Causality constraints on graviton three point functions

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Based on: Camanho, Edelstein, JM. , Zhiboedov. arXiv:1407.5597.

General Motivation

- Understand classical theories (weakly coupled).
- What are the possible classical lagrangians ? For particles with spin ?
- Can be studied by looking at on shell amplitudes and demanding that they are unitary, crossing symmetric and lorentz invariant.

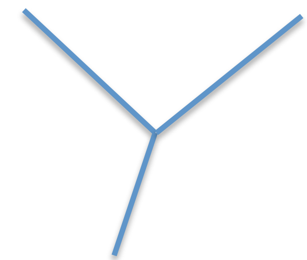
Some questions

- Q1: what is the most general form of a ``pure gravity'' theory ? . Only massless gravitons.
- Q2: what is the form of the most general classical "gravity" theory? (m=0 gravitons) + higher spins.
- Q3: what is the form of the general classical theory for interacting massive higher spin particles ? (Large N QCD)

Why do we care ?

- Classical → simpler than generic quantum.
- By classical we mean weakly coupled at the energies under consideration (might become strongly coupled at very high energies).
- How unique is the structure of string theory ?
- Example: Holographic dual of a large N gauge theory. Coupling is small by general large N arguments, $g \approx 1/N$. Stress tensor → graviton. What are its interactions ? How are interactions related to the single particle spectrum of the theory ?

Simplest question



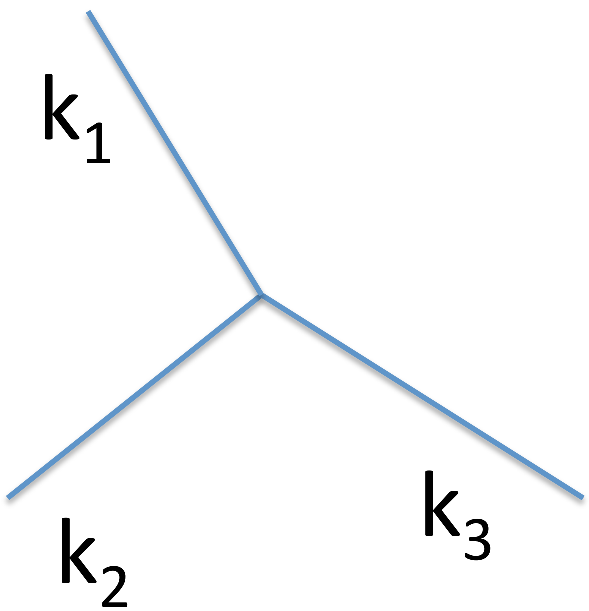
- Consider the graviton three point interactions.
- It has 3 different possible structures. One is the Einstein gravity one.
- We will argue that the theories that have the other structures have a causality problem if no other particles are present.
- This causality problem can be solved by adding massive higher spin particles.

Plan

- Review of graviton three point functions
- High energy scattering thought experiment.
- Propagation through shock waves and time delays.
- How the problem can be solved. Why do we need higher spin particles ?.
- Fun application.

Three point amplitudes

Spin 0



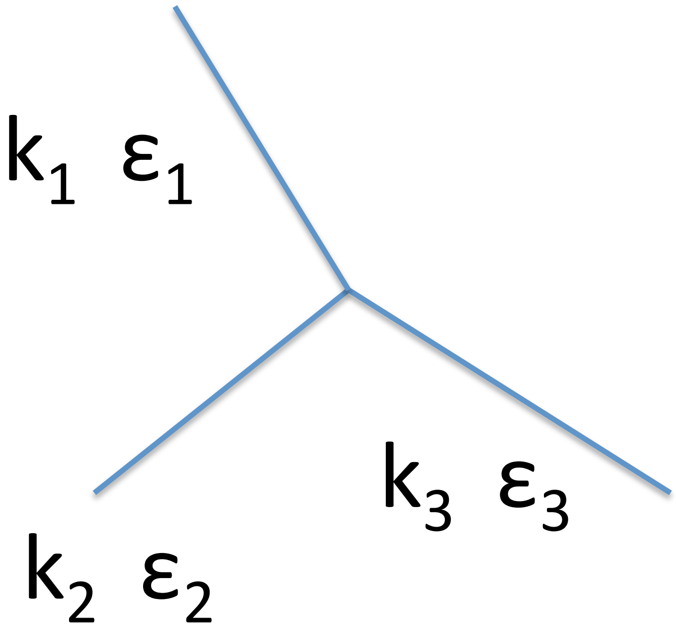
No kinematic invariants.

$$(k_1 + k_2)^2 = k_3^2 = 0$$

Only a single constant =
the coupling constant $\sqrt{G}$

Three point amplitudes

Spin 1



Now we also have the polarization vectors ε .

$k_i \, . \;\; \varepsilon_i = 0$

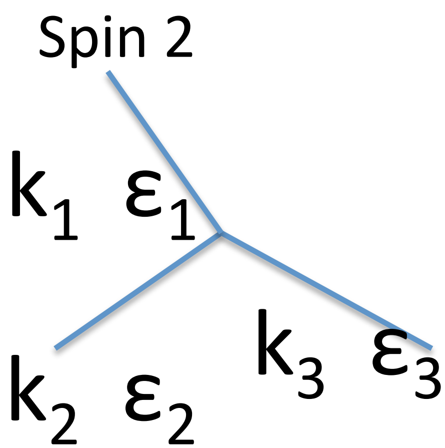
$\varepsilon_i \approx \varepsilon_i + k_i$

