Comments on FI-terms

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Introduction
(not needed here)

- We need to spontaneously break supersymmetry (SUSY):
  - Tree breaking
  - Dynamical breaking

- Tree breaking
  - O'Raifeartaigh (O’R) model – F-term breaking
  - Fayet-Iliopoulos (FI) model – D-term breaking

More advanced questions

- All calculable dynamical models of SUSY breaking look like O'R models at low energies. Are there models which are effectively FI models?
- Is there an invariant distinction between F-term and D-term breaking? Can a strongly coupled theory continuously interpolate between these two phenomena?
- What about the coupling of FI-terms to supergravity (SUGRA)? Is it consistent? Note that there is no example of an FI-term in string theory…

Outline

- Review of the Ferrara-Zumino (FZ) multiplet
  - The multiplet
  - An ambiguity
  - The multiplet in the Wess-Zumino (WZ) model
  - The multiplet in FI models – it is not gauge invariant

- Consequences in rigid SUSY

- Consequences in SUGRA

The Ferrara-Zumino (FZ) multiplet

Both the energy momentum tensor and the SUSY current are in reducible representations of the Lorentz group.

They reside in a real superfield $\mathcal{J}_\mu \sim \mathcal{J}_{\alpha\dot{\alpha}}$ which satisfies the conservation equation

$$\bar{D}^{\dot{\alpha}} \mathcal{J}_{\alpha\dot{\alpha}} = D_\alpha X$$

with X a chiral superfield (an irreducible representation of SUSY).

The FZ multiplet in components

Solving $\bar{D}^{\dot{\alpha}}\mathcal{J}_{\alpha\dot{\alpha}} = D_\alpha X$

$$\begin{aligned}\mathcal{J}_\mu &= j_\mu + \theta^\alpha\left(S_{\mu\alpha} + \frac{1}{3}(\sigma_\mu\bar{\sigma}^\rho S_\rho)_\alpha\right) + h.c. \\ &\quad +(\theta\sigma^\nu\bar{\theta})\left(2T_{\mu\nu} - \frac{2}{3}\eta_{\mu\nu}T^\rho_\rho + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial^\rho j^\sigma\right) + \ldots \\ X &= x + \frac{2}{3}(\theta\sigma^\mu S^\dagger_\mu) + \theta^2(\frac{2}{3}T^\mu_\mu + i\partial_\mu j^\mu) + \ldots\end{aligned}$$

Here $S_{\mu\alpha}$, $T_{\mu\nu}$ are the conserved SUSY current and energy momentum tensor and j_μ is a (perhaps not conserved) R-current.

The FZ multiplet in components

$$\mathcal{J}_\mu = j_\mu + \theta^\alpha \left(S_{\mu\alpha} + \frac{1}{3}(\sigma_\mu \bar{\sigma}^\rho S_\rho)_\alpha \right) + h.c.$$
$$+(\theta\sigma^\nu\bar{\theta}) \left(2T_{\mu\nu} - \frac{2}{3}\eta_{\mu\nu}T^\rho_\rho + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial^\rho j^\sigma \right) + \ldots$$

$$X = x + \frac{2}{3}(\theta\sigma^\mu S^\dagger_\mu) + \theta^2(\frac{2}{3}T^\mu_\mu + i\partial_\mu j^\mu) + \ldots$$

$X = 0$ means that the theory is superconformal.

Otherwise, X represents the non-conservation of the R-current and the (super)conformal symmetry.

Ambiguity in the FZ-multiplet

$$\bar{D}^{\dot{\alpha}}\mathcal{J}_{\alpha\dot{\alpha}} = D_\alpha X$$

We can shift

$$\begin{aligned} \mathcal{J}_\mu &\rightarrow \mathcal{J}_\mu + i\partial_\mu\left(Y - Y^\dagger\right) \\ X &\rightarrow X - \frac{1}{2}\bar{D}^2 Y^\dagger \end{aligned}$$

with any chiral superfield Y .

This shifts $S_{\mu\alpha}$, $T_{\mu\nu}$ only by improvement terms…

Review of improvement terms

The shift by "improvement terms"

$$
\begin{aligned}
S_\alpha^\mu &\rightarrow S_\alpha^\mu + \sigma_{\alpha\beta}^{\mu\nu}\partial_\nu \Psi^\beta \\
T_{\mu\nu} &\rightarrow T_{\mu\nu} + (\partial_\mu \partial_\nu - \eta_{\mu\nu}\partial^2)F
\end{aligned}
$$

with any $\Psi, \ F$ does not affect the conservation of $S_{\mu\alpha}, \ T_{\mu\nu}$ and does not change the charges.

The ambiguity we discussed above is the SUSY version of these improvement terms.

The FZ multiplet in WZ models

For example, the multiplet in the WZ model

$$\mathcal{L} = \int d^4\theta K(\Phi, \Phi^\dagger) + \int d^2\theta W(\Phi) + h.c.$$

is

$$\begin{aligned} \mathcal{J}_{\alpha\dot{\alpha}} &= 2(\partial\bar{\partial}K) D_\alpha \Phi \bar{D}_{\dot{\alpha}} \Phi^\dagger - \frac{2}{3}[D_\alpha, \bar{D}_{\dot{\alpha}}]K \\ X &= 4W - \frac{1}{3}\bar{D}^2 K \end{aligned}$$

Its lowest component is an R-current with $R(\Phi) = \frac{2}{3}$

In general this R-current is not conserved.

