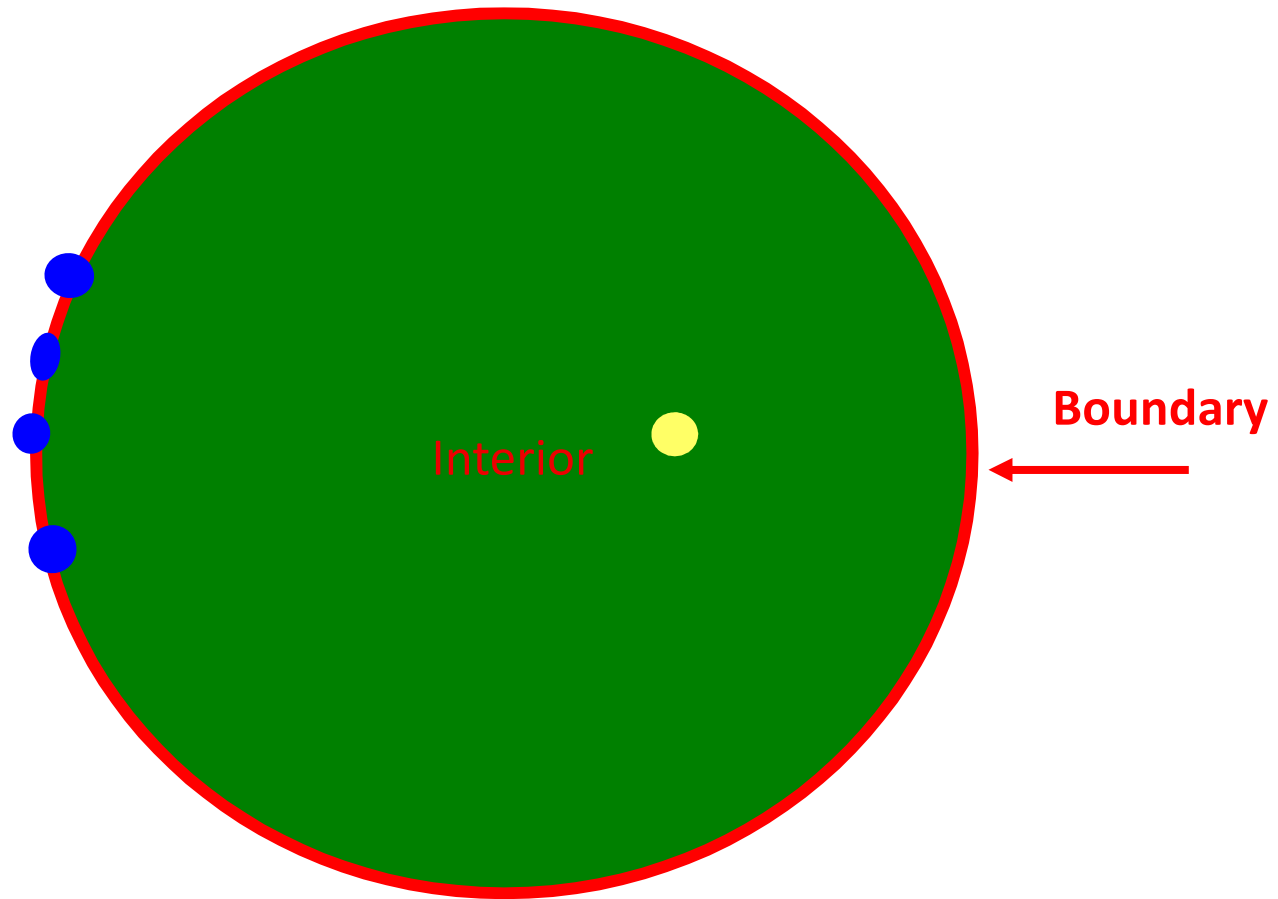


$$\text{string coupling} \sim G_N \propto \frac{1}{N^2}$$

Einstein gravity \rightarrow We need large N and strong coupling.

$3+1 \rightarrow \text{AdS}_5 \rightarrow \text{radial dimension}$





Einstein Gravity in the interior \rightarrow Described by very strongly interacting particles on the boundary.

BLACK HOLES = High energy, thermalized states on the boundary

- Entropy = Area of the horizon = Number of states in the boundary theory.

Strominger, Vafa,...

- Falling into the black hole = thermalization of a perturbation in the boundary theory.
- Unitary as viewed from the outside.

Black holes and hydrodynamics

- Field theory at finite temperature = black brane in Anti-de-Sitter space

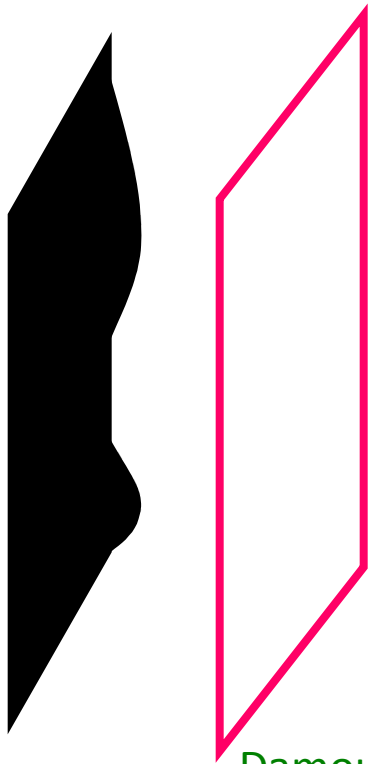
Ripples on the black brane =
hydrodynamic modes

Absorption into the black hole = dissipation,
viscosity.