$$A_0 = \epsilon_1 . \epsilon_2 \, \epsilon_3 . k_1 + \cdots \qquad \text{YM}$$

$$A_2 = \epsilon_1 . k_2 \, \epsilon_2 . k_3 \, \epsilon_3 . k_1 \qquad F^3$$

Two more factors of energy

Graviton three point functions



$$G_0 = A_0 A_0 \qquad \frac{1}{G_N}\int R \quad \text{Einstein}$$

$$G_2 = A_0 A_2 \qquad \frac{1}{G_N}\int \alpha_2 R^2_{\mu\nu\rho\sigma}$$

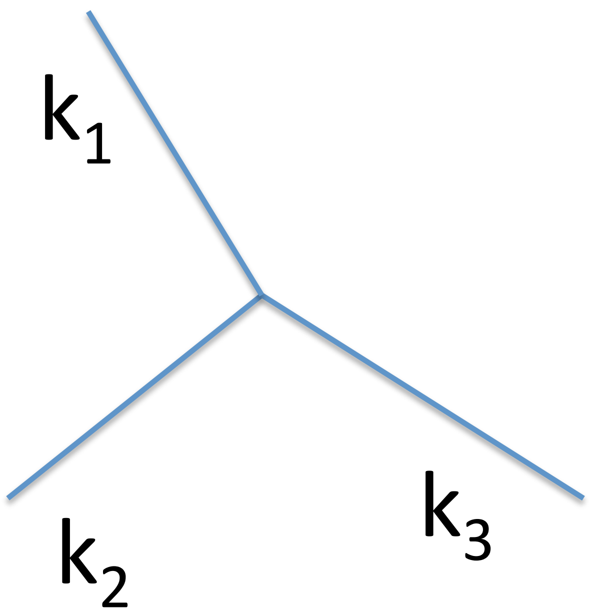
$$G_4 = A_2 A_2 \qquad \frac{1}{G_N}\int \alpha_4^2 R^3_{\mu\nu\rho\sigma}$$

$\alpha_2, \ \alpha_4$ units of (length)^2 . Total strength of this structure from all terms in the Lagrangian

Overall small coupling, $\sqrt{G_N}$, very, very small.

At distances comparable to α , the three structures are comparable.
But they are all small!

In 4 dimensions



Helicity can be + or - .

The ++- and --+ amplitudes → Usual

The +++ and ---- amplitudes → New ones, F^3 or R^3 . (difference between +++ and --- → parity violating structures).

The R^2 term is topological and this structure is not present in d=4. $\alpha_2 = 0$

Example

Perturbative string theory g→ 0 , α' finite. $\alpha_i \gg l_p^2$

$$\alpha_2 \sim \alpha_4 \sim \alpha' \ , \qquad \text{bosonic}$$

$$\alpha_2 \sim \alpha' \ , \quad \alpha_4 = 0 \ , \qquad \text{heterotic}$$

$$\alpha_2 = \alpha_4 = 0 \ , \qquad \text{type II}$$

In string theory, there are also new massive higher spin particles at this scale.

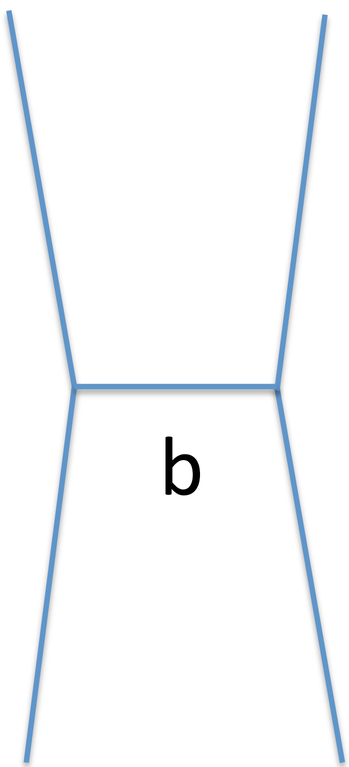
The size of the higher curvature corrections that we are considering is <u>much larger</u> than the value suggested effective field theory applied to the Einstein action.

In that case $\alpha_i \sim l_p^2$, and our considerations do not apply because the theory is now weakly coupled at that scale.

$$\frac{1}{l_{pl}^{D-2}} \int R + l_{pl}^2 R^2 + \cdots$$ ← Effective field theory (minimum value)

$$\frac{1}{l_{pl}^{D-2}} \int R + \alpha R^2 + \cdots$$ ← Our case, or weakly coupled string theory

Thought experiment



High energy $s \gg t \sim 1/b^2$

Fixed impact parameter, b

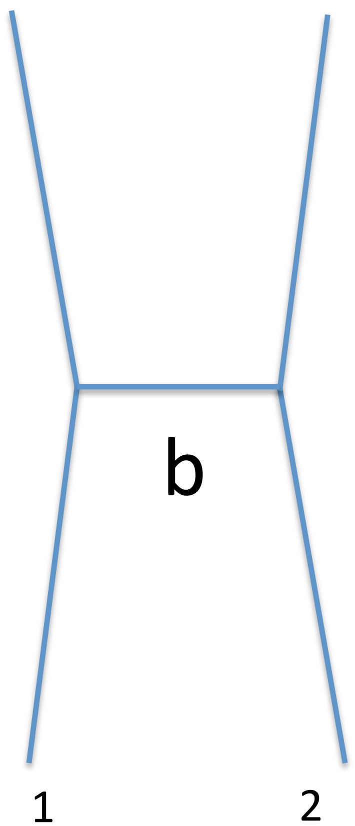
Weakly coupled regime. (s not too high)

But we will take

$$b^2 \sim \alpha_2 \quad \text{or} \quad b^2 \sim \alpha_4$$

We can obey all of these, thanks to our assumptions

First: General Relativity



View it as follows:

