Planet Formation mini-course

Assignment 1

Due: Friday, June 8

1. If the encounter hypothesis for planet formation is correct, roughly how many planetary systems should there be in the Galaxy? Assume that an encounter between stars with periastron distance $< 2R_\odot$ is needed to make planets, and don't forget gravitational focusing. You may approximate the Galaxy as a slab of $N = 10^{11}$ solar-type stars with number density $n = 0.1\,\mathrm{pc}^{-3}$, mass $1\,\mathrm{M}_\odot$, and rms velocity $50\,\mathrm{km\,s}^{-1}$. Your answer need only be correct to within an order of magnitude.
2. The minimum solar nebula is usually assumed to have the surface-density distribution

$$\Sigma(R) = \Sigma_0 \left(\frac{1\ \mathrm{AU}}{R} \right)^k .$$

   Derive estimates for Σ_0 and k from the properties of the planets.
3. Assuming that the gaseous protoplanetary disk is isothermal, show that its density can be written in the form

$$\rho(R, z) = \rho_0(R) \exp\left(-\frac{z^2}{2h^2} \right),$$

   where z is the height above the midplane, and derive the scale height h in terms of the solar mass $M_\odot$ and the temperature T.
4. What is the approximate mean free path of a molecule in the minimum solar nebula at 1 AU? What is the geometric optical depth of the disk, assuming that 0.5% of the mass is in dust grains of density $\rho_p = 3\ \mathrm{g\ cm}^{-3}$ and radius $r = 0.2\mu$? (The geometric optical depth is the optical depth that the disk would have if the cross-section were πr^2.)
5. If we can approximate a planet as a black-body heated by radiation from its parent star, then the planetary surface temperature should be $T \propto r^{-b}$ where r is the distance of the planet from the star. What is the exponent b?
6. How do we know that Jupiter has a rock-ice core?