Transport coefficients → Solving wave equations
on the black brane.

Einstein equations → hydrodynamics
(Navier Stokes equations)

Discovery of the role of anomalies in
hydrodynamics



Damour, Herzog, Son, Kovtun, Starinets, Bhattacharyya, Hubeny, Loganayagam,
Mandal, Minwalla, Morita, Rangamani, Reall, Bredberg, Keeler, Lysov, Strominger...

Black holes as a source of information!

- Strongly coupled field theory problems → Simple gravity problems.


Heavy ion collisions, high temperature superconductors, etc..

- Geometrization of physics !

Emergent geometry

- The exact description lives on the boundary.
- Good general relativity observables are defined on the boundary.
- The spacetime interior “emerges” in the large N and strong coupling limit.
- We do not have an exact description in terms of bulk variables...

Entanglement and geometry

A circular fractal pattern, resembling a Sierpinski triangle, is shown. The pattern is composed of black and white geometric shapes. A thick red line forms a circle around the entire pattern. On the right side of the circle, a blue arc is drawn, and along this arc, there are several orange dots. The text "Local boundary quantum bits are highly interacting and very entangled" is written in orange above the blue arc.

Local boundary
quantum bits are
highly interacting and
very entangled

$$S(R) = \frac{A_{\min}}{4G_N}$$

Ryu, Takayanagi,
Hubbeny, Rangamani

Entanglement and geometry

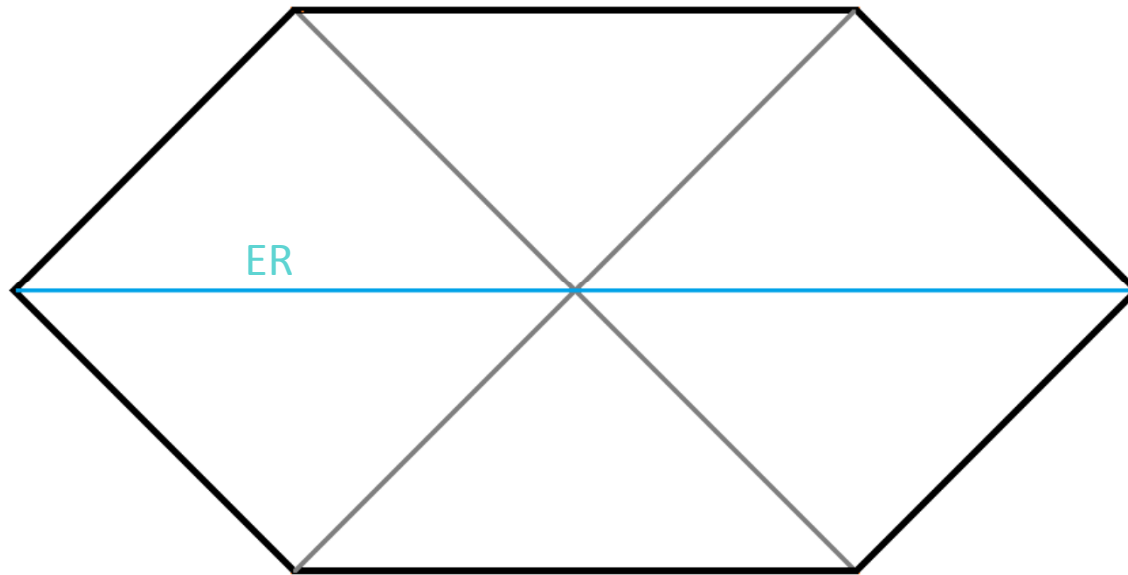
- The entanglement pattern present in the state of the boundary theory can translate into geometrical features of the interior.

Van Raamsdonk

- Spacetime is closely connected to the entanglement properties of the fundamental degrees of freedom.
- Slogan: Entanglement is the glue that holds spacetime together...
- Spacetime is the hydrodynamics of entanglement.