First particle creates a shock wave

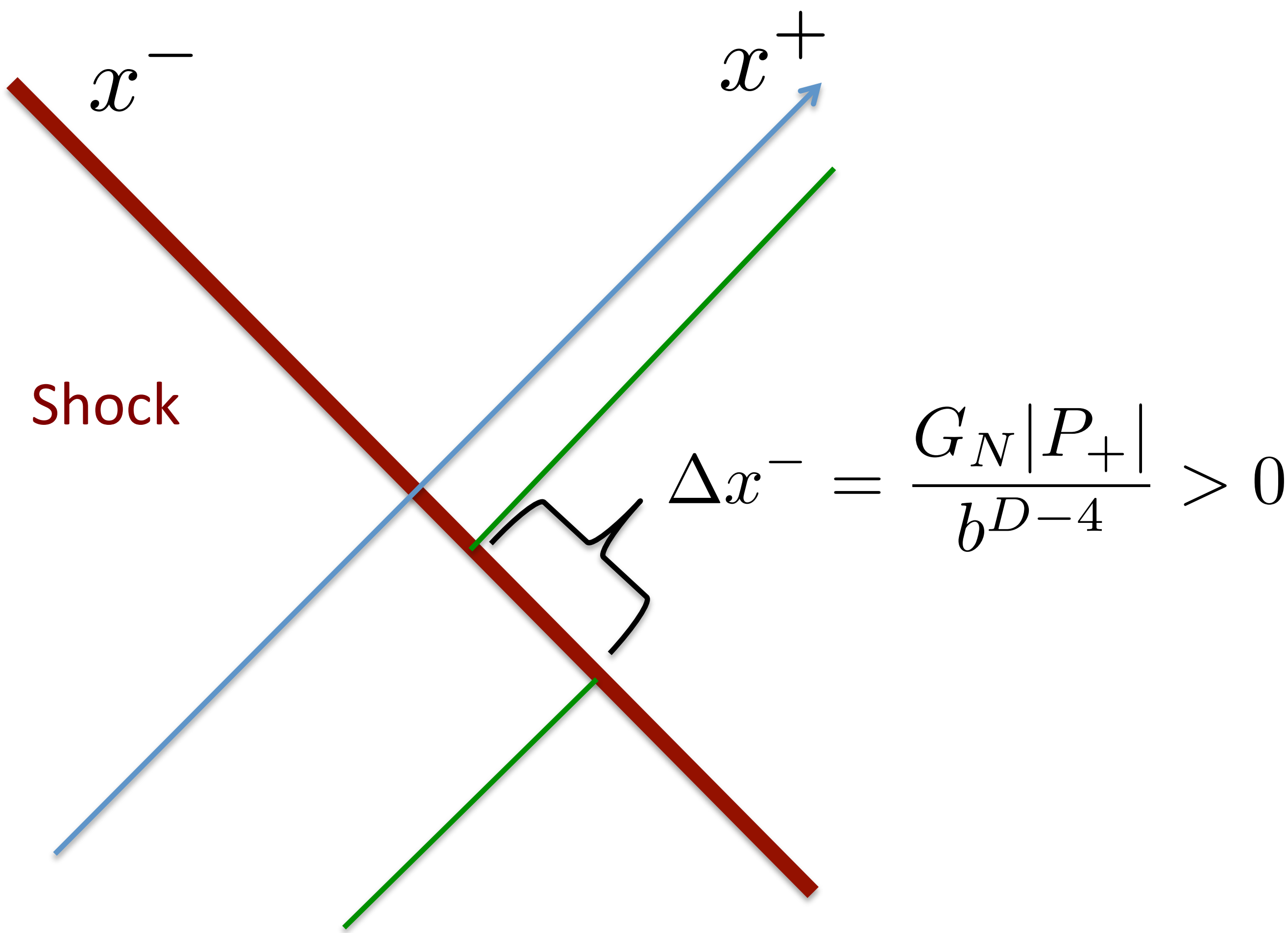
$$ds^2 = -dx^+ dx^- - h(x^+, r)(dx^+)^2 + d\vec{r}^2$$

$$h = \delta(x^+) \frac{G_N |P_+|}{r^{D-4}}$$

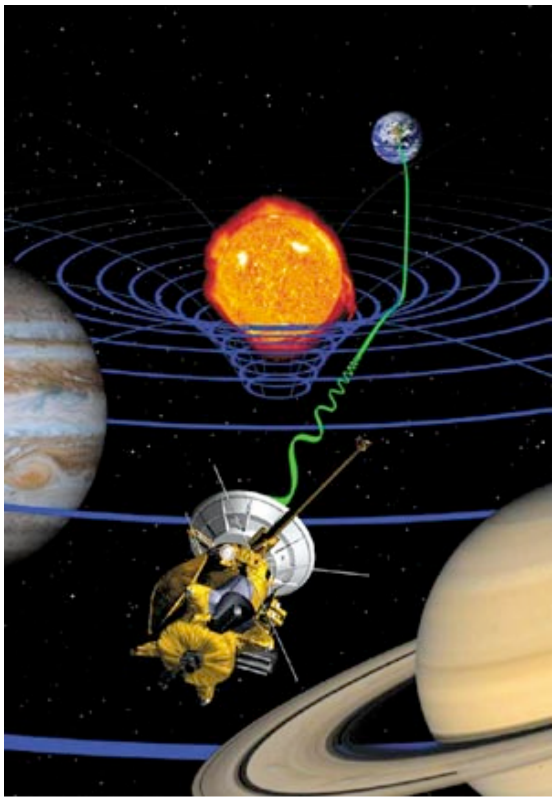
Treat the second particle as a probe.