Kahler transformations of the FZ-multiplet

The multiplet

$$\mathcal{J}_{\alpha\dot{\alpha}} = 2(\partial\bar{\partial}K)D_\alpha\Phi\bar{D}_{\dot{\alpha}}\Phi^\dagger - \frac{2}{3}[D_\alpha, \bar{D}_{\dot{\alpha}}]K$$

is not invariant under Kahler transformations

$$K \to K + f(\Phi) + f^\dagger(\Phi^\dagger)$$

This is the ambiguity we mentioned earlier.
It changes improvement terms in $S_{\mu\alpha}, \; T_{\mu\nu}$.
They are still conserved currents and their charges are not affected by this.

The FZ-multiplet in FI-models

Consider a free theory of a single vector superfield V with an FI term

$$\mathcal{L} = \frac{1}{4g^2}\int d^2\theta W_\alpha^2 + h.c. + \int d^4\theta \xi V$$

Here

$$\begin{aligned}\mathcal{J}_{\alpha\dot{\alpha}} &= -\frac{4}{g^2}W_\alpha W^\dagger_{\dot{\alpha}} - \frac{2}{3}\xi[D_\alpha, \bar{D}_{\dot{\alpha}}]V \\ X &= -\frac{\xi}{3}\bar{D}^2 V\end{aligned}$$

The FZ-multiplet in FI-models

$$\mathcal{J}_{\alpha\dot{\alpha}} = -\frac{4}{g^2} W_\alpha W^\dagger_{\dot{\alpha}} - \frac{2}{3}\xi [D_\alpha, \bar{D}_{\dot{\alpha}}] V$$

$$X = -\frac{\xi}{3}\bar{D}^2 V$$

Notice the similarity to the WZ model

$$\mathcal{J}_{\alpha\dot{\alpha}} = \ldots - \frac{2}{3}[D_\alpha, \bar{D}_{\dot{\alpha}}] K$$

$$X = \ldots - \frac{1}{3}\bar{D}^2 K$$

with $K \to \ldots + \xi V$

Checking gauge invariance

The superspace integrand of the FI-term is not invariant under gauge transformations

$$V \rightarrow V + \Lambda + \Lambda^\dagger$$

It transforms by a Kahler transformation (the Lagrangian is invariant).

Just as the FZ-multiplet is not invariant under Kahler transformations, it is also not gauge invariant!

Checking gauge invariance

$$\mathcal{J}_{\alpha\dot{\alpha}} = -\frac{4}{g^2} W_\alpha W^\dagger_{\dot{\alpha}} - \frac{2}{3}\xi[D_\alpha, \bar{D}_{\dot{\alpha}}]V$$

$$X = -\frac{\xi}{3}\bar{D}^2 V$$

Clearly, the terms proportional to ξ are not gauge invariant.

This is true in any model with an FI-term.

Gauge variation of the FZ-multiplet

We have already seen that Kahler transformations change $S_{\mu\alpha},\ T_{\mu\nu}$ by improvement terms.

Similarly, in the presence of an FI-term these improvement terms change under gauge transformations.

$S_{\mu\alpha},\ T_{\mu\nu}$ are still conserved and their charges are gauge invariant.

What happens in WZ gauge?

The choice of WZ gauge breaks SUSY, and therefore the problem cannot go away.

The remaining gauge freedom is ordinary gauge transformations.
Now $S_{\mu\alpha}, \ T_{\mu\nu}$ are gauge invariant.

However, the R-current $j_\mu = \frac{1}{g^2}\lambda^\dagger\bar{\sigma}_\mu\lambda - \frac{2}{3}\xi A_\mu$ is not gauge invariant.

Sometimes there is a way out

If a theory has a global $U(1)_R$ symmetry, its conserved current is in another superfield which includes the energy momentum tensor and the supersymmetry current.

These $S_{\mu\alpha},\ T_{\mu\nu}$ differ from those in the FZ-multiplet by improvement terms. Even if there is an FI-term, these are gauge invariant.

This option is not available in generic theories.

Consequences

- Clearly, the lack of gauge invariance follows from the FI-term. It is present in any model with such a term.
- It does not make the theory inconsistent.
- Starting with an FI-term, it cannot be perturbatively or non-perturbatively renormalized. This gives a new perspective on an old result [Witten; Fischler et al; Shifman and Vainshtein; Dine; Weinberg].

  (Exception: anomalous theories where the sum of the charges does not vanish.)

Consequences

- Starting without an FI-term, it cannot be generated. Again, a new derivation to an old result of [Witten; Fischler et al; Shifman and Vainshtein; Dine; Weinberg].
- The same applies to emergent gauge fields. (This can also be shown using the old methods.)
- This explains why all calculable models of dynamical SUSY breaking have F-term breaking. The FI-model never arises from the dynamics.