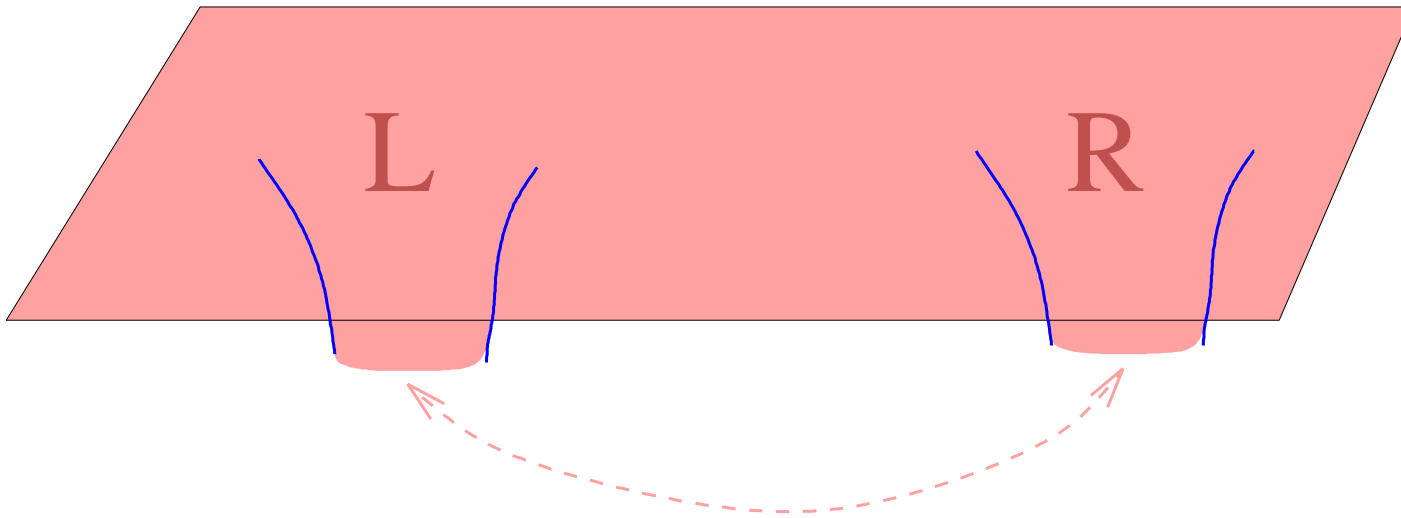
Two sided Schwarzschild solution



Eddington, Lemaitre,
Einstein, Rosen,
Finkelstein
Kruskal

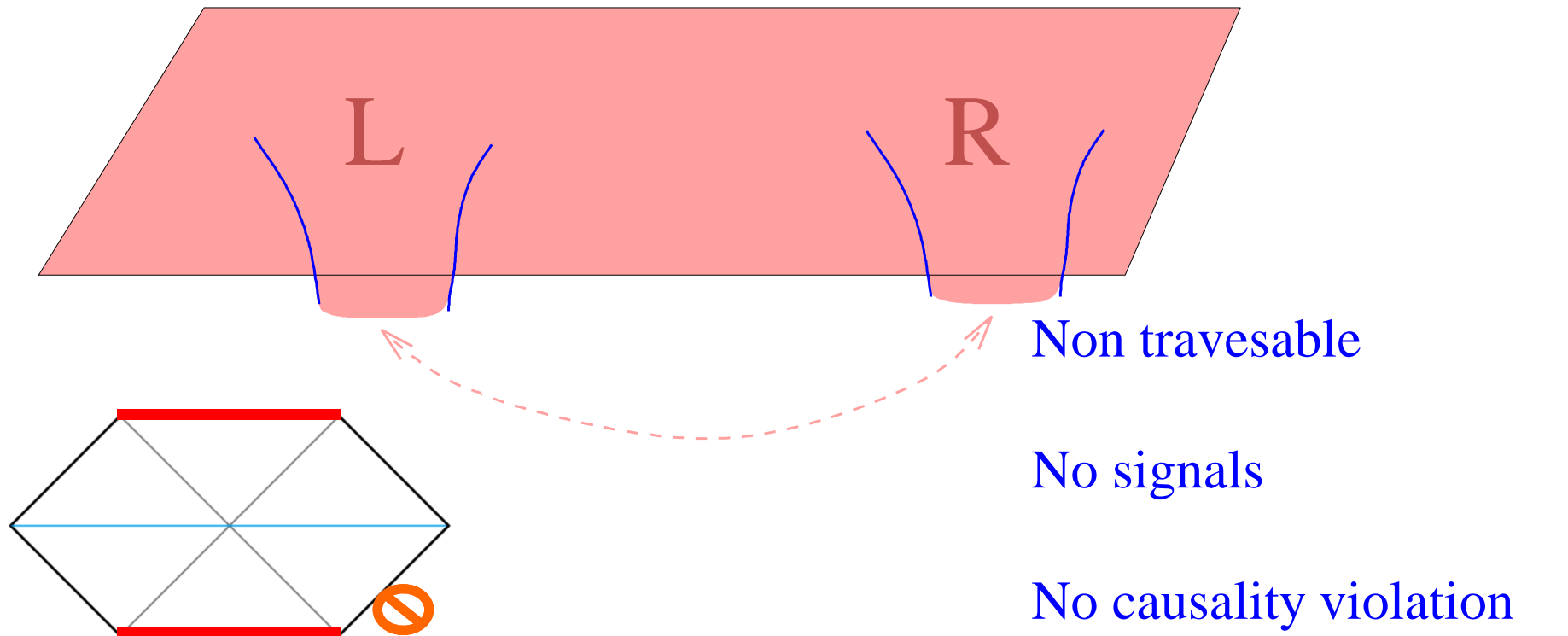
Simplest spherically symmetric solution of pure Einstein gravity
(with no matter)

Wormhole interpretation.