(In D=4 , we get a log(b) → some dependence on IR , not important for us)

Time delay



Shapiro time delay, the 4th test of GR



Cassini

Besides the bending of light.

Also a time delay

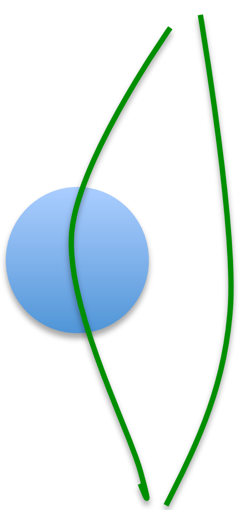
Gravity always slows you down

Null energy condition, $T_{++} \geq 0$, plus Einstein's equations →

Gao – Wald

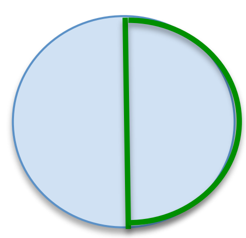
Asymptotic causality is respected.

You cannot send a signal faster through the ``interior'' than through the far away region.



Faster

We impose this as a principle for any theory of gravity (if not → time machines)



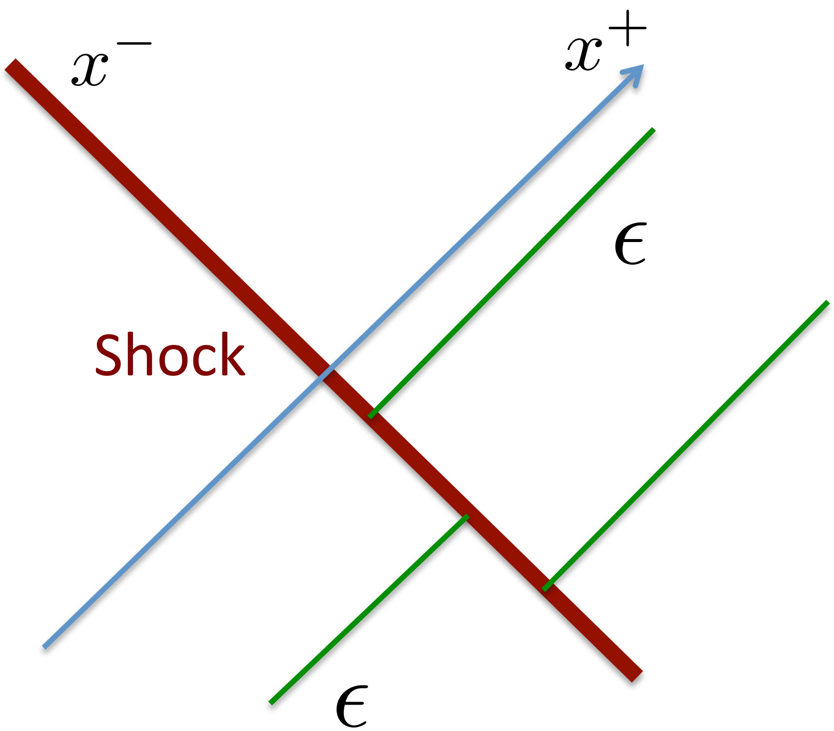
In AdS, bulk causality should be compatible with boundary causality

Page, Surya, Woolgar

Same but with higher derivative vertex

$$\Delta x^- = \left(1 + \alpha_2 (\epsilon.\partial_b)^2 + \alpha_4^2 (\epsilon.\partial_b)^4\right) \frac{G_N |P_+|}{b^{D-4}}$$

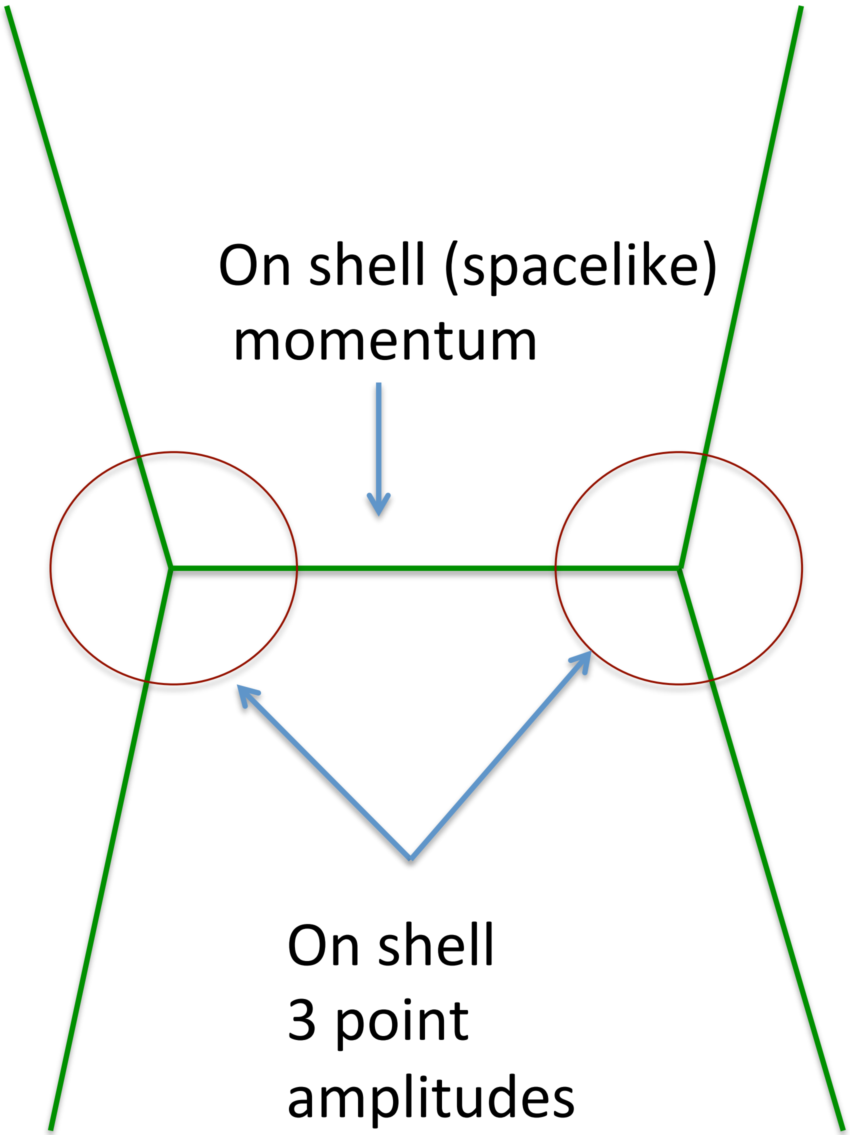
$$= \left(1 \pm \frac{\alpha_2}{b^2} \pm \frac{\alpha_4^2}{b^4}\right) \frac{G_N |P_+|}{b^{D-4}}$$



