Planet Formation mini-course

Assignment 2

Due: Wednesday, June 13

1. A raindrop falls in a quiescent fog. Assume that the drop is spherical, and that it accretes all of the much smaller water droplets in its path as it falls. Prove that the drop accelerates uniformly, at a rate αg, where g is the acceleration due to gravity, and find α. The physics is essentially identical to the growth of rocks from small dust grains in the protoplanetary disk.
2. These are related short questions:
   (a) A massive planet on a circular orbit excites the eccentricities of planetesimals orbiting just inside and outside it. (The surface density and other properties of the disk can be assumed to be such that the planet migrates neither in nor out, so its orbit can be taken to be fixed.) The planetesimals subsequently collide, losing energy in the inelastic collisions and eventually returning to conjunction with the planet, to be excited again. Where does the energy come from?
   (b) Suppose that the International Space Station contains a spherical room with smooth, perfectly elastic walls, and that the room is uniformly filled with a large number of small, inelastic balls ("apples") traveling in random directions. The balls gradually lose energy as they collide with one another. After the balls have lost their kinetic energy and come to rest, will they be (i) uniformly spread through the room (there is no gravity, so what other natural state is there?); (ii) collected into a compact cluster at the center of the room (they lose relative velocity every time they collide, so the natural final state is for them all to be together); (iii) half clustered on the portion of the wall closest to the Earth, and half on the opposite wall? Choose one (or a different state), and justify your choice. It may be comforting to know that this problem has led even Nobel-Prize winners astray; see Science 173, 522.
3. The IAU definition of a "planet" says that it must be large enough for self-gravity to overcome rigid-body forces, so that it assumes a spherical shape.
   (a) The typical strength of rock is about 100 MPa (Megapascals). Use this result to estimate the minimum size of a planet.
   (b) Look at images of the satellites of the giant planets and estimate at what radius they become approximately spherical. Compare the answers to (a) and (b).
4. A planetesimal traveling on a circular orbit of radius R in the protoplanetary gas disk is subject to a drag force per unit mass $\mathbf{F}/m = -X(\mathbf{v}-\mathbf{v}_g)$ where $\mathbf{v}$ is its velocity and $\mathbf{v_g}$ is the velocity of the gas, which is assumed to be in circular motion at an angular speed $\Omega_g = \Omega_K - \Delta\Omega$, where Ω_K is the Keplerian angular speed. Find an expression for the rate of inward drift of the planetesimal, in terms of R, Ω_K, X, and $\Delta\Omega$. Hint: as derived in class, the limiting cases are

$$\dot{R} = -2R\Delta\Omega \times \begin{cases} \Omega/X & \text{large drag} \\ X/\Omega & \text{small drag.} \end{cases}$$