Trying to fix the problem

If the gauged symmetry is Higgsed by a vev of a charged field Φ , we can redefine the FI-term in the Lagrangian and in the currents as

$$\xi V \to \xi (V + \log |\Phi|^2)$$

This restores gauge invariance of the Kahler potential and the currents, while not affecting the Lagrangian and changing the currents only by improvement terms.

However, this introduces a singularity at $\Phi = 0$ where the gauge symmetry is restored.

Trying to fix the problem

This is OK only if the singularity at $\Phi = 0$ is not in the field space; e.g. if it is at infinite distance.

However, in that case the gauge symmetry is always Higgsed and the FI-term is ill-defined.

Genuine FI-terms are present only when there is a region in field space where $\xi \neq 0$ and the gauge symmetry is unbroken there.

“Field dependent FI-terms”

“Field dependent FI-terms” are common in field theory and string theory.

Some charged field Φ Higgses the gauge symmetry. with $\langle\Phi\rangle = 0$ at infinite distance.
Expanding $K(V + \log|\Phi|^2)$ around some $\langle\Phi\rangle$ leads to an approximate FI-term whose coefficient is $\langle\Phi\rangle$ dependent.

These are not genuine FI-terms – the gauge symmetry is everywhere Higgsed at or above the mass of Φ .

Another "field dependent FI-term"

High dimension operators like $\int d^4\theta \Phi^\dagger \Phi D^\alpha W_\alpha$

can lead to FI-terms, if the F-component of Φ has a nonzero vev.

Equivalently, SUSY is broken by this vev and a D-term is induced. Example: $U(1)_Y$ acquires such a D-term in the MSSM.

However, the redefinition $V \to V + c\Phi^\dagger\Phi$ with an appropriate constant c can remove this D-term.

Coupling to supergravity: history

- [Freedman (77)] coupled the FI-model to SUGRA
- [Barbieri, Ferrara, Nanopoulos, Stelle (82); Ferrara Girardello, Kugo, Van Proeyen (83)] showed that this construction is possible only when the rigid theory has a global $U(1)_R$ symmetry.
- The gauge charges are shifted by an amount proportional to $r\xi/M_P^2$ where r is that R-charge.
- Hence the gravitino is charged and the theory is gauged supergravity.
- …

Coupling to supergravity: history

- [Witten (89)] pointed out that in the presence of magnetic monopoles this shift of electric charges is inconsistent with Dirac quantization.
- [Chamseddine, Dreiner (96); Castano, Freedman, Manuel (96); Binetruy, Dvali, Kallosh, Van Proeyen (04); Elvang, Freedman, Kors (06)] considered the restrictive conditions on the charges due to anomaly cancelation.
- No example in string theory.

- [Many people]: perhaps it simply does not exist…

Coupling to SUGRA (new)

- We focus on $\xi \ll M_P^2$. If $\xi \sim M_P^2$, a field theory description is not valid.
- The complexity of coupling these theories to SUGRA stems from the lack of gauge invariance in the SUSY current multiplet.
- One way to find SUGRA is to couple the SUSY current multiplet to gauge fields (metric, gravitino,…).
- Since the R-current is not conserved, one introduces compensator fields which are charged under it.

Coupling FI terms to linearized SUGRA (for experts)

- These compensators can be used to fix the lack of gauge invariance of the current by assigning to them gauge charge and turning the FI-term into a "field dependent FI-term."
- Consequences of this charge assignment:
  - This is possible only when the rigid theory has a global $U(1)_R$ symmetry.
  - The vev of the compensators mixes the original gauge field with the auxiliary field which couples to j_μ
  - When the dust settles, the original gauge symmetry charges are shifted by an amount proportional to $r\xi/M_P^2$

Generalizing beyond the linearized theory

- It is straightforward to extend this analysis to the full SUGRA and not only the linearized theory.
- It is also easy to include all possible high derivative terms.

- We learn that in addition to the gauge $U(1)$ symmetry, the full theory must have a global $U(1)$ symmetry. It can be taken to be either an ordinary symmetry (the original gauge symmetry) or an R-symmetry.

Consequences of the global symmetry

- However, considerations based on black hole physics make continuous global symmetries incompatible with quantum gravity.

- We learn that a SUGRA with FI-terms (which is also equivalent to gauged SUGRA) is quantum mechanically inconsistent.
- Clearly, this conclusion does not apply to “field dependent FI-terms.”

Conclusions

- The energy momentum tensor and the SUSY current are members of the FZ-multiplet.
- In the presence of an FI-term this multiplet is not gauge invariant.
- This gives a new perspective on the lack of renormalization of the FI-term.
- It explains why all calculable models of dynamical SUSY breaking have F-term breaking.
- This is the root of the difficulties of having an FI-term in SUGRA.

Conclusions

- The only theories with an FI-term which can be coupled to SUGRA have a global continuous R-symmetry The resulting theory is gauged supergravity.
- This theory has an exact continuous global symmetry. Hence it must be inconsistent.
- This explains why string theory never leads to models with genuine FI-terms.
- There is no problem with “field dependent FI-terms.”
- There are many consequences in models of particle physics and cosmology and in string constructions.