We can have propagation faster than light as seen from infinity.

Violates asymptotic causality.

e.g. in AdS → violates causality of the boundary theory.

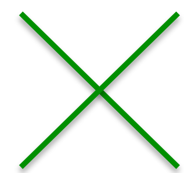


At fixed impact parameter, $b > 0$,
only the on shell intermediate
particle contributes
(gives rise to a long distance force)

(Three point on shell vertices are
nonzero because the kinematics
is effectively that of (2,D-2) signature.)

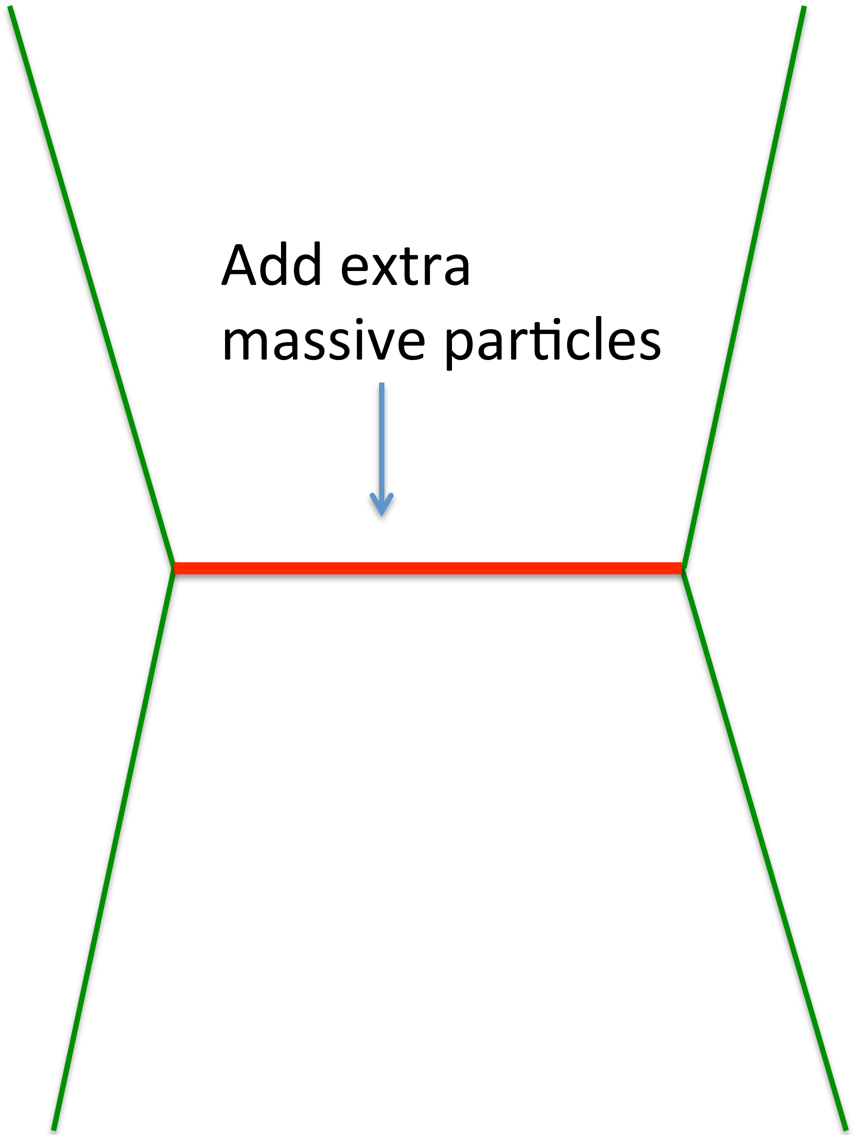
All other vertices can be neglected!