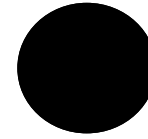
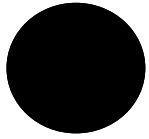


Note: If you find two black holes in nature, produced by gravitational collapse, they will not be described by this geometry

No faster than light travel



Fuller, Wheeler, Friedman, Schleich, Witt, Galloway, Wooglar



In the exact theory,
each black hole is described by a set of microstates from the outside

Wormhole is an entangled state

$$|\Psi\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L \times |E_n\rangle_R$$

EPR

Israel
JM

Geometric connection
from entanglement. ER = EPR

Susskind JM
Stanford, Shenker, Roberts, Susskind

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

EPRA. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in

quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

The Particle Problem in the General Theory of Relativity

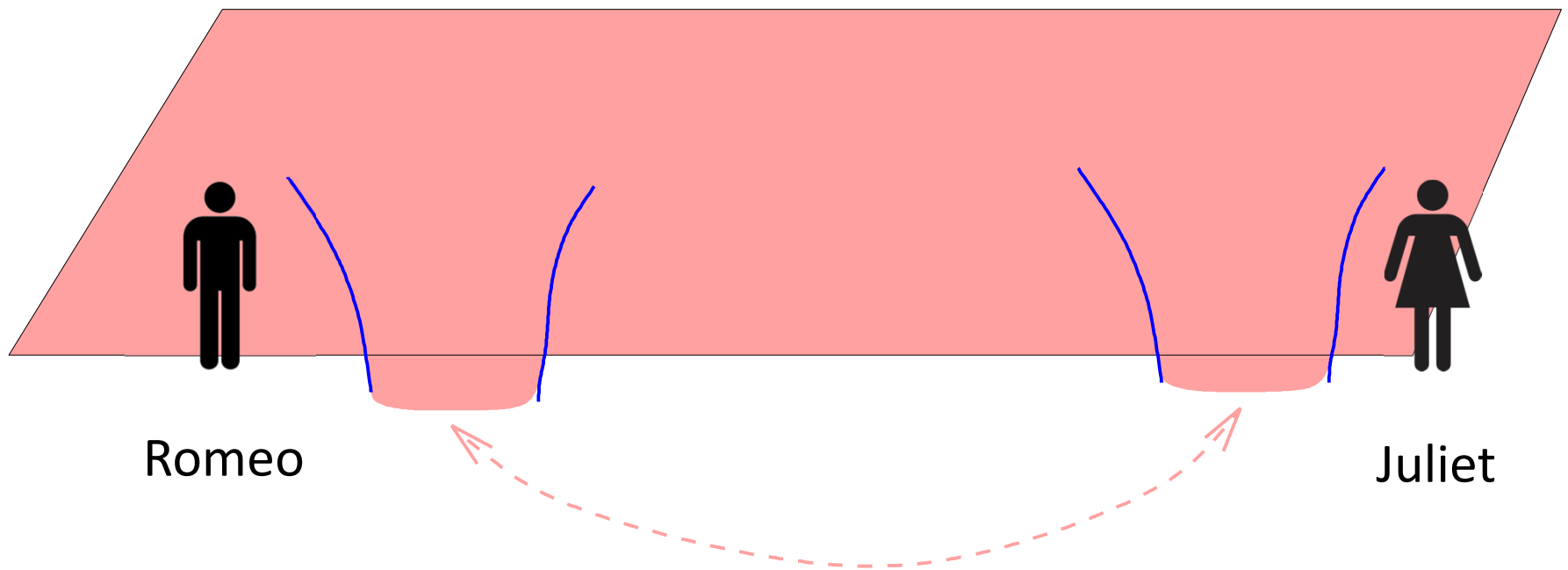
ERA. EINSTEIN AND N. ROSEN, *Institute for Advanced Study, Princeton*

