Field Theory: The Past 25 Years

Nathan Seiberg (IAS)

The Future of Physics

A celebration of 25 Years of

October, 2004

The Nobel Prize in Physics 2004

David J. Gross, H. David Politzer and Frank Wilczek

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Applications of Field Theory

- Condensed matter physics
- Particle physics – the standard model, beyond the standard model (*e.g.* GUT)
- Advances in mathematics
- String theory
  - theory on worldsheet
  - effective (low energy theory) in spacetime
  - holography
  - …
- …

The Early Days

- Definition of field theory
- Well defined perturbation expansion – power series in $\hbar$

- Some nonperturbative information

The Exact Era

Advances (not distinct, not complete):

- Solvable two dimensional models
    - Conformal field theory
    - Integrable systems
- Supersymmetric field theory and duality
- Large N
- Many others

No references

Two Dimensional Conformal Field Theory

These are scale invariant 2d field theories.

They enjoy an infinite symmetry algebra – the conformal algebra.

The spectrum is in representations of the conformal algebra (similar to spectrum of hydrogen atom in representations of $O(3)$, or even $O(4,1)$).

More symmetry ⇒ more control

(There exist generalizations with even larger algebras)

Simplest example: minimal (rational) models

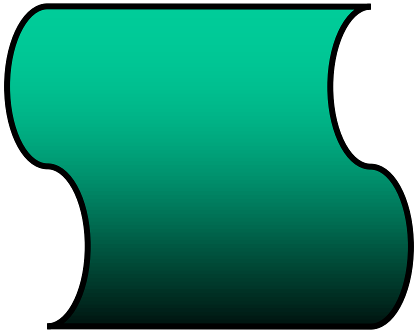
Finite number of representations

All observables $\langle 0|\mathcal{O}_1\mathcal{O}_2\ldots|0\rangle$ are exactly computable

- The exact interacting conformal field theory exists
- Applications to statistical mechanics
- Useful in string theory (see below)
- Relation to mathematics (infinite dimensional algebras, three dimensional topology...)

Typical example: sigma models

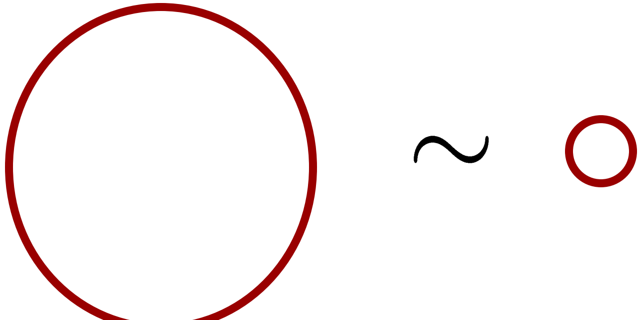
The two dimensional string worldsheet is mapped to target space $\mathcal{M}$



Action = Area swept by the string

Simplest case: $\mathcal{M}$ is a circle of radius *R* (2d *XY* model)

T-duality: the same results with $\widetilde{\mathcal{M}}$ a circle of radius *1/R*

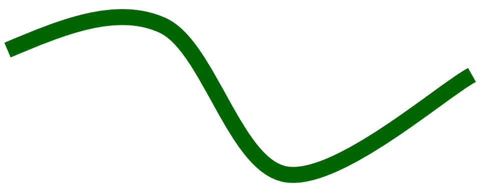


Many interesting generalizations (*e.g.* mirror symmetry $\mathcal{M} \sim \widetilde{\mathcal{M}}$ with different topology)

Ordinary geometry is probed by point particles •

Here we learn about a generalization:

``stringy geometry" – geometry probed by strings



The extended nature of the string makes the geometry more subtle and more interesting.

Supersymmetric Field Theories

Here the simplification is not because of infinite symmetry algebra, but:

Some observables in supersymmetric theories vary holomophically with coupling constants and fields.

Using analyticity (*e.g.* Cauchy's theorem) such observables are exactly calculable.

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Main dynamical lesson – duality

It generalizes

- $p \longleftrightarrow q$ in the harmonic oscillator
- $E \longleftrightarrow B$ in electrodynamics
- High *T*/low *T* duality in Ising Model
- $R \longleftrightarrow 1/R$

More than one description of the same physics. Often, one description is easier and more natural than others.

New phenomena

- Strong/weak coupling duality
- Massless composite particles
- Notion of elementary/fundamental particle becomes ambiguous
- Composite gauge bosons – gauge symmetry is not fundamental (gauge symmetry is not a symmetry)
- Many conformal field theories in four (and even higher) dimensions – no hbar
- Confinement and chiral symmetry breaking via monopole condensation (dual superconductivity)
- Applications to topology

Generalization to string theory

- Only one string theory with several dual descriptions – the theory is unique
- Branes
- No $\hbar$ – the theory is intrinsically quantum mechanical
- Fundamental formulation without general covariance
- …

Large *N*

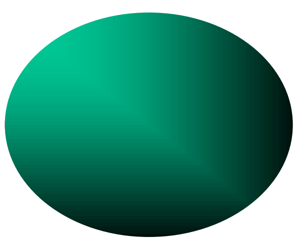
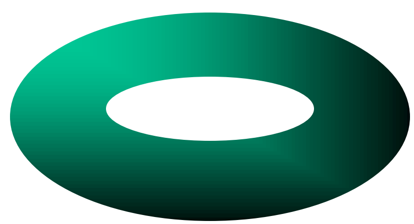
Simplest example – **matrix models**

Study the partition function

$$e^{-F} = \int dM e^{-N \mathrm{Tr}\, V(M)}$$

with M an $N \times N$ matrix, and take the limit $N \to \infty$

$1/N$ expansion – diagrams with topology of sphere, torus, etc.

$F = N^2$  $+$  $+ \mathcal{O}(N^{-2})$

Looks like a string theory!

Many applications:

- Nuclear physics
- Condensed matter physics (quantum chaos, …)
- Simple model of quantum field theory/statistical mechanics
- Supersymmetric gauge dynamics
- Mathematics (number theory, ...)
- Random surfaces/noncritical strings
- …

Four dimensional version:

$SU(N)$ gauge theory (QCD) as $N \to \infty$ – infinite number of colors

Expectation: the limiting theory exhibits confinement

What is the string theory dual of QCD?

AdS/CFT is an explicit realization of these ideas:

Precise map between a certain field theory and a certain string theory

- Theory with gravity is dual to theory without gravity – like gauge symmetry, general covariance is a derived concept
- Precise definition of string theory (as least in negatively curved spacetime)
- New insights into gauge theory and string theory, in particular, holography and issues in the black hole information puzzle
- Solution of a long standing problem in field theory

Conclusions

- There has been enormous progress in field theory
- Many applications in different branches of physics (and mathematics)
- Field theory is gradually becoming a central tool in theoretical physics, with diverse and vital applications (like calculus)

Prediction for the next 25 years:

It will be even more exciting!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