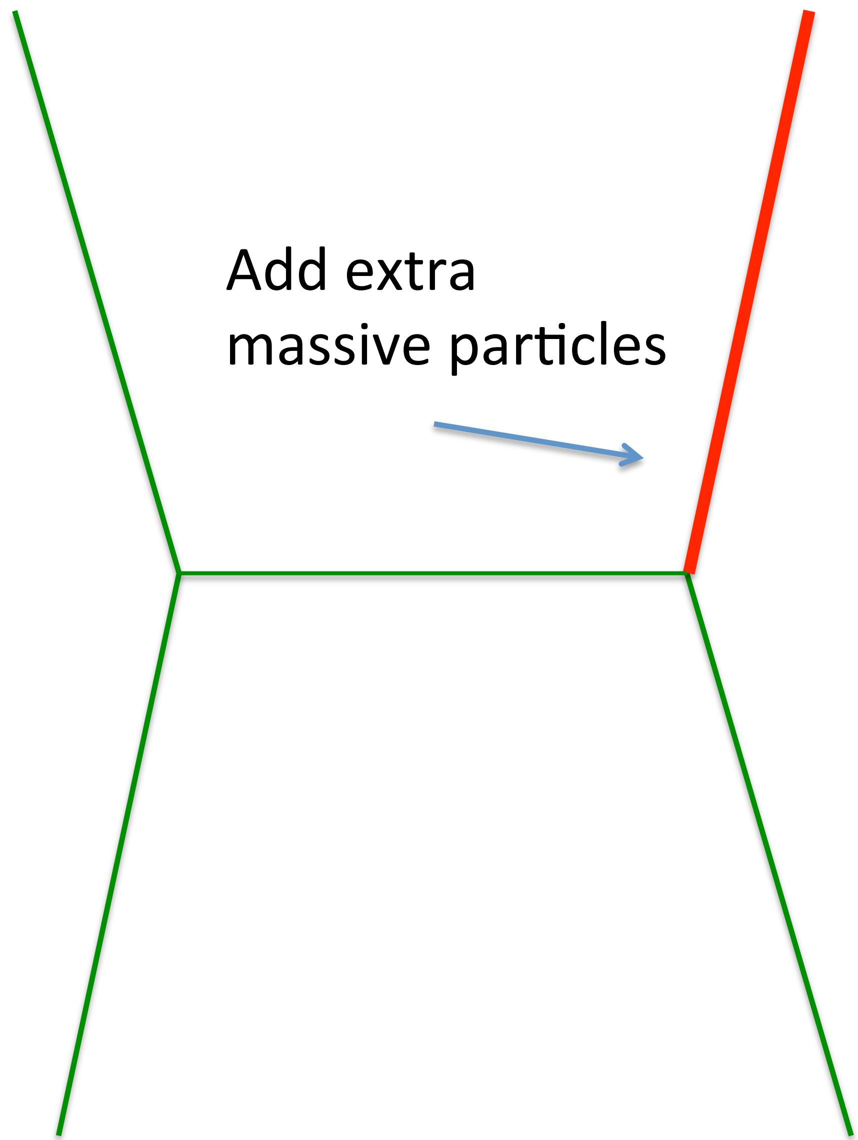
The problem cannot be fixed by
adding higher order terms to the
lagrangian.



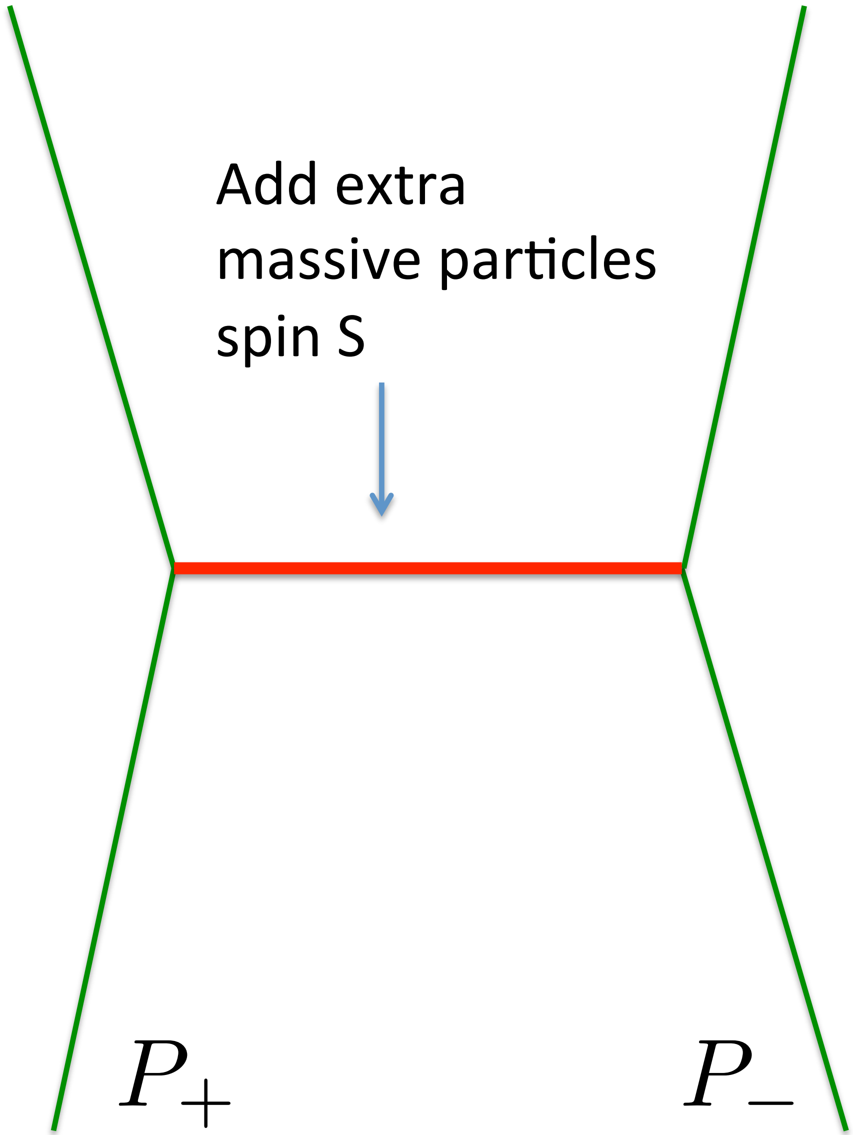
If we want to solve this in the
classical theory:
We need extra particles.