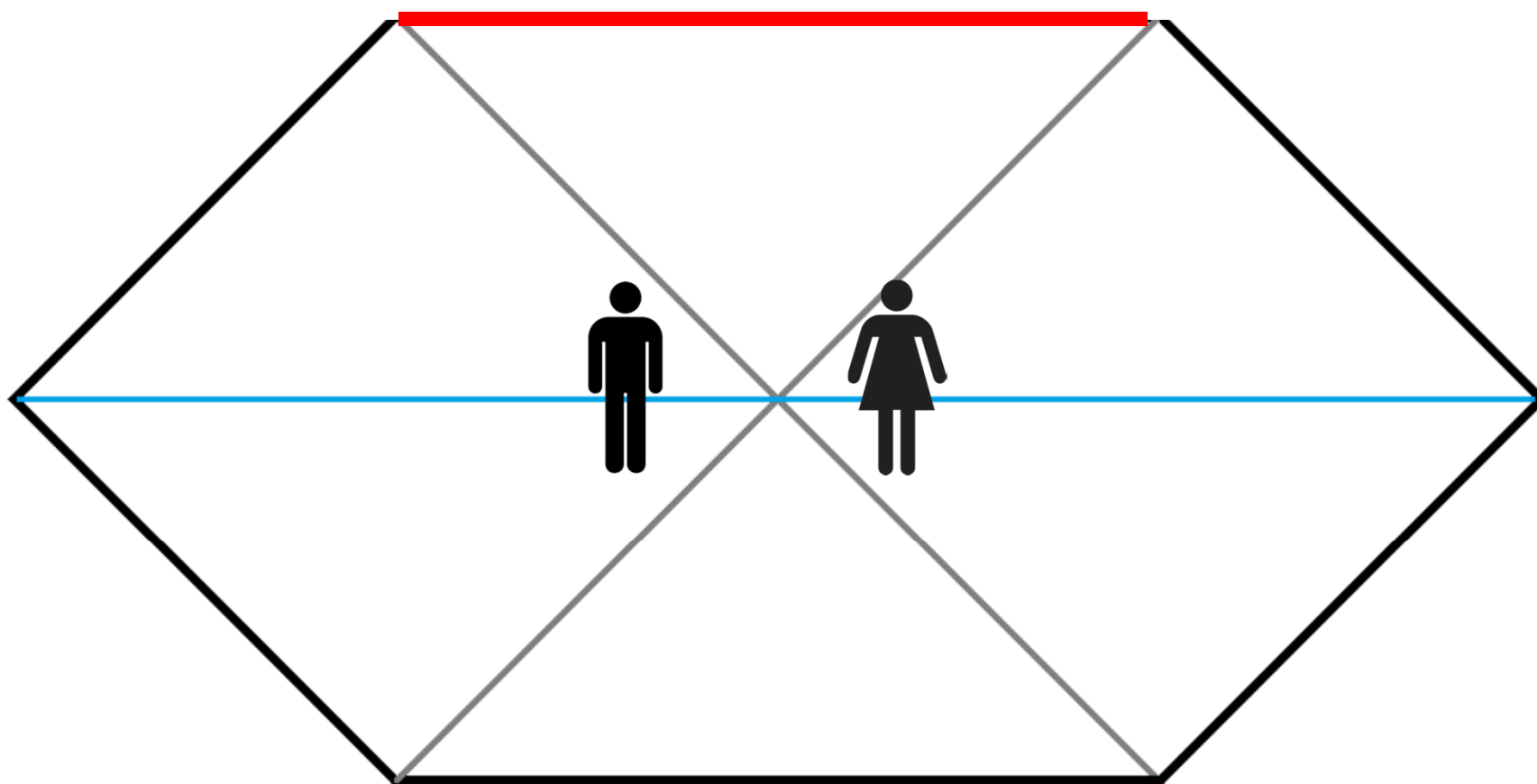
(Received May 8, 1935)

The writers investigate the possibility of an atomistic theory of matter and electricity which, while excluding singularities of the field, makes use of no other variables than the $g_{\mu\nu}$ of the general relativity theory and the φ_μ of the Maxwell theory. By the consideration of a simple example they are led to modify slightly the gravitational equations which then admit regular solutions for the static spherically symmetric case. These solutions involve the mathematical representation of physical space by a space of two identical sheets, a particle being represented by a "bridge" connecting these sheets. One is able to understand why no neutral particles of negative mass are to be

found. The combined system of gravitational and electromagnetic equations are treated similarly and lead to a similar interpretation. The most natural elementary charged particle is found to be one of zero mass. The many-particle system is expected to be represented by a regular solution of the field equations corresponding to a space of two identical sheets joined by many bridges. In this case, because of the absence of singularities, the field equations determine both the field and the motion of the particles. The many-particle problem, which would decide the value of the theory, has not yet been treated.

A forbidden meeting



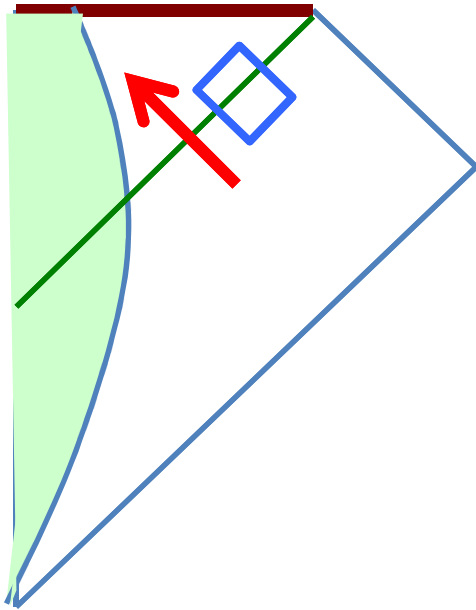


Mysteries

- Black hole interior
- Exact descriptions for spacetimes with no boundaries.
- Big bang singularity
- Multiverse.

Black hole interior

Equivalence principle



From outside: infalling observer never crosses the horizon. It just thermalizes.

Inside: No problem with crossing the horizon. There is an afterlife.

Same thought experiment that Einstein did !

The Multiverse

- Quantum effects might grow larger at bigger distances than the currently observable universe.



The Multiverse

- The ultimate theory can have different macroscopic solutions, with different laws of physics. Eg: string theory → many solutions due to different choices of internal manifolds.
- Inflation, eternal inflation, could naturally populate them all. Even if inflation doesn't, they could all be connected at the initial singularity.

- A multiverse + anthropic constraints, could explain the small value of the cosmological constant and other coincidences.
- We do not know yet how to compute probabilities in the multiverse. (Several conjectures, some ruled out by experiment...).
- We do not yet understand the theory well enough...

- We can predict the small fluctuations from inflation.
- But we cannot predict the constant quantities: the values of the constants (of both standard models) in our universe. We do not have even a probabilistic prediction.
- This is related to understanding the initial singularity.