The full argument involves starting with the leading amplitude and then ensuring we can iterate the process to make it measurable without running into the strong coupling region


Add extra
massive particles



Do not solve the
problem.



$$s = P_+ P_-$$

Graviton time advance grows like s .

Effect produced by the new particles should also grow, at least, like s.

$$s^{S-1}$$

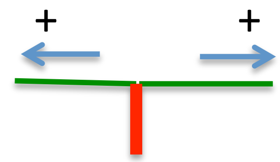
Comes from momenta contracted across the two sides, thanks to the sum over polarizations for the intermediate state.

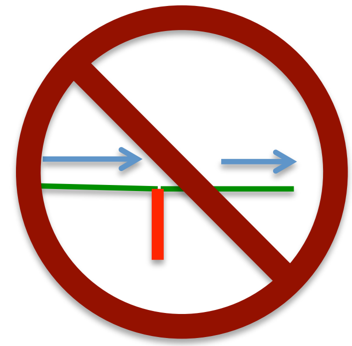
Need new particles with spins

$$S \geq 2$$

Massive spin 2

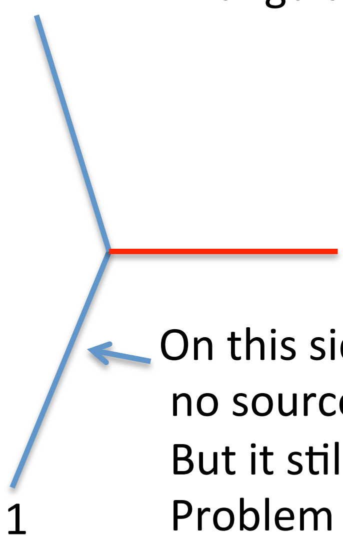
Four dimensional
massive amplitude





(No stress tensor in a theory of gravity: Weinberg-Witten theorem)

Go to rest frame for massive particle. ++ or - - are the only allowed by angular momentum conservation along the momentum of the decay products.



On this side, set up a coherent state of particles of a given spin → no spin flip →
no source for massive intermediate particles.
But it still sources the ordinary massless graviton.
Problem still remains when we scatter the second graviton if there is a +++ or --- vertex.

In D>4 , we have a slightly longer argument.

Massive spin S>2

Phase shift of the second particle as it crosses the shock:

$$e^{i\delta} = e^{iC(P_+P_-)^{S-1}}$$

As a function of P_-

Causality + unitarity → analytic in upper half plane
→ Bounded in upper half plane $|e^{i\delta}| \leq 1$

If S-1 >1 , there is some sector where it increases exponentially.

→ Finite number of higher spin particles would not work.
→ But an infinite number with delicate cancellations might work.
→ It would also work if we have extended objects so that we can produce particles in the s-channel
→ It works in string theory!.

In string theory

Take the four graviton string tree amplitude.
Take the large s limit.
Go to impact parameter representation.

$$\delta \sim \frac{g^2 s}{(\log s)^{\frac{D-4}{2}}}$$

For $b^2 < \alpha' \log s$

Amati, Ciafaloni, Veneziano (88)

(δ also develops an imaginary part due to the possibility of exciting massive strings in the s channel).

Regge behavior is important $\mathcal{A} \propto \frac{s^2}{t} s^{\alpha' t/4}$

Conclusion

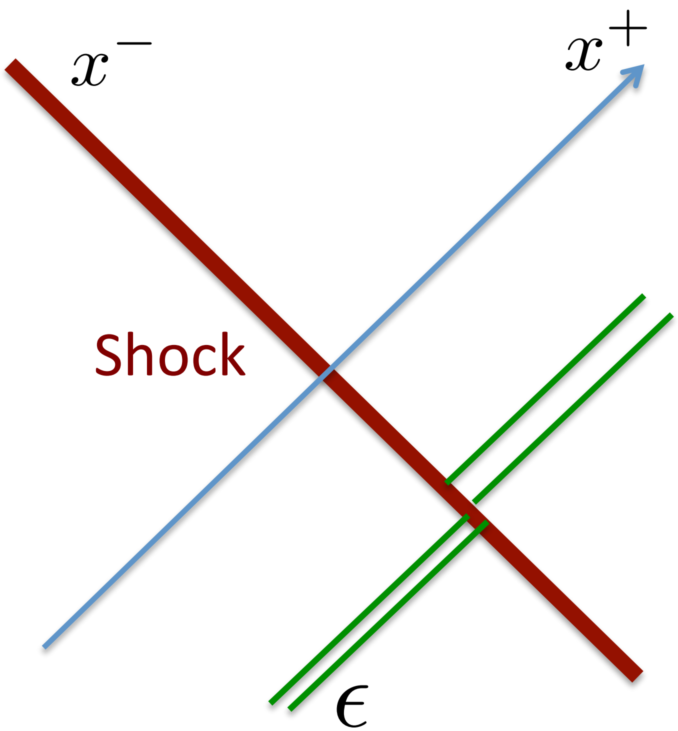
If the graviton three point function differs from the one in the Einstein theory → we get a causality problem.

To fix it we need an infinite number of massive spin > 2 particles !

The masses of those particles should be roughly $m^2_{S>2} \lesssim \frac{1}{\alpha_4}$

Some comments

Extended graviton and 3pt function



If the graviton is extended.