Conclusions

- Quantum mechanics in curved spacetime gives rise to interesting effects: Hawking radiation and primordial inflationary fluctuations.
- These effects are crucial for explaining features of our universe.
- Black hole thermodynamics poses interesting problems: Entropy, Unitarity, Information problem.

Conclusions

- Exploration of these problems led to connections between strongly coupled quantum systems and gravity.
- General relativity applied to other fields of physics (condensed matter).
- Patterns of entanglement are connected to geometry.

Questions

- Equivalence principle and the observer falling into the black hole ?
- Big bang singularity ?
- Probabilities in the multiverse ?



Still young!

Still full of surprises!

Extra slides

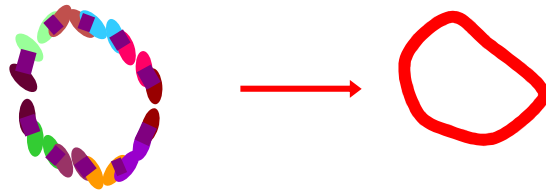
Incorporating Quantum Mechanics

A simple approach

- General relativity \rightarrow is a classical field theory
- We should quantize it
- It is hard to change the shape of spacetime
- For most situations \rightarrow quantum fields in a fixed geometry is a good approximation
- General relativity as an effective field theory
 \rightarrow systematic low energy approximation.

How well established is the gauge/gravity duality ?

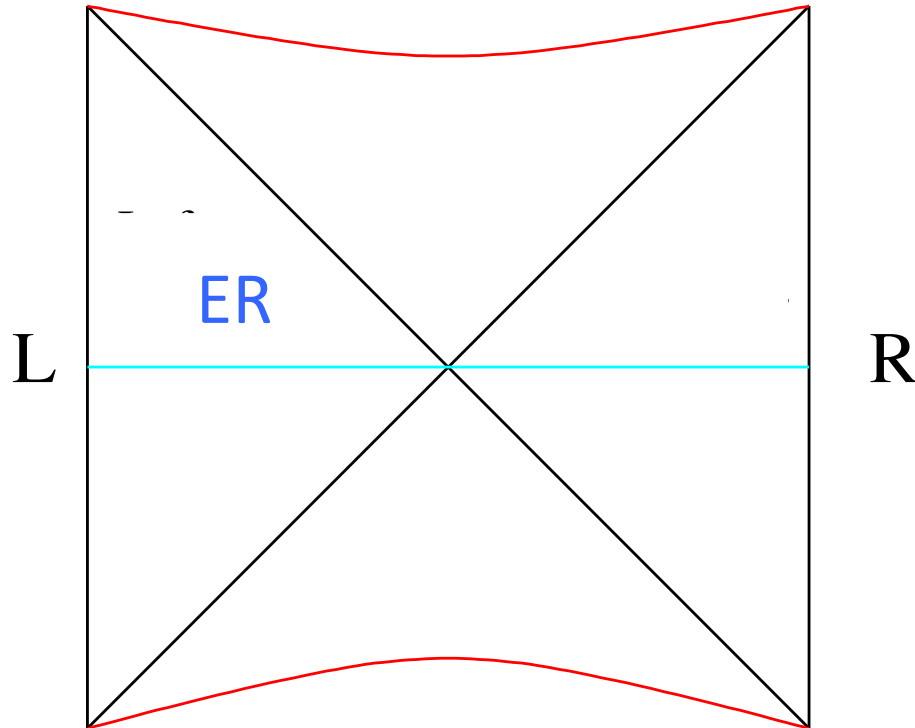
- Lots of evidence in the simplest examples.
- Large N: Techniques of integrability → computations at any value of the effective coupling.



Minahan, Zarembo,
Beisert, Eden, Staudacher
Gromov, Kazakov, Vieira
Arutynov, Frolov
Bombardeli, Fioravanti, Tateo
....

- No explicit change of variables between bulk and boundary theories (as in a Fourier transform).

Two sided AdS black hole



Entangled state in
two non-interacting
CFT's.