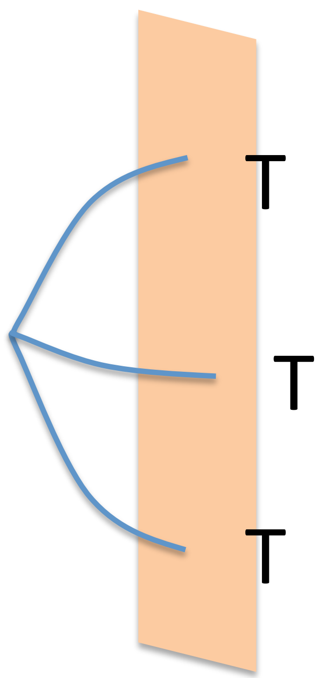
Different ``pieces'' suffer different time delays. (They cross the shock at different values of b)

Leads to a term proportional to derivatives of the Einstein time delay

$$\frac{1}{2}[\delta(b-\epsilon)+\delta(b+\epsilon)] \propto \delta(b)+\frac{\epsilon^2}{2}\partial_b^2\delta(b)$$

In this case, this also implies that the graviton can get excited → higher spins…

AdS duals of large N gauge theories



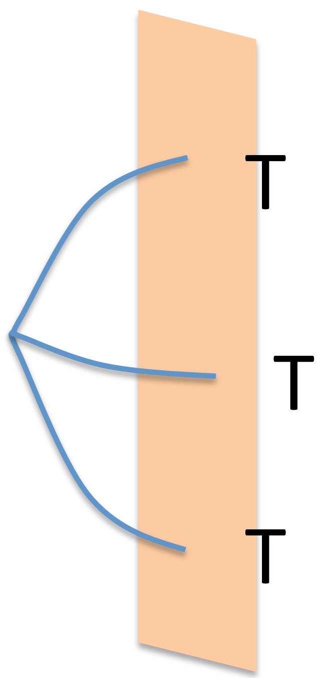
Three different structures for the stress tensor three point function. (SUSY → 2 of them).

In CFT_4 they also determine a, c trace anomaly coefficients.

$$T^\mu_\mu = cW^2 + aE$$

In Einstein gravity: a=c .

$$\frac{a-c}{c} = \frac{\alpha_2}{R^2_{AdS}}$$



The condition $$m_{S>2}^2 \lesssim \frac{1}{\alpha_{2,4}}$$

with $$\Delta_{S>2} = m_{S>2} R_{AdS}$$

Implies

$$\frac{|c-a|}{c} \lesssim \frac{1}{\Delta_{S>2,\ \mathrm{min}}}$$

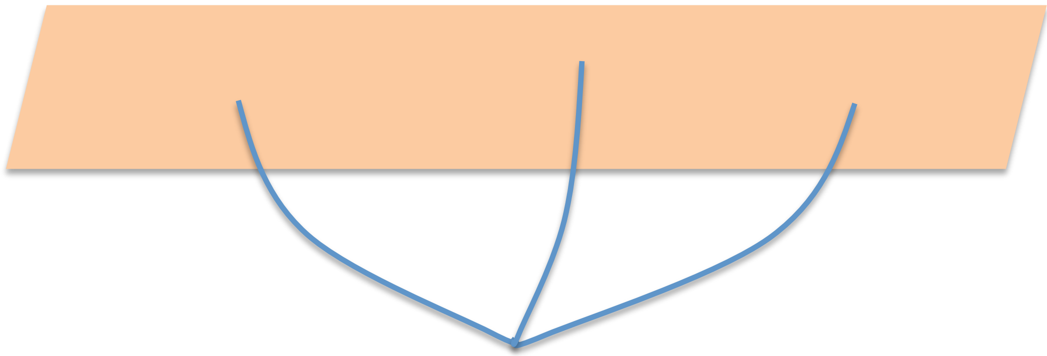
Can this be derived by the conformal bootstrap ?

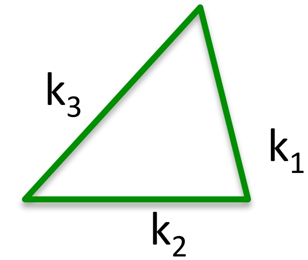
10^{14} Gev graviton collider

- H during inflation *could* be this high. (If BICEP signal is not only dust)
- Gravitons get produced by inflation with energies E ≈ H .
- They are weakly coupled. The two point function is a direct measurement of the coupling.

$$\langle hh \rangle \sim \frac{H^2}{M_{pl}^2} \sim g_{eff}^2 \lesssim 10^{-9}$$

- They can ``collide''. There are signatures of the graviton three point interaction:
- Non –gaussianities in the gravity wave spectrum of primordial fluctuations.





$$\frac{\langle hhh \rangle}{\langle hh \rangle^{3/2}} \sim \frac{H}{M_{pl}} \left[F_E(\text{shape}) + \alpha_4^2 H^4 F_2(\text{shape}) + o(\epsilon) \right]$$

JM, Pimentel

Overall small coupling

This is allowed by the approximate scale and conformal invariance of inflation

Detecting the new structure in gravity wave non-gaussianity + this discussion → something like string theory during inflation.

(The converse is not true since we can have string theories with no correction to the graviton 3pt function at the Hubble scale. In fact, most stringy inflationary models have small corrections).

In this scenario: $\alpha' \sim H^{-2} = R_H^2$

Conclusions

- Deviations of the graviton three point function from the Einstein value are impossible in causal field theories with particles of spin S ≤2
- A deviation in the graviton three point function signals the existence new higher spin particles. (If the theory is weakly coupled).
- If such deviations were seen in the cosmological gravity wave three point function → indication of a structure like string theory during inflation.