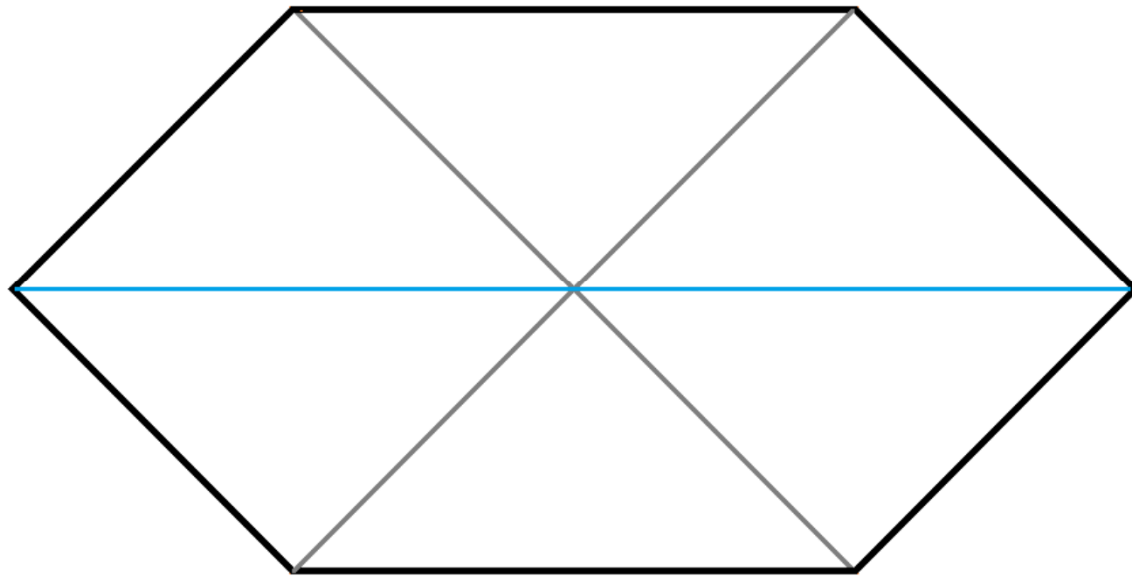
Geometric connection
from entanglement

Israel
JM

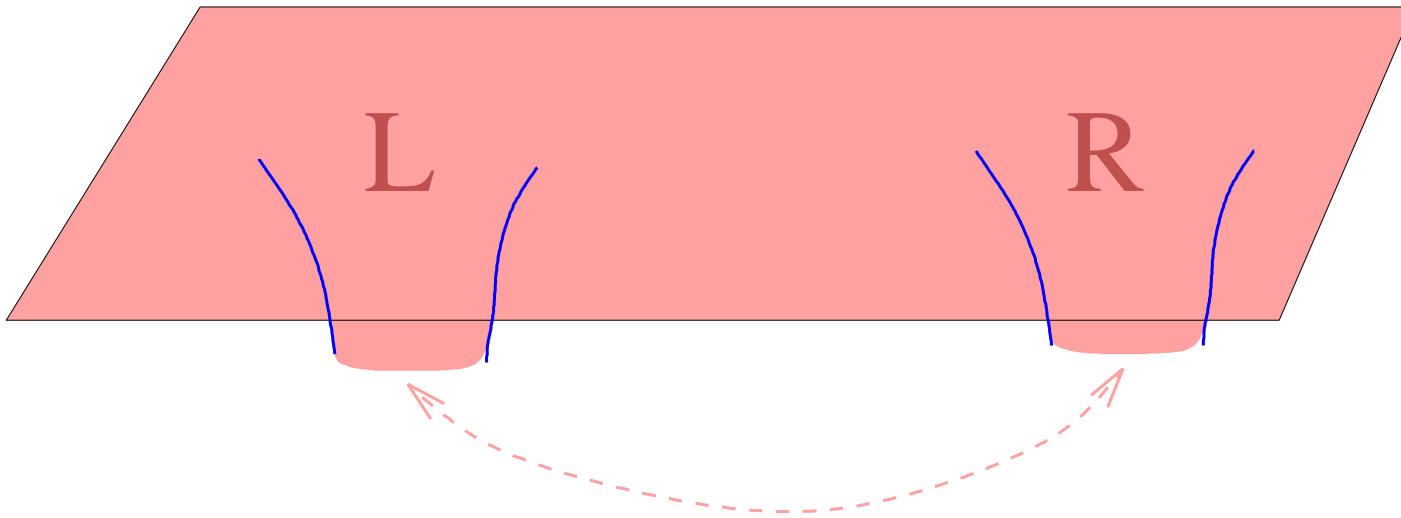
$$|\Psi\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L \times |E_n\rangle_R$$

EPR

Back to the two sided Schwarzschild solution

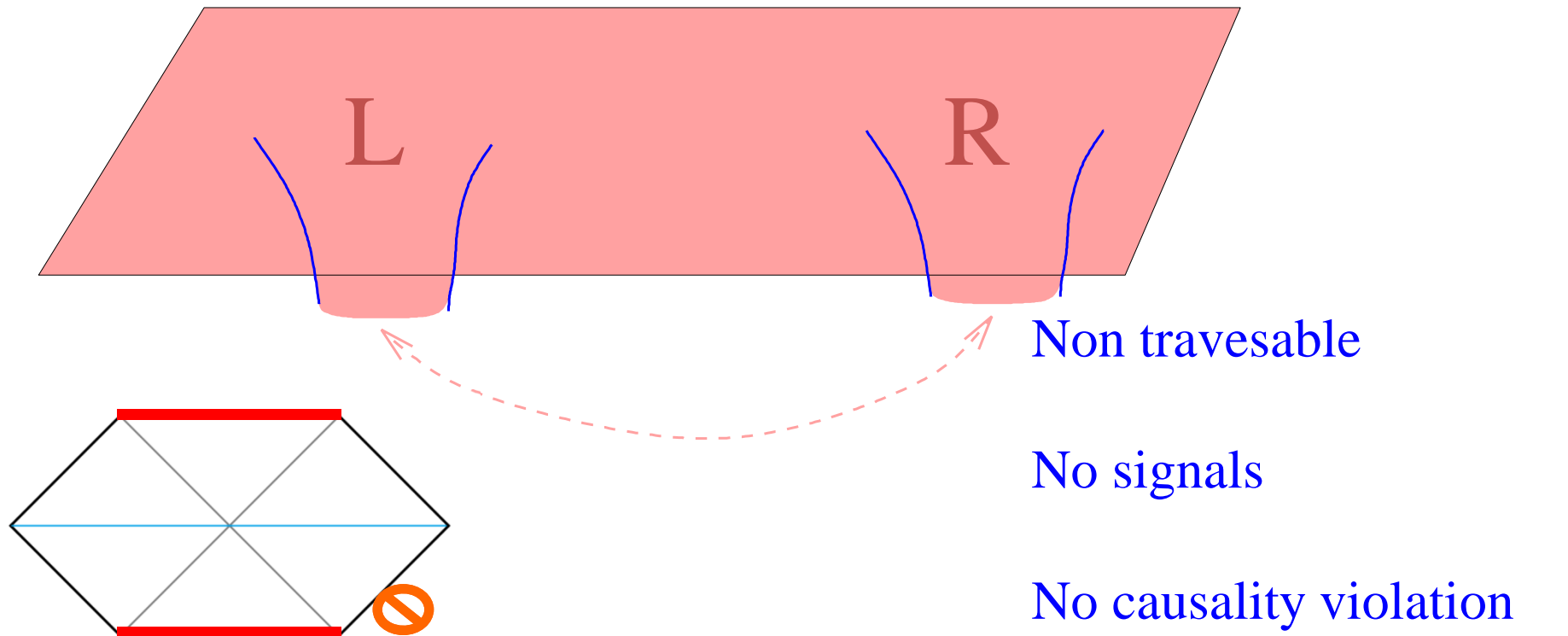


Wormhole interpretation.



Note: If you find two black holes in nature, produced by gravitational collapse, they will not be described by this geometry

No faster than light travel



Fuller, Wheeler, Friedman, Schleich, Witt, Galloway, Wooglar

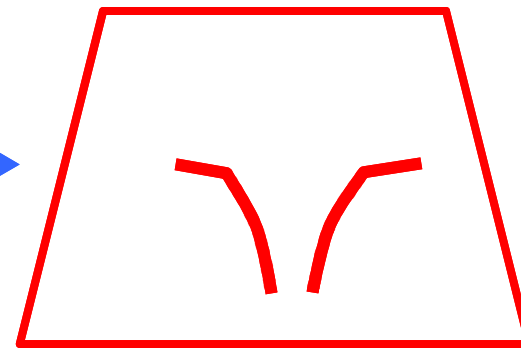
Brane argument

JM 1997

Polchinski



Collection of N 3-branes



Horowitz
Strominger

Geometry of a black 3-brane

Low energies



=

SU(N) Super Quantum
Chromodynamics in four
dimensions

string theory
on $\text{AdS}_5 \times S^5$

$$ER = EPR$$

- Wormhole = EPR pair of two black holes in a particular entangled state:
- Large amounts of entanglement can give rise to a geometric connection. J.M., Susskind
- We can complicate the entanglement of the two sided black hole → get longer wormhole

Stanford, Shenker, Roberts, Susskind

Black hole interior

- We do not understand how to describe it in the boundary theory.
- General relativity tells us that we have an interior but it is not clear that the exterior is unitary.
- Some paradoxes arise in some naïve constructions
 - Hawking,
Mathur, Almheiri, Marolf,
Polchinski, Sully, Stanford
- Actively explored... Under construction...

Error correcting codes

Nonlinear quantum mechanics

Entanglement

Firewalls/Fuzzballs

Non-locality

Final state projection

General relativity and thermodynamics

- Viewing the black hole from outside, this suggests that that general relativity is giving us a thermodynamic (approximate) description of the system if we stay outside.
- Quantum mechanics suggests that there should be an exact description where entropy does not increase. (As viewed from outside). And where Hawking radiation is not mixed.
- 2nd law already suggests that information is not lost (if information were lost, why should the 2nd law be valid ?).

Unitarity from outside ?

- Identify the degrees of freedom that give rise to black hole entropy.
 - Black hole entropy depends only on gravity → fundamental degrees of freedom of quantum gravity.
 - Should reveal the quantum structure of spacetime.
 - Understand their dynamics.
-
- This seems to requires going beyond perturbation theory.

Strings from gauge theories

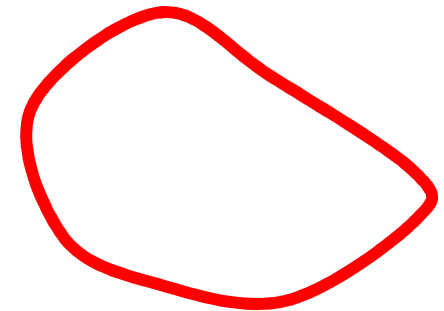
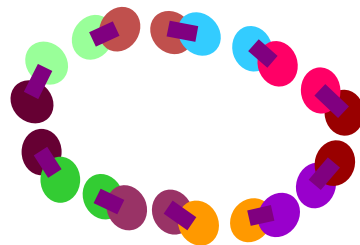
Gluon: color and anti-color



Take N colors instead of 3, $SU(N)$

t' Hooft '74

Large N limit



$g^2 N$ = effective interaction strength.

Keep it fixed when $N \rightarrow \text{infinity}$

Closed strings \rightarrow glueballs

String coupling $\sim 1/N$

Unitarity from the outside

- We can form a black hole and predict what comes out by using the boundary theory.
- If you assume the duality \rightarrow unitary evolution for the outside observer, no information loss.